

# Explaining ECB and Fed interest rate correlation: Economic interdependence and optimal monetary policy

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## Abstract

This paper studies whether the observed high correlation between monetary policy in the U.S. and the Euro area can be explained by economic fundamentals, i.e. by macroeconomic interdependence between the two regions. We show that an optimal monetary policy reaction function for the ECB that accounts explicitly for economic interrelationships between the two economies reproduces substantial parts of the observed patterns of interest rate correlation and represents a good approximation to the actually observed monetary policy of the ECB. It implies strong reactions to shocks to US variables, particularly to shocks to the Federal Funds Rate.

*Keywords:* optimal monetary policy, monetary policy reaction function, vector autoregressions

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# 1 Introduction

This paper studies whether the observed high correlation between monetary policy in the U.S. and the Euro area can be explained by economic fundamentals, i.e. by macroeconomic interdependence between the two regions. Using a vector autoregression (VAR) framework we derive an optimal monetary policy reaction for the European Central Bank (ECB) that accounts explicitly for the effects of U.S. macroeconomic variables on the Euro area economy. We show that this optimal reaction function implies that the ECB responds to shocks both within the Euro area and in U.S. The optimal reaction to shocks to the U.S. economy often turns out to be even stronger than actually estimated and this applies to the reaction to the Federal Funds Rate, in particular. This optimal reaction function for the ECB does not only fit the actually observed path of monetary policy in the Euro area remarkably well but succeeds also in replicating the observed correlation patterns between short-run interest rates in the Euro area and in the U.S. for leads and lags up to one year.

Figure 1 displays the U.S. Federal Funds Rate - the overnight interest rate closely controlled by the Federal Reserve - and its counterpart in the Euro area the EONIA. Both time series are monthly averages of daily data. The figure suggests that monetary policy in the Euro area follows that of the U.S. with a lag. The cross correlation coefficient peaks at a lag of seven months for the Federal Funds Rate. This relationship between policy interest rates in the Euro area and the U.S. has been studied empirically by Belke and Gros (2003, 2005, 2006). They investigate the dynamic interrelationship between Euro area and U.S. short-term interest rates using Granger causality tests with daily and weekly observations. For the time period before September 2001 they find a symmetric relationship with either bi-directional Granger causality or no Granger causality at all depending on the chosen lag length. Using observations from after September 2001 only they present evidence for an asymmetric relationship with Granger causality running from the Federal Funds Rate to the Euro area interest rate. The analysis of the short-run interest-rate interactions between the Euro and the U.S. is extended in Belke and Cui (2010) to simultaneously account for a possible long-run relationship. Using a vector-error-correction model (VECM) and monthly data they estimate a cointegrating relationship between the EONIA and the Federal Funds Rate which indicates

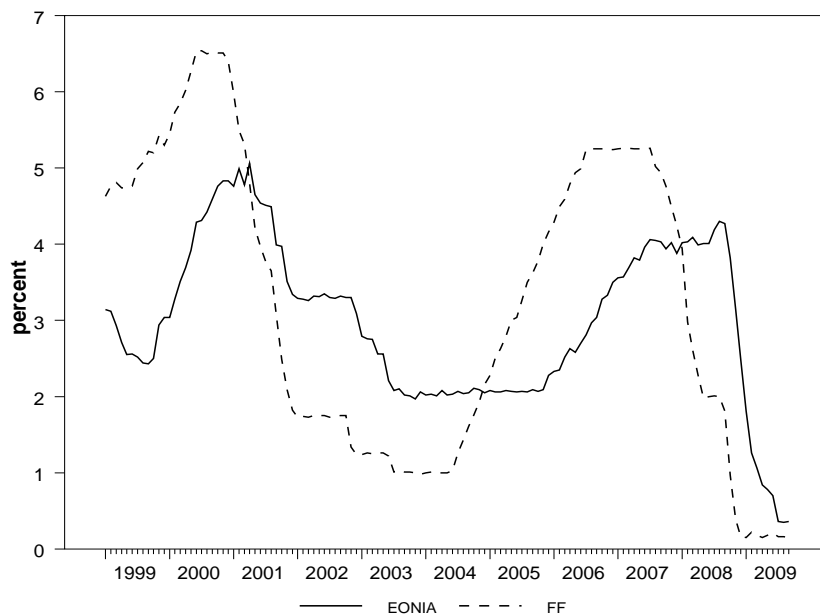


Figure 1: EONIA and Federal Funds Rate

the existence of a long-run equilibrium relation between both interest rates. For the short-run interest-rate dynamics their estimates indicate for both interest rates similar speeds of adjustment to deviations from this long-run equilibrium.

One possible explanation for the lead-lag pattern in Figure 1 is that both central banks change their interest rates according to movements in the business cycle and that the Euro area business cycle lags that of the U.S. (e.g. Begg et al., 2002). This effect can be accounted for by estimating interest rate reaction functions for the central banks that include macroeconomic variables and control for the stage of the business cycle. Breuss (2002) and Ullrich (2005) estimate Taylor rules for the ECB augmented by the lagged Federal Funds Rate and show that the U.S. interest rate enters the Euro area monetary policy reaction function in a statistically significant way. Belke and Cui (2010) augment their VECM by Euro area and U.S. inflation rates and output gaps. Their results still indicate a cointegrating relationship between the EONIA and the Federal Funds Rate. However, the U.S. interest rate is estimated to be weakly exogenous to the VECM indicating an asymmetry in the relationship between the ECB and the Fed by which only the ECB and not the Fed responds to deviations from the cointegrating relation. Scotti (2006) analyses the interdependence of the timing of interest-rate changes by the ECB and the Fed and controls for the effects of output and inflation

on the interest-rate decisions. She estimates a bivariate conditional hazard model on weekly data and finds evidence for contemporaneous interdependence in the timing of interest rates changes by the two central banks.

The patterns found empirically in the time-series of policy interest rates are the result of the monetary policy reaction functions of the Fed and the ECB. These reaction functions link the setting of the short-term interest rate by the central banks to other macroeconomic variables. Correlation between the interest rates set by both central banks can be caused by one or both central banks reacting directly to the interest-rate chosen by the other one or by both central banks reacting to the same or similar macroeconomic variables, perhaps with different time lags and intensities. Theoretical analyses of monetary policy reaction functions (e.g. Clarida et al. (1999), Galí, 2008; Svensson (1997), Woodford, 2003a) show that an optimum reaction function makes the central bank respond to all variables and shocks that help in forecasting the central bank's goal variables. This implies, that it will be optimal for the ECB to react to U.S. macroeconomic variables if these have predictive power for Euro area inflation and economic activity, either because these variables directly or indirectly affect the Euro area economy or because they convey information about shocks that are relevant to the Euro area.

In this paper, we study how far this explanation takes us in reproducing the observed correlations of short-term interest rates in the U.S. and the Euro area. From an empirically estimated VAR model of the U.S. and Euro area we construct an optimal monetary policy reaction function for the ECB and investigate how important the optimal responses to U.S. variables and shocks are in determining the time path of the EONIA. By means of simulations we show that the optimal reaction function can reproduce the observed interest-correlation pattern to a large extent and that its reactions to the various shocks in the model are close to those for the reaction function estimated on the observed data. Our results suggest that the observed interest rate correlation between the U.S. and the Euro area results from the optimal reaction of the ECB to U.S. variables and U.S. shocks.

The main results of the paper are as follows: (1) It is shown that the observed monetary policy function of the ECB can be approximated by an optimal reaction function derived from a small VAR model with very few restrictions imposed. (2) The optimal

monetary policy implies strong reactions to shocks to U.S. variables, particularly to the Federal Funds Rate and induces a high correlation between the interest rates set by the ECB and the Fed. (3) These results are robust with respect to changes in the identification assumptions of the VAR and to the introduction of uncertainty about the monetary transmission mechanism.

We construct the optimal monetary policy reaction function for the ECB from an estimated structural VAR model of the Euro area and the U.S. economies using a methodology proposed in Sack (2000). The VAR framework is a natural way to model the implications of macroeconomic interdependence on the monetary policy of both central banks. A VAR that includes both policy interest rates together with macroeconomic variables that are important determinants of monetary policy such as unemployment, output and inflation is a flexible and relatively unrestricted framework that can account for both the systematic responses of the central banks to the macroeconomy and for the macroeconomic interdependence of the U.S. and the Euro area. It allows to estimate the monetary policy reaction functions of the ECB and the Fed and to study the central banks' reaction functions by means of impulse response analyses and variance decompositions.

Furthermore, VAR models have already been successfully applied to studies of monetary policy interdependence, although mostly for small open economies.

The interrelation between the U.S. Federal Funds Rate and interest rates in other countries has been studied with structural vector autoregressions by Grilli and Roubini (1995). They show that unexpected innovations in the Federal Funds Rate lead to significant changes in the short-term interest rates of G-7 countries. The transmission channels of U.S. monetary policy shocks to the other G-7 countries are studied in Kim (2001) using a sample period from 1974 to 1996. His study uses VAR model for the U.S. augmented with additional variables for the other countries one at a time. Although output and production in the other countries increase significantly after an expansionary monetary policy shock in the U.S. he finds these shocks to have little effect on the other countries' trade balances and short-term interest rates. He concludes that expansionary monetary policy impulses are transmitted from the U.S. to the other economies via their effects on world interest rates. Neri and Nobili (2010) use

a structural VAR to study the effects of U.S. monetary policy shocks on the Euro area. They estimate a significantly positive response of the Euro area short-term nominal interest rate to an exogenous increase in the Federal Funds Rate.

Other authors have not focussed explicitly on the effects of U.S. monetary policy shocks on other countries' interest rates. Nevertheless, they have incorporated in their VAR models the assumption of a dependence of the country of interest's monetary policy on the U.S. For example, Cushman and Zha (1997) study the effects of monetary policy shocks on the Canadian economy and account explicitly for the dependence of Canadian monetary policy on the U.S. Federal Funds Rate. Kim and Roubini (2000) investigate the effects of monetary policy shocks on exchange rates for small open economies and use the Federal Funds Rate to control for the effects of foreign monetary policy. Brischetto and Voss (1999) adapt the structural VAR model of Kim and Roubini to estimate the effects of monetary policy shocks on the Australian economy and, again, include the Federal Funds Rate as an indicator of foreign monetary policy. This VAR literature does, however, only shed limited light on the question of interest-rate correlation. Its focus is mostly on the unsystematic part of monetary policy, i.e. the monetary policy shocks, whereas interest rate correlation between the economies is driven by the systematic reactions of monetary policy to the whole range of shocks. Hence, in this paper we will focus on how monetary policy responds to a variety of shocks as opposed to how the other variables respond to monetary policy shocks. Furthermore, our approach enables us to consider optimal interest rate reactions and to use these to evaluate the actually observed behavior of the central banks.

The next section derives an optimal monetary policy reaction function for the ECB from an estimated structural VAR (Section 2.1) and presents results for the importance of U.S. variables in the optimal reaction function and for its ability in reproducing the observed time series of the EONIA and its correlation with the Federal Funds Rate (Section 2.2). Section 3 investigates the robustness of these results by considering alternative identification schemes for the structural VAR. Section 4 presents results for a model which introduces uncertainty about the structural relationships in the economy. The results from the accordingly adjusted optimal monetary policy reaction function resemble closely those from Section 2. The importance of direct reactions of the EONIA to the Federal Funds Rate is considered in Section 5. Section 6 summarizes

the results and concludes.

## 2 Optimal policy with additive uncertainty

### 2.1 The optimal monetary policy reaction function of the ECB

The starting point of the analysis is an estimated structural vector autoregression (VAR)

$$Z_t = \mathbf{k}_Z + \sum_{i=0}^q \mathbf{A}_i Z_{t-i} + \sum_{i=0}^q \mathbf{b}_i R_{t-i} + \ell_Z t + \nu_t^Z \quad (1)$$

$$R_t = k_R + \sum_{i=0}^q \mathbf{c}'_i Z_{t-i} + \sum_{i=1}^q d_i R_{t-i} + \ell_R t + \nu_t^R. \quad (2)$$

$Z_t$  is an  $(n \times 1)$ -vector of non-policy variables,  $R_t$  is the ECB policy interest rate,  $q$  is the number of lags in the VAR,  $\nu_t^Z$  is a  $(n \times 1)$ -vector of uncorrelated structural shocks that is also uncorrelated with the structural policy disturbance  $\nu_t^R$ .  $\mathbf{k}_Z$  is a vector of constants,  $\ell_Z$  a vector of coefficients on time trends and  $k_R$  and  $\ell_R$  the constant and time trend in the estimated monetary policy reaction function.  $\mathbf{A}_i$  are  $(n \times n)$  coefficient matrices and  $\mathbf{b}_i$  and  $\mathbf{c}_i$  are  $(n \times 1)$  coefficient vectors.  $\mathbf{A}_0$  describes the contemporaneous interactions of the non-policy variables while  $\mathbf{b}_0$  gives the immediate (if any) reactions of the variables in  $Z_t$  to the monetary policy instrument  $R_t$ . Non-zero elements in  $\mathbf{c}_0$  indicate to which of the variables in  $Z_t$  monetary policy responds to within the same period.

In this paper's application the variables in  $Z_t$  are the deviations of the U.S. and Euro area unemployment rates from their natural levels ( $UNUS$ ,  $UNEMU$ ), the growth rates of industrial production in the U.S. and in the Euro Area ( $IPUS$ ,  $IPEMU$ ), rates of consumer price inflation in the U.S. and in the Euro Area ( $INFLUS$ ,  $INFLEMU$ ), a smoothed rate of commodity price inflation  $PCOM$ , the Federal Funds Rate ( $FF$ ) and the nominal U.S.-Dollar/Euro exchange rate ( $EXCHR$ ). The monetary policy indicator  $R_t$  is approximated by the EONIA rate, the average overnight interest rate in the Euro area interbank market. As shown in (1) the VAR includes constants and time

trends. The specification in inflation rates and growth rates is chosen in accordance with the variables included in the central bank's loss function below.<sup>1</sup>

Equation (2) represents a backward-looking monetary policy reaction function (MPRF) of the ECB. Its estimate can be obtained from the estimated structural VAR that results from stacking the non-policy variables  $Z_t$  and the monetary policy indicator  $R_t$ .

Equations (1) of the structural VAR can be rewritten in state-space form as a transition equation for the state vector  $X_t$

$$X_{t+1} = \mathbf{F}X_t + \mathbf{H}R_t + \mathbf{J} + \mu_{t+1}, \quad (3)$$

where the coefficients in  $\mathbf{F}$ ,  $\mathbf{H}$ ,  $\mathbf{J}$  can be derived from the coefficients in (1). The state vector  $X_t$  contains current and lagged values of the variables in  $Z_t$  and lags of the EONIA. The immediate effects of monetary policy on the state variables are captured by the vector  $\mathbf{H}$ . Details of the construction of (3) are given in Appendix B.

The state-space representation of the structural model of the economy (3) can be used to derive an optimal monetary policy reaction function (e.g. Mandler, 2009; Sack, 2000): The ECB is assumed to maximize a quadratic objective function

$$-\frac{1}{2}E_t \left\{ \sum_{i=1}^{\infty} \beta^i [(\pi_{t+i} - \pi^*)^2 + \lambda_u (u_{t+i} - u^*)^2 + \lambda_R (R_{t+i-1} - R_{t+i-2})^2] \right\}. \quad (4)$$

This is a standard objective function used in monetary policy analysis that penalizes the central bank for deviations of unemployment and inflation from their target values  $u^*$  and  $\pi^*$  (e.g. Walsh, 2010). The presence of the squared change in the interest rate  $R_{t+i-1} - R_{t+i-2}$  represents an aversion to interest rates changes and leads to interest-rate smoothing by the central bank (e.g. Woodford, 2003b).<sup>2</sup>  $\lambda_u$  and  $\lambda_R$  are the weights attached to the employment and interest-rate objectives relative to the inflation objective and  $\beta$  is a discount factor. Since the unemployment variables in  $Z_t$  are already defined as deviations of unemployment rates from their natural levels,  $u^*$  is equal to zero, i.e. the ECB is assumed to target the natural rate of unemployment.

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<sup>1</sup>Detailed information on the data is given in Appendix A.

<sup>2</sup>Empirically, interest rate smoothing manifests in a statistically significant and quantitatively important autoregressive element in estimated interest rate rules.



The part of the objective function in square brackets can be written in a notation compatible with (3)

$$(X_{t+i} - X^*)' \mathbf{G} (X_{t+i} - X^*). \quad (5)$$

$\mathbf{G}$  has as non-zero entries only the elements corresponding to the weights attached to the relevant variables in  $X_{t+i}$  in the objective function.  $X^*$  is the vector of target values for the state variables. In this model the only non-zero element in  $X^*$  corresponds to the inflation target  $\pi^*$ <sup>3</sup>

The optimal policy reaction function determines the policy instrument  $R_t$  as a function of the state variables  $X_t$  and maximizes (4) subject to (3). This linear-quadratic dynamic programming problem can be solved using standard methods (e.g. Ljungqvist and Sargent, 2004, Ch. 4; Sack, 2000). The optimal monetary policy reaction function solves the Bellman equation

$$V(X_t) = \max_{R_t} \left\{ - (X_t - X^*)' \mathbf{G} (X_t - X^*) + \beta E_t [V(X_{t+1})] \right\}, \quad (6)$$

subject to (3). For a linear quadratic dynamic programming problem like this the value function has the form

$$V(X) = X_t' \mathbf{\Lambda} X_t + 2X_t' \omega + \rho, \quad (7)$$

with constants  $\mathbf{\Lambda}$ ,  $\omega$  and  $\rho$ . The solution for the optimal policy reaction function is<sup>4</sup>

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<sup>3</sup>A slight departure from the standard specification is the use of  $R_{t+i-1} - R_{t+i-2}$  instead of  $R_{t+i} - R_{t+i-1}$  in (3). Through this modification, the objective function can be written in terms of the state variables as in (5). The difference of (4) to the standard specification caused by this modification is twofold: First, in the infinite sum in equation (4) the term  $R_{t+i} - R_{t+i-1}$  is multiplied by  $\beta^{i+1}$  instead of  $\beta^i$  as in the standard formulation and, second, the objective function (4) includes the term  $-(1/2)\beta(R_{t-1} - R_{t-2})$  which would not be present when using the standard specification.  $\beta$  is close to one and  $R_{t-1} - R_{t-2}$  does not depend on the setting of the interest rate in period  $t$ . Hence, these two differences will have a negligible effect on the optimal monetary policy reaction function.

<sup>4</sup>See Appendix C for details.

$$R_t^* = -(\mathbf{H}'\mathbf{\Lambda}\mathbf{H})^{-1}(\mathbf{H}'\mathbf{\Lambda}\mathbf{F}X_t + \mathbf{H}'\mathbf{\Lambda}\mathbf{J} + \mathbf{H}'\omega), \quad (8)$$

where the symmetric matrix  $\mathbf{\Lambda}$  is defined implicitly by the Riccati equation

$$\mathbf{\Lambda} = -\mathbf{G} + \beta\mathbf{F}'\mathbf{\Lambda}\mathbf{F} - \beta\mathbf{F}'\mathbf{\Lambda}\mathbf{H}(\mathbf{H}'\mathbf{\Lambda}\mathbf{H})^{-1}\mathbf{H}'\mathbf{\Lambda}\mathbf{F}. \quad (9)$$

The vector  $\omega$  is given by

$$\begin{aligned} \omega &= \left( \mathbf{I} - \beta\mathbf{F}' \left( \mathbf{I} - \mathbf{\Lambda}\mathbf{H}(\mathbf{H}'\mathbf{\Lambda}\mathbf{H})^{-1}\mathbf{H} \right) \right)^{-1} \\ &\quad \times \left( \mathbf{G}X^* + \beta\mathbf{F}'\mathbf{\Lambda} \left( \mathbf{I} - \mathbf{H}(\mathbf{H}'\mathbf{\Lambda}\mathbf{H})^{-1}\mathbf{H}'\mathbf{\Lambda} \right) \mathbf{J} \right). \end{aligned} \quad (10)$$

Under the optimal monetary policy reaction function the dynamics of the economy are given by (3) and (8). (8) shows how the value of the monetary policy instrument  $R_t$  is determined by the current state of the economy  $X_t$ .  $X_t$  and  $R_t$  in turn determine the state vector in the next period  $X_{t+1}$  according to (3). The only sources of uncertainty are the shocks  $\mu_{t+1}$  to the transition equation (3). The optimal monetary policy reaction function (8) is much less restrictive than a Taylor-type rule and allows the policy rate to react to current and lagged values of all of the non-policy variables and to lags of the policy interest rate.

In deriving the optimal ECB reaction function (8) we treat (3) as a structural representation of the economy, i.e. we assume the coefficients in (3) that are derived from the estimated structural VAR to be invariant with respect to changes in the ECB reaction function. This assumption can be questioned in the light of the Lucas (1976) critique. Since the parameters in lagged representations of an economic model as in (1) and (2) depend on agents' expectations of monetary policy they will change when the central bank is assumed to follow a monetary policy different from that in the estimation period.

However, the empirical relevance of the critique depends on the size and on the economic significance of the changes in the reduced form parameters that are caused by alternative policies. For example, even though much evidence has been presented for

pronounced changes in the Fed's monetary policy reaction function, empirical VAR and backward-looking non-VAR models appear to be stable, see, for example Rudebusch and Svensson (1999), Bernanke and Mihov (1998), and Estrella and Fuhrer (2003).<sup>5</sup> Rudebusch (2005) conducts a thorough investigation into the empirical relevance of the Lucas critique. He simulates structural economic models that contain expectational variables and finds only very modest changes in the reduced form coefficients. In most cases, he is unable to reject the null hypothesis of stability in the reduced form parameters after having changed the policy rule. That structural invariance in face of plausible policy changes often cannot be rejected is argued as well by Estrella and Fuhrer (2003). In the following section we will show that the optimal monetary policy reaction function is actually a good approximation to the actual one and that the behavior of the economy model under the optimal ECB reaction function is close to that of the estimated VAR. Hence, that the possible effect of the Lucas critique will be very limited.

The first step in the construction of the optimal monetary policy reaction function for the ECB is the estimation of the structural VAR (1) and (2) and its transformation into the state space model (3). Structural identification of the VAR is achieved by imposing zero restrictions in the matrix  $\mathbf{A}_0$  of contemporaneous interactions of the non-policy variables in (1), in the vector  $\mathbf{b}_0$  in (1) indicating the immediate reactions of the non-policy variables to the monetary policy instrument, and in the vector  $\mathbf{c}_0$  in (2) which represents the within-period response of monetary policy to the non-policy variables. Of special importance are the restrictions imposed on  $\mathbf{c}_0$  since these restrict to which non-policy variables monetary policy can respond immediately, i.e. of which non-policy variables the current observations are in the central bank's information set. This has implications for the construction of the state vector  $X_t$  in (3) since comparisons of the optimal and the estimated monetary policy reaction functions must be based on identical information sets: The current observations of all of the variables with non-zero elements in  $\mathbf{c}_0$  must be included in  $X_t$ .

In our model the identification of the structural VAR is based on the following assump-

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<sup>5</sup>In the context of estimated Taylor rules see, for example, Boivin (2006), Clarida, Galí, and Gertler (2000), Taylor (1999), and Judd and Rudebusch (1998).

tions:<sup>6</sup> For both the EMU and the U.S. variables I assume recursive orderings in which the unemployment rate is ordered first, followed by the the growth rate of industrial production and by the inflation rate. There is no contemporaneous interaction of these variables across country blocks which is a reasonable assumption for monthly data. As in Kim and Roubini (2000) I include an indicator of commodity price inflation to capture global inflationary shocks. Commodity price inflation reacts to all other variables with a lag. The nominal exchange rate is affected within the same period by all other variables including the interest rates in the Euro Area and in the U.S.

Of particular importance are the identification assumptions concerning monetary policy in the U.S. and in the Euro Area. In the first version of the model I assume that U.S. monetary policy reacts within the same period to unemployment, industrial production and inflation in both regions and to commodity price inflation but not to the ECB's monetary policy and to the exchange rate. The ECB is assumed to react within the same period to all variables except for the exchange rate, i.e. the ECB is allowed to respond immediately to U.S. monetary policy. Since this implies an asymmetry in the treatment of both central banks we will also present results in Section 3 for slightly different identification schemes and show that our results are robust with respect to these changes. In particular, we also consider a model in which the ECB does not react to U.S. monetary policy within the current period while the Federal Reserve immediately reacts to changes in Euro area interest rates.<sup>7</sup>

The structural VAR is estimated on monthly data from 1995M7 to 2007M12.<sup>8</sup> Since a VAR with ten variables is fitted to a relatively short sample period the VAR is estimated with only six lags.

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<sup>6</sup>These variables are in part derived from the standard standard recursive identification scheme common in the literature (e.g. Christiano et al., 1999). This structure for variables within one economy is also used in Kim (2001).

<sup>7</sup>Kim (2001) considers various structural and recursive identification schemes in his VAR study on the international transmission of monetary policy shocks and shows his results to be very robust with respect to these changes.

<sup>8</sup>Including observations up to the collapse of Lehman Brothers in September 2008 resulted in a large increase in the imprecision of the estimates indicating the possibility of a structural break.

## 2.2 Results

The optimal monetary policy reaction function (8) contains four free parameters: the discount factor  $\beta$ , the relative weights of unemployment and interest-volatility in the central bank's objective function,  $\lambda_u$  and  $\lambda_R$ , and the inflation target  $\pi^*$ . As suggested by Sack (2000) we impose  $\beta = 0.996$  and estimate  $\lambda_u$ ,  $\lambda_R$  and  $\pi^*$  by minimizing the sum of squared deviations of the interest rate implied by (8) from the actually observed interest rate.<sup>9</sup> For any combination of  $\lambda_u$ ,  $\lambda_R$  and  $\pi^*$  on a grid we compute the optimal monetary policy reaction function (8) and use it to obtain a time series of optimal interest rates based on the historically observed values for the state variables in  $X_t$ . The particular combination of  $\lambda_u$ ,  $\lambda_R$  and  $\pi^*$  selected is the one which minimizes the sum of squared deviations of the optimal from the observed EONIA rate for 1999M1 to 2007M12.<sup>10</sup> The results of this search procedure showed a tendency for  $\lambda_R$  to become excessively large while the estimates for the other two parameters were mostly independent of  $\lambda_R$ . Hence, we fixed  $\lambda_R = 2.5$  at a value which provided a reasonably good approximation to the observed time series of the EONIA and searched over  $\lambda_u$  and  $\lambda_R$  only. Higher values for  $\lambda_R$  led to only very small improvements in the fit of the model. The resulting estimate for the weight on unemployment algorithm is  $\lambda_u = 0.0575$  and the estimate for the inflation target is  $\pi^* = 2.20$  percent which is only slightly above the ECB's official inflation target of two percent. The estimated weight on the unemployment objective is relatively low but in line with other results in the literature. Using quarterly data for the period 1980:3-1998:3 Favero and Rovelli (2003) find a weight of 0.00125 on the output gap for the Federal Reserve. Dennis (2001, 2004) reports statistically insignificant estimates of the weight on the output gap while Collins and Siklos (2004) estimate a weight of 0.001.

Figure 2 shows the observed time series for the EONIA together with the one constructed from the optimal monetary policy reaction function. Except for the first and last few months the optimal interest rate path tracks the observed one very closely

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<sup>9</sup>The estimation of structural parameters in an optimal monetary policy rule by fitting it to observed U.S. monetary policy is also performed in Rudebusch (2001).

<sup>10</sup>Since no explicit solution for the matrix  $\mathbf{A}$  in (9) exists this search procedure is employed. As a consequence information on the precision of the estimates is not available.

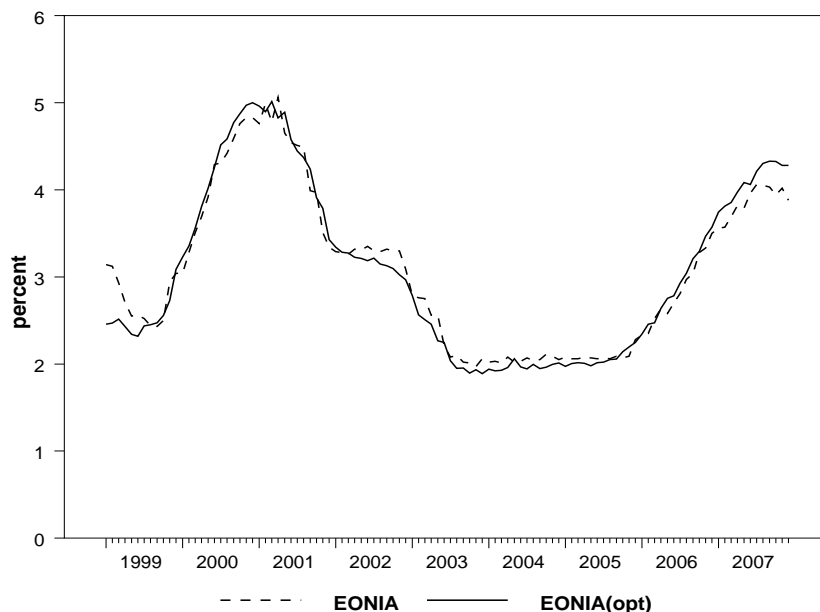


Figure 2: Actual and fitted optimal EONIA

with a sum of squared deviations of 3.50. The volatility of the optimal interest rate is slightly above the volatility of the observed EONIA (standard deviations of 0.98 and 0.90, respectively). The optimal monetary policy reaction function is able to reproduce the cross correlation structure of the EONIA and the observed Federal Funds Rate at leads and lags up to two two years as presented in Figure 3.

Figure 4 compares impulse responses of the EONIA for the estimated monetary policy reaction function to those for the optimal one. The impulse responses for the optimal monetary policy reaction functions (MPRF) (solid lines) are obtained from simulating the structural equations for the non-policy variables in (3) together with the optimal monetary policy reaction function (8). The impulse responses for the system with the optimal reaction function are simulated with the same structural shocks as used for the estimated impulse responses. The reason for this is that the identification assumption concerning the ECB's information set that is used in computing the optimal reaction function is identical to the one imposed in the estimation of the structural VAR. For the structural shocks to the EONIA itself we use the estimate from the structural VAR (1) and (2). The dashed lines are the impulse responses for the EONIA in the estimated structural VAR ((1) and (2)) and the dotted lines represent 90% probability bands around the estimated impulse responses and were constructed by Monte Carlo

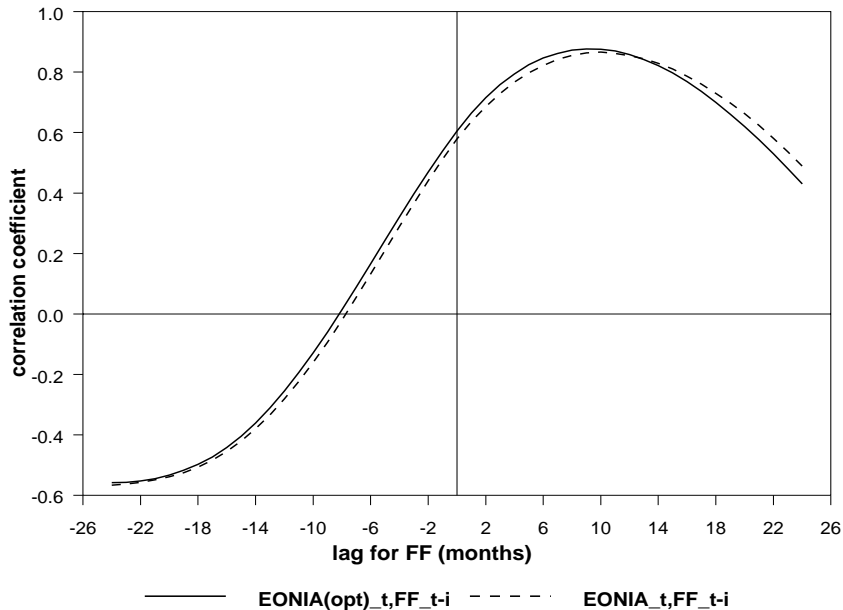


Figure 3: Correlation of actual and fitted optimal EONIA with observed Federal Funds Rate

simulation.<sup>11</sup> For many of the shocks the impulse responses of the model with the optimal reaction function imposed are close to the estimated impulse responses and follow very similar trajectories. The optimal impulse responses match the estimated ones very successfully in the first year after shocks to the EMU unemployment rate, U.S. industrial production and to the EONIA itself. The optimal reaction function leads to a quicker but shortly lived negative (positive) interest rate response to shocks to U.S. (EMU) inflation compared to the estimated reaction of the EONIA. The response of the EONIA to Federal Funds Rate shocks within the first few months is more pronounced for the optimal reaction function but less persistent. Strong differences can be observed for the response to U.S. unemployment, Euro area industrial production and to commodity price inflation. Overall, the impulse responses from the optimal monetary policy reaction function provide a reasonably good approximation to the estimated responses of the ECB. Figures 5 and 6 offer the same comparisons for the impulse responses of the EMU unemployment and inflation rates. Again, the

<sup>11</sup>The dynamical stability of the model obtained from combining (3) and (8) was checked by computing the largest absolute eigenvalue of the system as 0.996. The largest absolute eigenvalue of the estimated VAR is 0.998.

*impulse responses for EONIA: with optimal MPRF (solid), with estimated MPRF (dashed)*

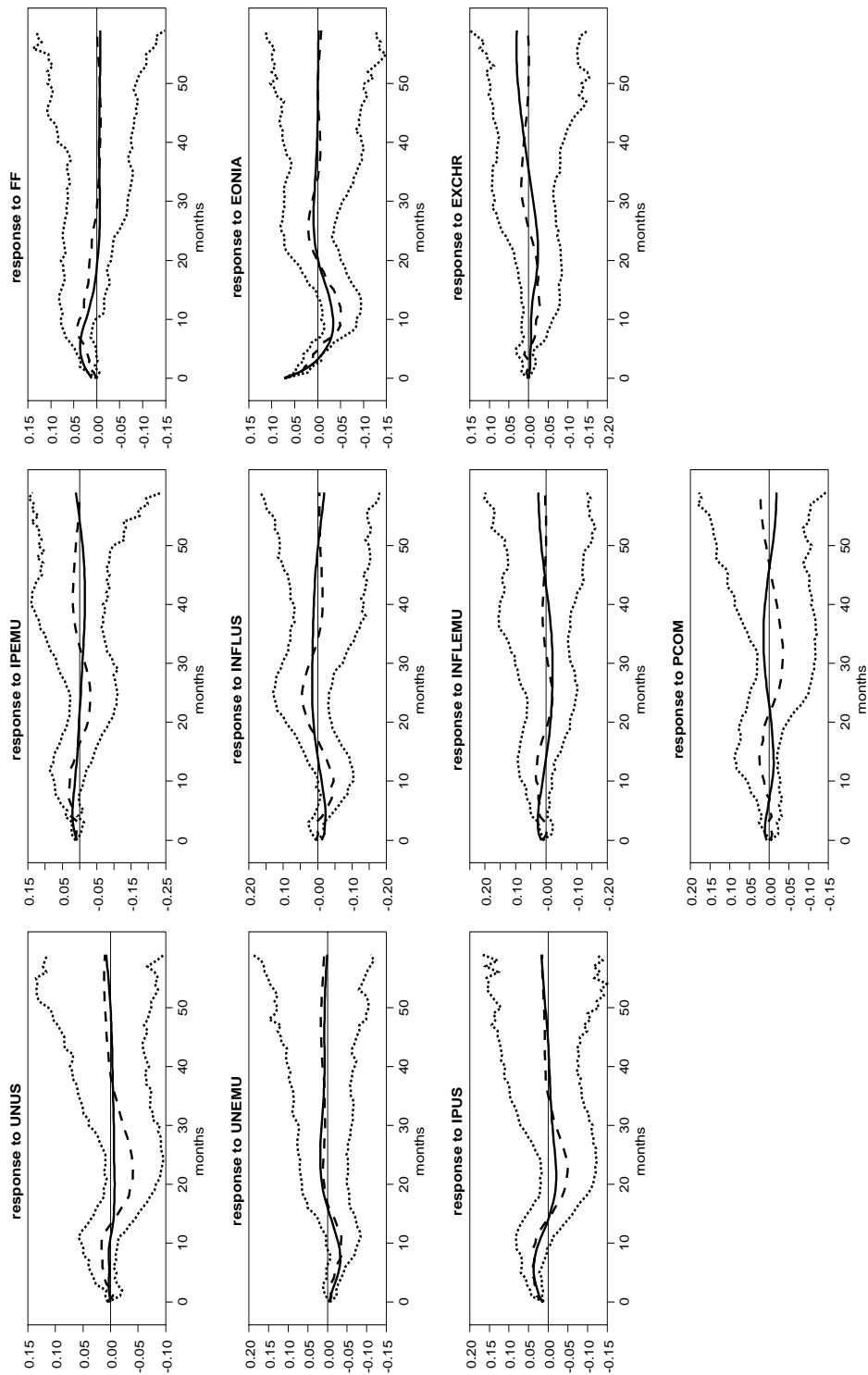


Figure 4: Impulse responses of EONIA



impulse responses from the VAR with the optimal monetary policy reaction function imposed are very close to the estimated ones.

Table 1 presents decompositions of the EONIA forecast variance for the model with, first, the estimated and then the optimal ECB reaction functions imposed. Again, because of the identical identification assumptions we use the VAR estimates of the structural shocks to construct the variance decompositions for the system including the optimal reaction function.

A few interesting facts emerge: The contribution of inflation shocks both in the U.S. and in the EMU is higher under the optimal ECB reaction function than that under the estimated one for forecast horizons up to six months. The importance of shocks to industrial production in explaining unexpected EONIA changes declines. U.S. unemployment shocks explain less variation in the EONIA under the optimal ECB reaction function than under the estimated one while the explanatory power of EMU unemployment shocks increases for forecast horizons of three months and more. In addition, the contribution of Federal Funds Rate innovations to the EONIA forecast variance increases strongly while that of the EONIA's own shocks declines in the short run. The last two rows show the aggregated variance contribution of all U.S. variables together. For forecast horizons up to three months the optimal monetary policy reaction function recommends assigning a greater importance to U.S. shocks than actually observed. After three months the aggregate contribution of U.S. shocks to the EONIA is similar for the optimal and for the estimated reaction function.

The time series for the optimal EONIA in Figure 1 was derived by computing the optimal EONIA rate for the actually observed values of the state variables in  $X_t$  at each point in time. Figure 7 shows the time series for EONIA and Federal Funds Rate that result from simulating (3) with the optimal monetary policy reaction function (8) and imposing the historical structural shock series from the estimated VAR. This assumes all variables in the model to evolve according to their dynamics in (3) and (8) and does not reset them to their observed values in each period.<sup>12</sup> In Figure 7 the

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<sup>12</sup>The starting values for the state vector  $X_t$  are the observations of the variables at 1999M1. The historical series for the structural shocks are recovered from the estimated VAR using the identification scheme described in section 2.

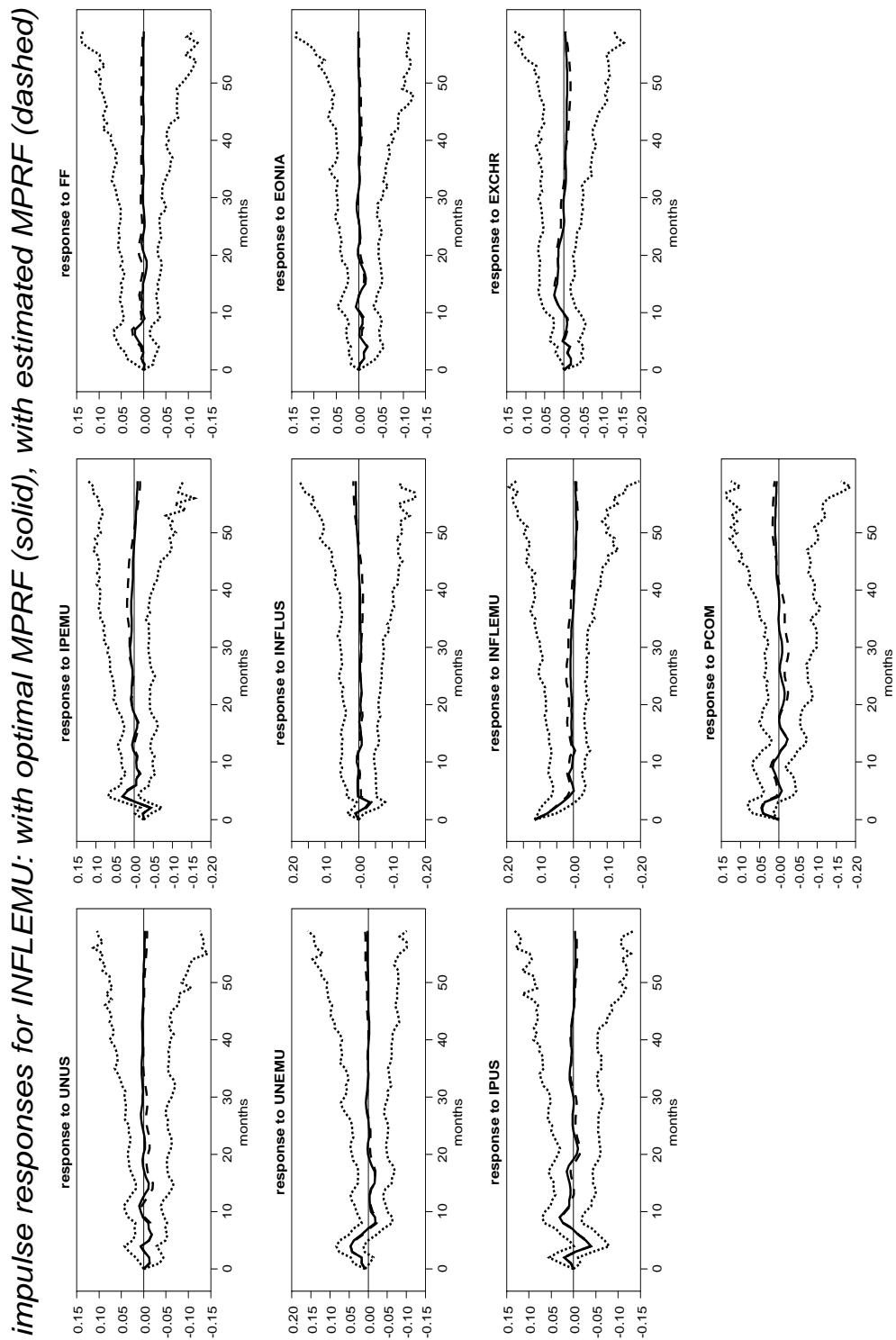


Figure 5: Impulse responses of EMU inflation rate

*impulse responses for UNEMU: with optimal MPRF (solid), with estimated MPRF (dashed)*

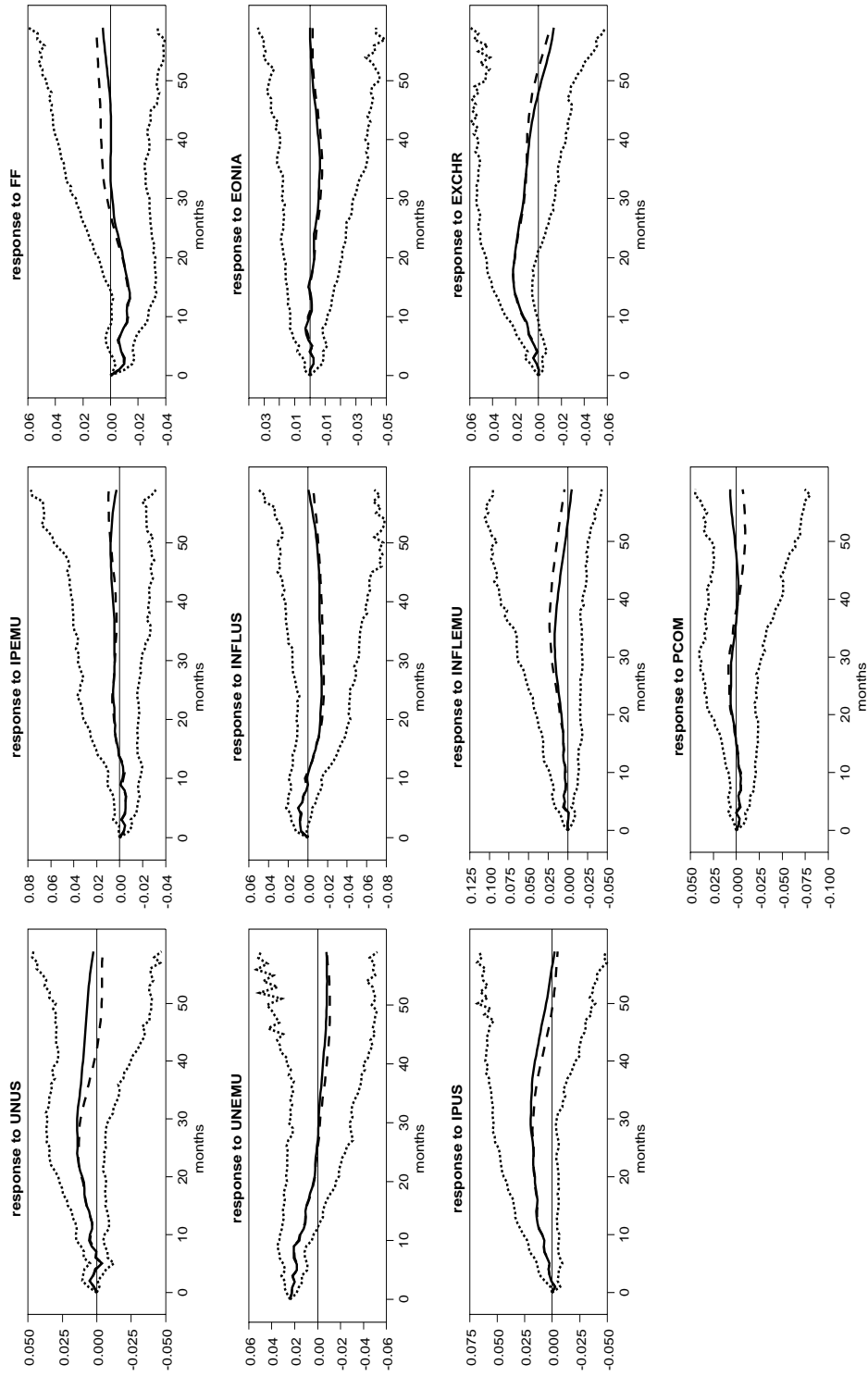


Figure 6: Impulse responses of EMU unemployment rate

Table 1: Comparison of variance decompositions under estimated and optimal MPRF

Percentage contribution to k-month ahead EONIA forecast error variance						
	k=0	k=1	k=3	k=6	k=12	k=24
UNUS	0.53 (0.00)	0.57 (0.02)	1.17 (0.11)	2.46 (0.12)	2.34 (0.14)	9.07 (0.82)
UNEMU	0.61 (0.28)	1.65 (0.99)	1.84 (3.23)	4.89 (8.47)	9.52 (13.33)	6.47 (12.90)
IPUS	2.60 (3.18)	11.89 (7.16)	24.80 (14.16)	31.21 (19.20)	17.15 (18.87)	19.74 (18.33)
IPEMU	2.37 (0.74)	2.80 (2.12)	2.57 (5.07)	6.45 (6.35)	8.02 (5.97)	7.82 (4.93)
INFLUS	0.33 (2.41)	0.29 (5.07)	0.41 (8.16)	6.51 (8.66)	14.52 (6.97)	14.40 (7.20)
INFLEMU	2.04 (4.22)	1.46 (8.21)	1.07 (12.84)	6.21 (12.59)	7.58 (9.45)	6.85 (9.76)
PCOM	0.22 (0.78)	0.78 (1.63)	0.67 (2.24)	0.50 (1.41)	1.58 (1.39)	3.07 (2.05)
FF	0.02 (1.99)	0.57 (5.06)	6.22 (12.76)	10.39 (18.19)	12.26 (17.79)	9.45 (14.27)
EONIA	91.28 (86.30)	79.61 (69.69)	61.00 (41.26)	30.21 (24.68)	23.49 (25.42)	17.68 (22.88)
EXCHR	0.00 (0.00)	0.37 (0.05)	0.25 (0.18)	1.15 (0.33)	3.54 (0.66)	5.45 (6.85)
$\sum_{US}$	3.46 (7.58)	13.32 (17.32)	32.61 (35.19)	50.58 (46.18)	46.27 (43.78)	52.66 (40.62)

NOTES: Sample period is 1995:7-2007:12.

Numbers in brackets apply to the model including the optimal MPRF, number without brackets to the estimated VAR.

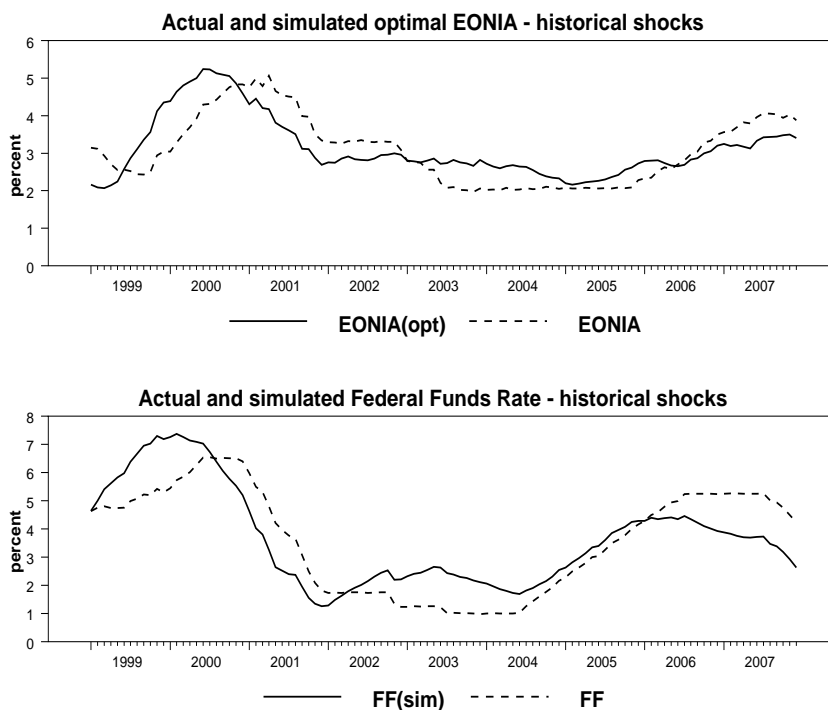


Figure 7: Simulation of EONIA and Federal Funds Rate with historical shocks

model is still able to capture broadly the evolution of the policy interest rates in the U.S. and in the EMU.<sup>13</sup>

Figure 8 displays the correlation coefficients between the simulated EONIA and Federal Funds Rates series in Figure 7 at various leads and lags and shows the correlation patterns to be very close to those of the observed time series for leads and lags of the Federal Funds Rate of up to about a year. However, the model has some problems in replicating the long-run correlations between the two interest rate series which are too weak compared to the observed ones.

All these results indicate that shocks to U.S. macroeconomic variables are important driving forces behind the dynamic behavior of the EONIA even though the U.S. variables enter the optimal monetary policy reaction function only because of their predictive power for inflation and unemployment in the Euro area. In fact, the optimal monetary policy reaction assigns an even greater importance to short-run reactions

<sup>13</sup>Note that only for the ECB an optimal monetary policy reaction function is used. For the Fed the model still includes the estimated reaction function.

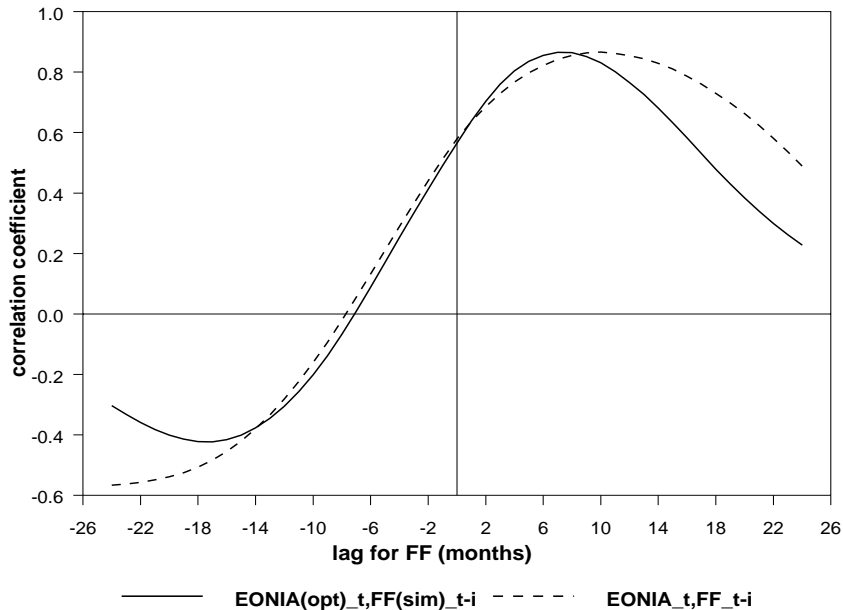


Figure 8: Correlation of simulated EONIA and simulated Federal Funds Rate

Table 2: Granger causality tests for observed and simulated interest rate series

causality			lags	p-value
EONIA	←	FF	12	0.00
EONIA	→	FF	12	0.01
EONIA(opt)	←	FF(sim)	12	0.04
EONIA(opt)	→	FF(sim)	12	0.16

to U.S. variables than estimated. Compared to the estimated ECB reaction function the optimal one implies a stronger reaction to unexpected Federal Funds Rate changes within the first months and emphasizes the importance of U.S. monetary policy shocks for the EONIA. The correlation pattern between the EONIA and the Federal Funds Rate is thus caused by the direct response of each central bank to the other's policy interest rate and by the two central banks reacting to changes in the macroeconomic variables in both the U.S. and the Euro area. In the context of the model presented in this paper, this behavior of the ECB is close to optimal.

As a final piece of evidence on how well the model captures the observed correlation between the EONIA and the Federal Funds Rate Table 2 presents results of Granger causality tests for these two variables as they have been employed, for example in

Belke and Gros (2003, 2005, 2006) Although the p-values are somewhat higher for the simulated than for the observed series the null hypothesis of Granger causality of the Federal Funds Rate for the EONIA cannot be rejected. In contrast to the observed data the simulated time series imply a rejection of Granger causality from the EONIA to the Federal Funds Rate.

### 3 Robustness

The results in the preceding model were derived from a structural VAR in which the Federal Reserve reacts to U.S. and Euro area unemployment, industrial production and inflation and to commodity price inflation within a given month. The Federal Reserve was assumed to respond to the EONIA with a lag of month. In contrast, the ECB was allowed to react immediately to unemployment, industrial production and inflation in both the Euro area and the U.S. and to commodity price inflation as well as to the Federal Funds Rate. This identification assumption was important in deriving the state equation for the economy (3) and the structural shocks used to construct impulse responses, variance decompositions and the simulated interest rate series in Figure 7.

In order to investigate how strongly our results depend on this assumption we derived results as in Section 2 for different identification schemes. The difference in model 1 to the benchmark model in Section 2 is that it restricts the set of variables the Federal Reserve is assumed to react to within the month to only U.S. unemployment, industrial production and inflation and to commodity price inflation. It retains the assumption that the Fed does respond to the ECB's policy decision with a lag of one month. Model 2 switches the information assumptions of the benchmark model between the Federal Reserve and the ECB around and assumes that the Federal Reserve reacts immediately to U.S. and Euro area unemployment, industrial production and inflation and to commodity price inflation as well as to the EONIA while the ECB does not react to the Federal Funds Rate within the month.<sup>14</sup>

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<sup>14</sup>We estimated also a third model which assumed that both central banks react to each other's interest rate changes immediately but to ensure identification, restricted the Federal Reserve to respond to the other Euro area variables except for the EONIA with a lag of one month. Unfortunately, there were difficulties in estimating the contemporaneous interaction coefficients in  $A_0$ ,  $b_0$  and  $c_0$  between

Table 3: Estimation results for various identification schemes

model	parameter values		SSD	$\sigma_{EONIA}$	$\max eig_{opt}$
benchmark	$\lambda_u = 0.0575$	$\pi^* = 2.20$	3.50	0.983	0.996
model 1	$\lambda_u = 0.0575$	$\pi^* = 2.20$	3.50	0.983	0.996
model 2	$\lambda_u = 0.0650$	$\pi^* = 2.20$	3.51	0.983	0.996

SSD: Sum of squared deviations of optimal from observed EONIA.

$\sigma_{EONIA}$ : Standard deviation of fitted EONIA (standard deviation of observed EONIA is 0.900).  $\max eig_{opt}$  is the largest absolute Eigenvalue of the model with the optimal MPRF imposed. The largest absolute eigenvalue of the estimated VAR is 0.998.

The estimates for the parameters in the central bank’s loss function (4) are almost the same across models.<sup>15</sup> The fit of the optimally set EONIA to the observed one is almost identical for for the benchmark model and models 1 and 2 as shown in Figure 9. For all models, the correlation between the fitted EONIA and the Federal Funds Rate comes close to the one of the observed EONIA (Figure 10).

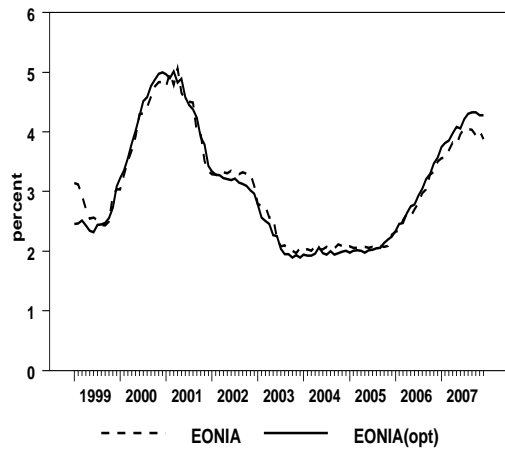
Figure 11 presents for all models the impulse responses of the EONIA with the optimal monetary policy reaction function combined with the appropriate version of the state equation (3) for the model. The structural shocks used in this simulation are that from the structural VAR with identification assumptions consistent with those used to construct the optimal monetary policy reaction function. The dotted lines are the estimated impulse responses and 90% probability around these were constructed by Monte Carlo simulation. The impulse responses of the optimal EONIA differ only very slightly across models and are for many shocks very close to the estimated ones.<sup>16</sup>

Table 4 compares the results of variance decompositions for the EONIA across the different models. As in Table 2 the structural shocks used to derive these results are the variables not only for this specification but also for slightly different models which with the assumption of immediate interaction between the ECB and the Fed. Furthermore the impulse responses indicated that the interest rate shocks were not identified appropriately. These problems could be avoided by replacing the U.S.-Dollar/Euro nominal exchange rate by a real effective exchange rate for the Euro area. The evidence from this model supports the general robustness of our results.

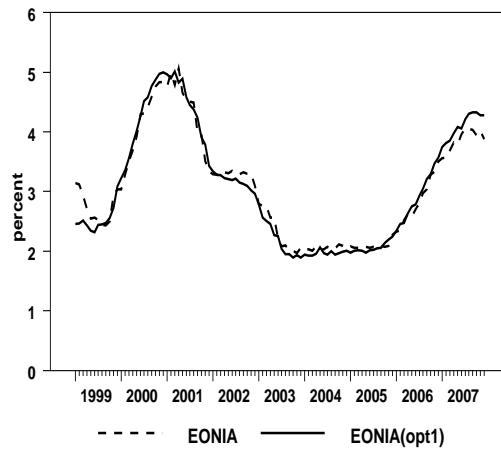
<sup>15</sup>For the weight on interest-rate smoothing  $\lambda_R$  a uniform value of 2.5 was imposed for all models.

<sup>16</sup>This applies as well to the impulse responses of Euro area unemployment and inflation which are not shown here.

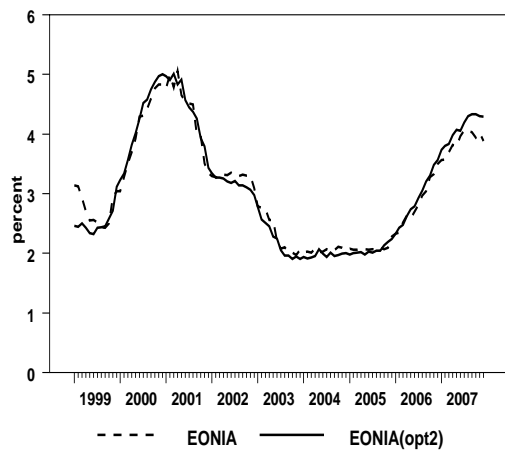




benchmark model



model 1



model 2

Figure 9: Actual and fitted optimal EONIA for all models

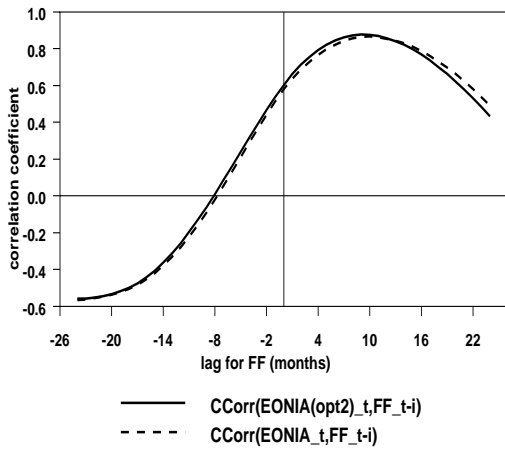
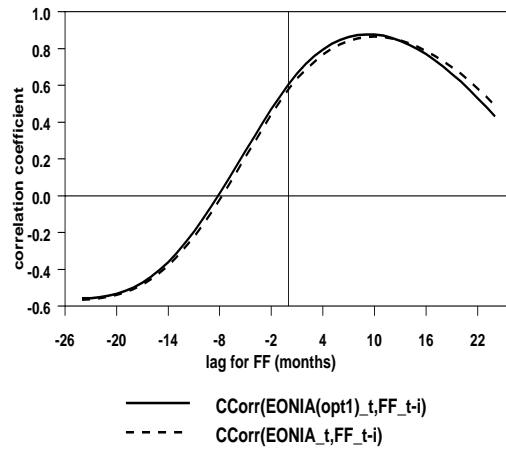
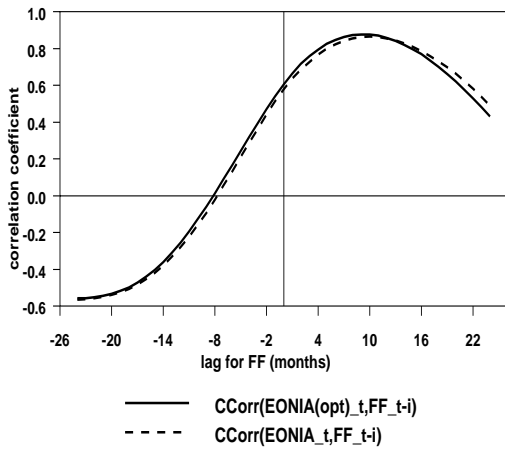


Figure 10: Correlation of actual and fitted optimal EONIA with observed Federal Funds Rate (all models)

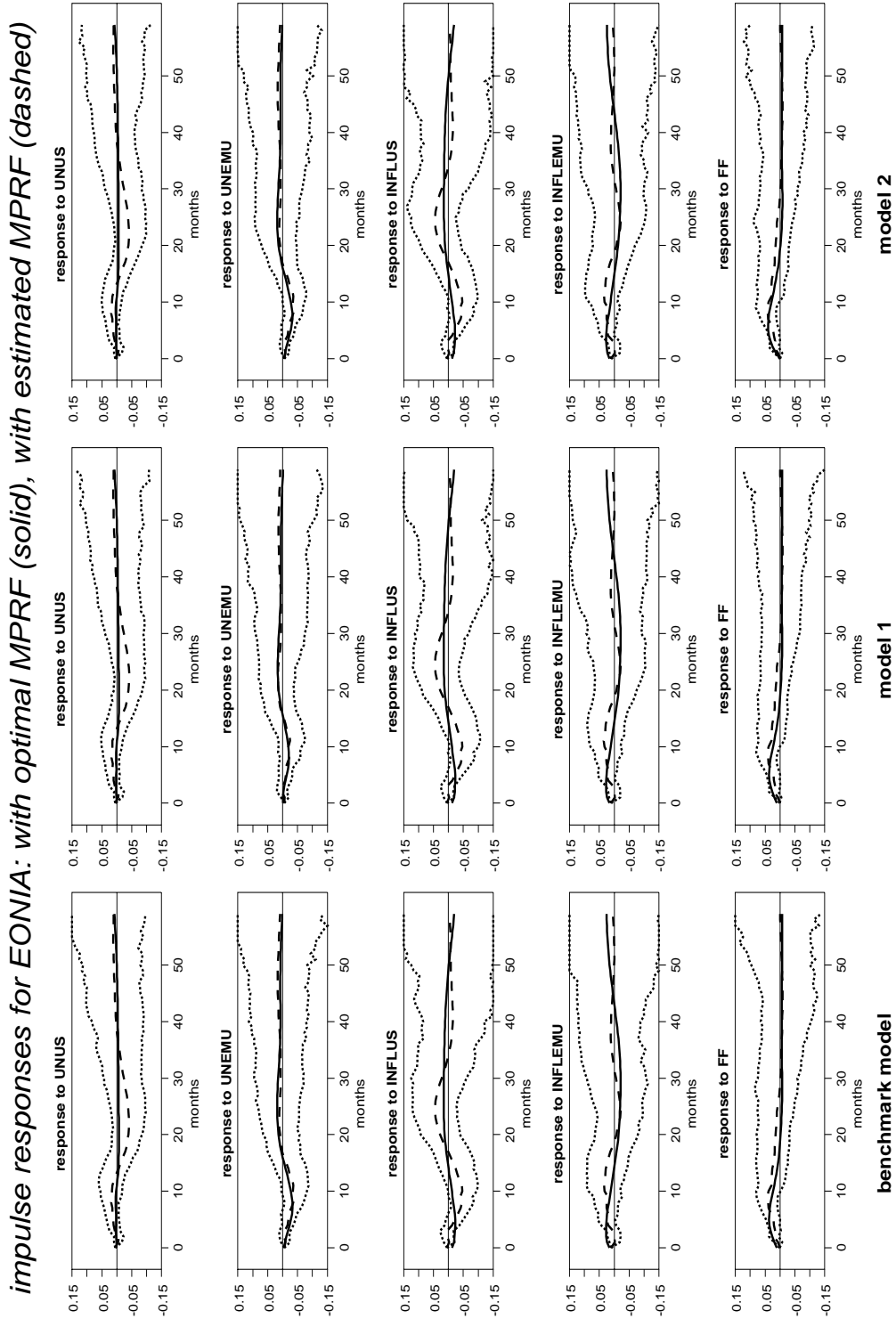


Figure 11: Impulse responses of EONIA for all models

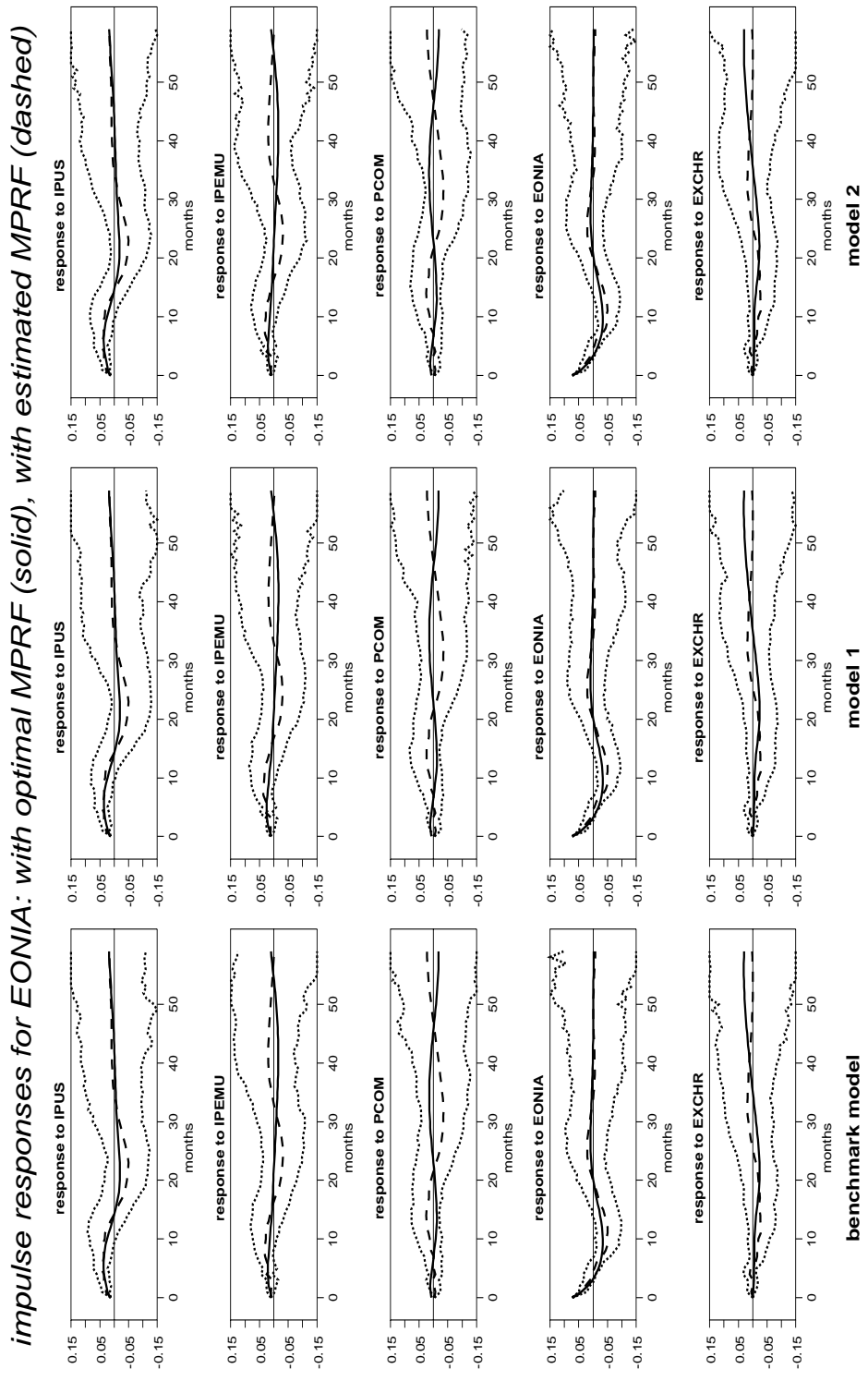


Figure 12: \*

Figure 10: Impulse responses of EONIA for all models (contd.)

Table 4: Comparison of variance decompositions for different models

Percentage contribution of U.S.-Variables to k-month ahead EONIA forecast error variance						
	k=0	k=1	k=3	k=6	k=12	k=24
benchmark (/FF)	3.46 (5.59)	12.75 (12.26)	26.38 (22.43)	40.19 (27.99)	34.01 (25.98)	43.21 (26.36)
benchmark (FF)	0.02 (1.99)	0.57 (5.06)	6.22 (12.76)	10.39 (18.19)	12.26 (17.79)	9.45 (14.27)
benchmark (all)	3.48 (7.58)	13.32 (17.32)	32.60 (35.19)	50.58 (46.18)	46.27 (43.78)	52.66 (40.62)
model 1 (/FF)	3.44 (5.46)	12.68 (11.91)	25.77 (21.63)	39.49 (27.48)	33.97 (26.01)	43.41 (26.47)
model 1 (FF)	0.03 (2.28)	0.65 (5.81)	7.16 (14.62)	12.08 (21.30)	14.48 (21.31)	10.97 (16.99)
model 1 (all)	3.47 (7.75)	13.33 (17.72)	32.92 (36.26)	51.57 (48.78)	48.44 (47.32)	54.38 (43.47)
model 2 (/FF)	3.46 (5.23)	12.75 (11.68)	26.38 (21.60)	40.19 (26.72)	34.01 (24.67)	43.21 (24.46)
model 2 (FF)	0.00 (0.00)	0.46 (1.87)	5.95 (10.57)	10.29 (19.07)	12.59 (20.64)	9.71 (17.27)
model 2 (all)	3.46 (5.23)	13.21 (13.55)	32.33 (32.17)	50.48 (45.79)	46.60 (45.31)	52.92 (41.73)

NOTES: Sample period is 1995:7-2007:12.

Numbers in brackets apply to the model including the optimal MPRF, number without brackets to the estimated VAR. (/FF) denotes the sum of the contributions of *UNUS*, *IPUS* and *INFLUS* to the EONIA forecast variance. (All) denotes the sum of these contributions plus the contribution of FF.

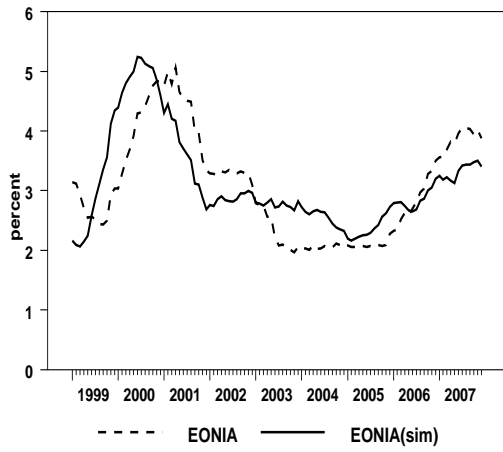
identical for the models with the estimated and the optimal reaction function and obtained from the estimated VAR with identification assumptions equivalent to those implicit in the optimal reaction function. For each model the sum of the contributions to the EONIA forecast variance of shocks to all U.S. variables except for the Federal Funds Rate is shown in the (*/FF*) line of each panel for the estimated VAR and in the second line for the model with the optimal monetary policy reaction function imposed for the ECB. The third and fourth lines show the contribution of the Federal Funds Rate while the last two lines give the sum of the contribution of all U.S. variables including the Federal Funds Rate.<sup>17</sup>

The table provides very similar results concerning the cumulative importance of shocks to U.S. unemployment, industrial production and inflation across models for both the estimated and the optimal ECB reaction function. The optimal reaction function in all models assigns a greater importance to these U.S. shocks in the impact period but a lower one afterwards relative to the estimated ECB reaction function. In turn, Federal Funds Rate shocks contribute stronger to unexpected EONIA movements under the optimal ECB reaction functions than under the estimated ones. The aggregate contributions of all U.S. shocks to the EONIA forecast variance under the assumption of the optimal monetary policy reaction function for the ECB are very close to their estimated counterparts apart from forecast over 24 months when it is considerably lower. This mirrors the difficulties of the optimally derived monetary policy reaction functions to capture the correlation between EONIA and Federal Funds Rate at longer leads and lags.

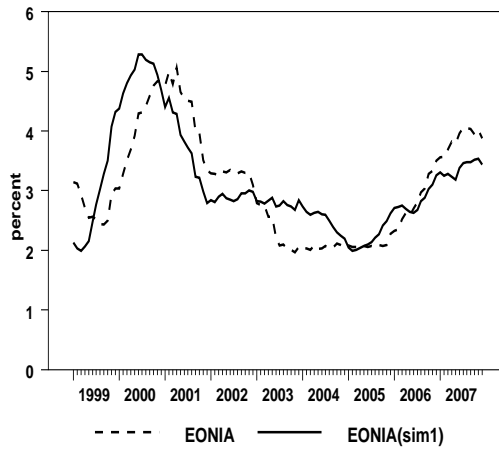
Figure 13 repeats the simulations from Figure 7 for each model. It shows the time series for the EONIA that result from simulating the model's version of (3) with the relevant optimal monetary policy reaction function (8) imposed. For each model the historical time series of the structural shocks were constructed from the reduced form VAR residuals using the model's identification assumptions. Beginning at the historically observed values for the state vector  $X_t$  in 1999M1 each model is simulated subject to the estimated time series of structural shocks. The simulated EONIA series are almost identical across the different models. As in Figure 7 the models are quite capable in

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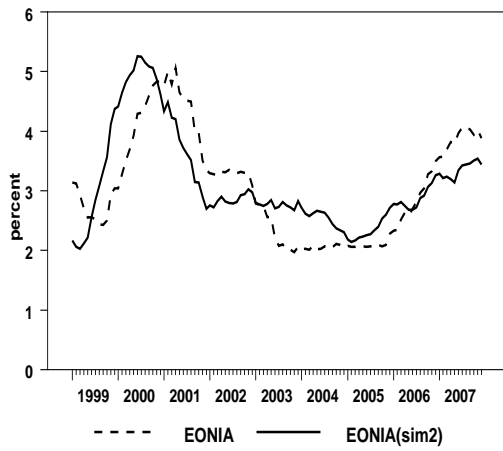
<sup>17</sup>While values in the first row of the top and bottom panels are identical, the individual contributions of the different U.S. shocks that enter these sums differ between the two models.



benchmark model



model 1



model 2

Figure 13: Simulation of EONIA with historical shocks (all models)

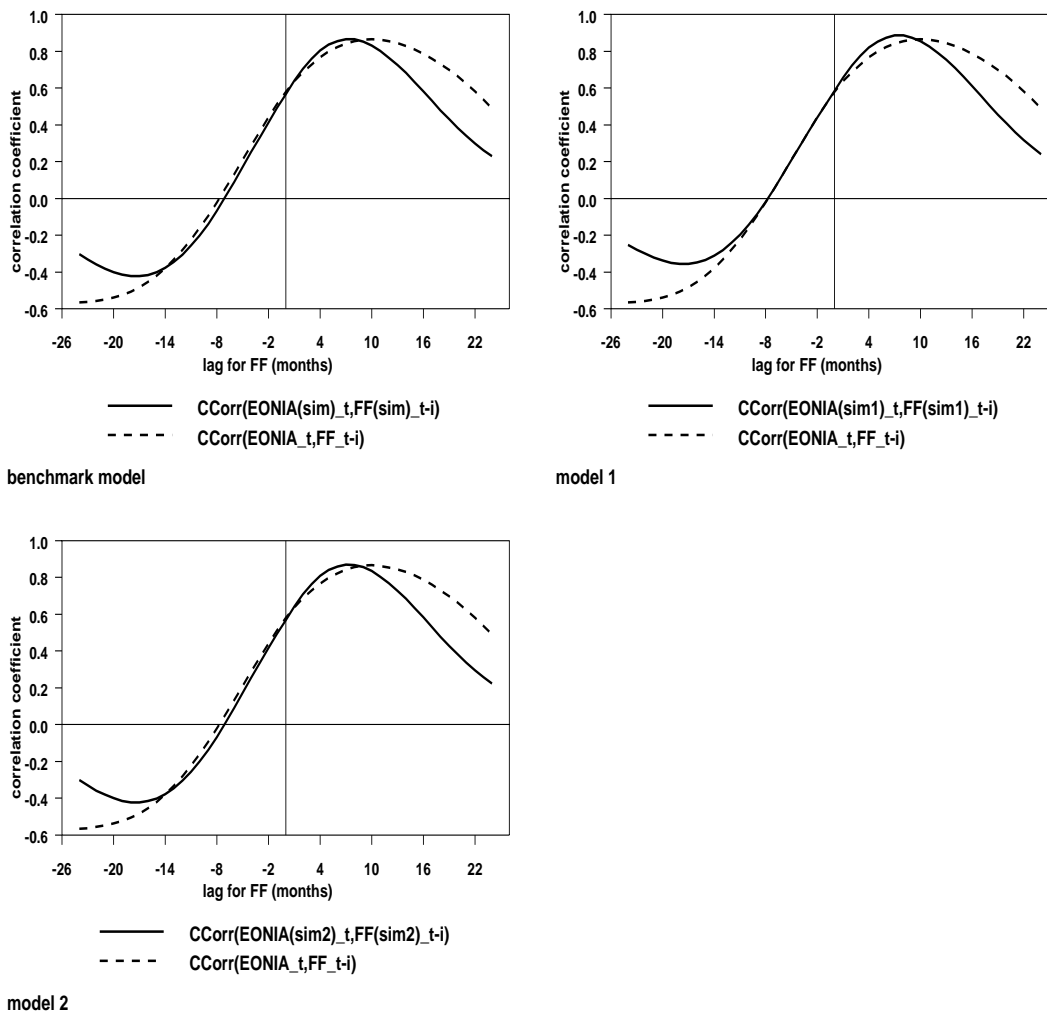


Figure 14: Correlation of simulated EONIA and simulated Federal Funds Rate (all models)

reproducing the general pattern of interest rate policy in the Euro area.

The correlation of the simulated times series of the EONIA and the Federal Funds Rate at various leads and lags are shown in Figure 11 for the different models. All models succeed in reproducing the correlations between the EONIA and the Federal Funds Rate for leads and lags up to about one year but imply weaker than observed correlations for longer leads and lags.



## 4 Optimal policy with model uncertainty

The following section derives an optimal monetary policy reaction function for the ECB which explicitly accounts for uncertainty about the monetary transmission mechanism. The results from this optimal reaction function are then compared to the results from section 2. In the last section we assumed that the central bank knew the true dynamic structure of the economy as represented in (3) and that all uncertainty was due to the stochastic disturbances  $\mu$ . Since the loss function (4) is quadratic and the constraints in (3) are linear certainty equivalence holds and uncertainty about the shocks  $\mu$  do not affect the shape and structure of the optimal monetary policy reaction function. In reality, however, central banks rely on estimated and, therefore, necessarily uncertain models of the structural relations within the economy. Brainard (1967) showed that uncertainty about the economic model's coefficients implies a less aggressive optimal policy reaction function compared to that under certainty equivalence. However, other studies have concluded that parameter uncertainty does not necessarily lead to monetary policy becoming more cautious (e.g. Söderström (2002)).

### 4.1 The ECB's optimal reaction function with parameter uncertainty

Sack (2000) proposes an approximate solution to the optimal policy problem under uncertainty. First (3) is replaced by

$$\hat{X}_{t+1} = \mathbf{F}\hat{X}_t + \mathbf{H}R_t + \mathbf{J} + \mu_{t+1}, \quad (11)$$

where  $\hat{X}_t = \mathbf{E}_{t-1}\hat{X}_t$  is the forecast of  $X_t$  based on time  $t-1$  information. The optimal policy sets the interest rate as a function of  $\hat{X}_t$ . This implies that the central bank reacts to shocks to the elements of  $X_t$  with a lag of one period.

Sack (2000) shows that an approximate solution to the minimization of the central bank's loss function subject to (11) is given by the Bellman equation

$$V(\hat{X}_t) = \max_{i_t} \left\{ - \left( \hat{X}_t - X^* \right)' \mathbf{G} \left( \hat{X}_t - X^* \right) - \left( \hat{X}_t' \mathbf{K} \hat{X}_t + 2\hat{X}_t' \mathbf{L} \right) + \beta E_t \left[ V(\hat{X}_{t+1}) \right] \right\} \quad (12)$$

together with (11). This transformation of the optimization problem leaves the dynamic structure of equation (3) unchanged and incorporates the effects of parameter uncertainty into the loss function. The matrix  $\mathbf{K}$  and the vector  $\mathbf{L}$  are weighted sums of the variance-covariance matrices of the parameters describing the dynamic behavior of the variables in the central bank's loss function.  $\mathbf{K} = \Sigma_{\beta(\pi)} + \lambda_u \Sigma_{\beta(u)} + \lambda_R \Sigma_{\beta(R)}$ , where  $\Sigma_{\beta(n)}$ ,  $n = u, \pi, R$ , is the covariance matrix of the coefficients within the equation of current unemployment, inflation and the lagged EONIA in (11).  $\mathbf{L}$  contains the covariances of the  $n$ -th equation's elements in  $\mathbf{F}$  with the  $n$ -th element of the vector  $\mathbf{J}$ .<sup>18</sup>

As explained in Sack (2000, pp. 247) the approximation in (11) and (12) implies that the variances of the shocks  $\mu$  increase through time due to accumulation effects.<sup>19</sup> He shows that the optimal solution for the policy instrument can be retrieved by assigning different weights to the first and the second terms in (12), that is by replacing  $\mathbf{G}$  with  $\hat{\mathbf{G}} = (1 - \rho)\mathbf{G}$ ,  $\mathbf{K}$  with  $\hat{\mathbf{K}} = \rho\mathbf{K}$ , and  $\mathbf{L}$  with  $\hat{\mathbf{L}} = \rho\mathbf{L}$ ,  $0 \leq \rho \leq 1$ .<sup>20</sup>

The optimal policy reaction function under model uncertainty has the same structure as before with  $X_t$  being replaced by  $\hat{X}_t$

$$R_t^* = -(\mathbf{H}'\mathbf{\Lambda}\mathbf{H})^{-1} \left( \mathbf{H}'\mathbf{\Lambda}\mathbf{F}\hat{X}_t + \mathbf{H}'\mathbf{\Lambda}\mathbf{J} + \mathbf{H}'\omega \right). \quad (13)$$

The Riccati equation becomes

$$\mathbf{\Lambda} = -\hat{\mathbf{G}} - \hat{\mathbf{K}} + \beta\mathbf{F}'\mathbf{\Lambda}\mathbf{F} - \beta\mathbf{F}'\mathbf{\Lambda}\mathbf{H}(\mathbf{H}'\mathbf{\Lambda}\mathbf{H})^{-1}\mathbf{H}'\mathbf{\Lambda}\mathbf{F}, \quad (14)$$

---

<sup>18</sup> $\mathbf{K}$  and  $\mathbf{L}$  are derived from the variance-covariance matrix of the VAR coefficient estimates over the complete sample period. This probably leads to an underestimation of the actual degree of uncertainty the central bank is facing.

<sup>19</sup>The forecast of the state vector  $\hat{X}_{t+1} = \mathbf{F}X_t + \mathbf{H}i_t + \mathbf{J}$  results from  $X_t$  and not from  $\hat{X}_t$  as suggested in (11). Hence, the shocks in  $\mu_{t+1}$  in (11) pick up terms related to  $X_t - \hat{X}_t$ . Despite this fact, the shock vector  $\mu$  remains uncorrelated with  $\hat{X}_t$  and the dynamics in (11) are unbiased representations of the true dynamics of  $\hat{X}$ . However, the accumulation of the effects of  $X_t - \hat{X}_t$  through time leads to an increasing variance of  $\mu$ . Since the Bellman equation (12) does not account for this fact it underestimates the true extent of model uncertainty. See Sack (2000), pp. 247.

<sup>20</sup> $\rho$  is chosen to minimize the central bank loss function. The exact procedure is given in Sack (2000), p. 248 and Table 1. Since the VAR used in the present paper is much larger than his the required simulations for estimating  $\rho$  turn out to be excessively lengthy. Results from a limited number of simulations indicate an estimate of  $\rho = 0.2$ . We experimented with different parameter values and found the results in this section to be very robust.

and

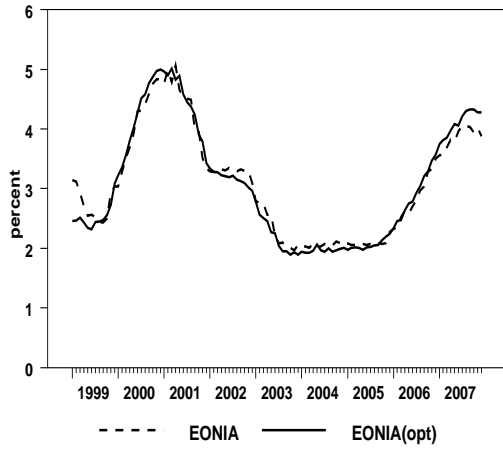
$$\begin{aligned} \omega &= \left( \mathbf{I} - \beta \mathbf{F}' \left( \mathbf{I} - \boldsymbol{\Lambda} \mathbf{H} (\mathbf{H}' \boldsymbol{\Lambda} \mathbf{H})^{-1} \mathbf{H} \right) \right)^{-1} \\ &\quad \times \left( \hat{\mathbf{G}} X^* - \hat{\mathbf{L}} + \beta \mathbf{F}' \boldsymbol{\Lambda} \left( \mathbf{I} - \mathbf{H} (\mathbf{H}' \boldsymbol{\Lambda} \mathbf{H})^{-1} \mathbf{H}' \boldsymbol{\Lambda} \right) \mathbf{J} \right). \end{aligned} \quad (15)$$

## 4.2 Results

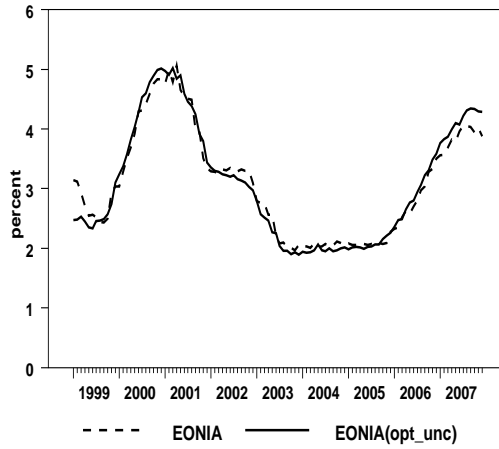
As shown in the optimal monetary policy reaction function (13) the EONIA is determined by the expectation of the state vector  $\hat{X}_t = \mathbf{E}_{t-1} X_t$ . As a consequence the ECB does not react to any structural shock within the same period. Translated into the VAR model (1) and (2) this implies an identification assumption in which all of the entries in  $\mathbf{b}_0$  are equal to zero, i.e. the ECB does not react contemporaneously to any other variable. To keep the assumptions underlying the optimal reaction function consistent with the estimated VAR the results that follow are derived from an estimated structural VAR that imposes this identification assumption but otherwise is identified as in the benchmark model.

After imposing the estimates from Section 2 for the parameters in the central bank's loss function ( $\lambda_u = 0.0575$ ,  $\lambda_R = 2.5$ ,  $\pi^* = 2.2$ ) the optimal monetary policy reaction function under uncertainty results in a fit almost identical to the benchmark model in section 3 with a sum of squared deviations of the optimal from the actual EONIA of 3.55 compared to 3.50. Figure 15 shows that the fitted time series for the EONIA and their correlation with the observed Federal Funds Rate is almost undistinguishable from that in the benchmark model.

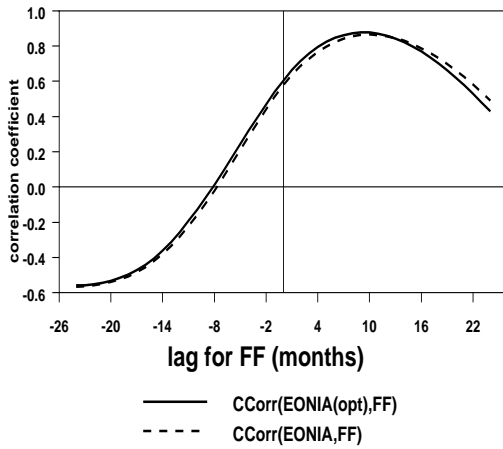
Figure 16 compares impulse responses of the EONIA if the optimal monetary policy reaction function from the benchmark model is simulated with (3) to those that result from simulating (13) with (11). The dashed lines represent the estimated impulse response functions and the dotted lines are 90% probability bands around these estimates. The only obvious differences between the impulse responses from the two optimal reaction functions can be observed in the immediate response to the various shocks. The optimal reaction function for the case of model uncertainty is constructed from the assumption that the ECB responds not to the actual values of the variables in the state vector but to their forecasts from last period  $\hat{X}_t$ . This implies that the



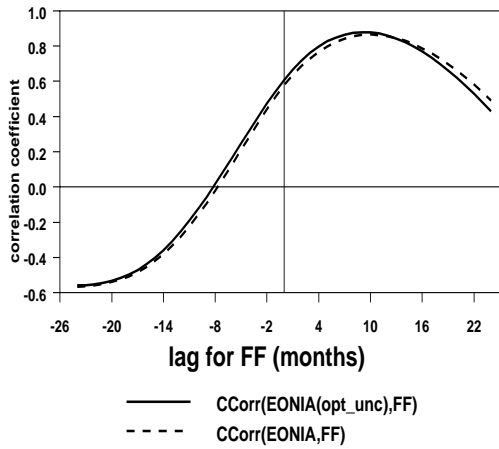
benchmark model



model unc



benchmark model



model unc

Figure 15: Actual and fitted optimal EONIA and correlation with Federal Funds Rate for additive and coefficient uncertainty

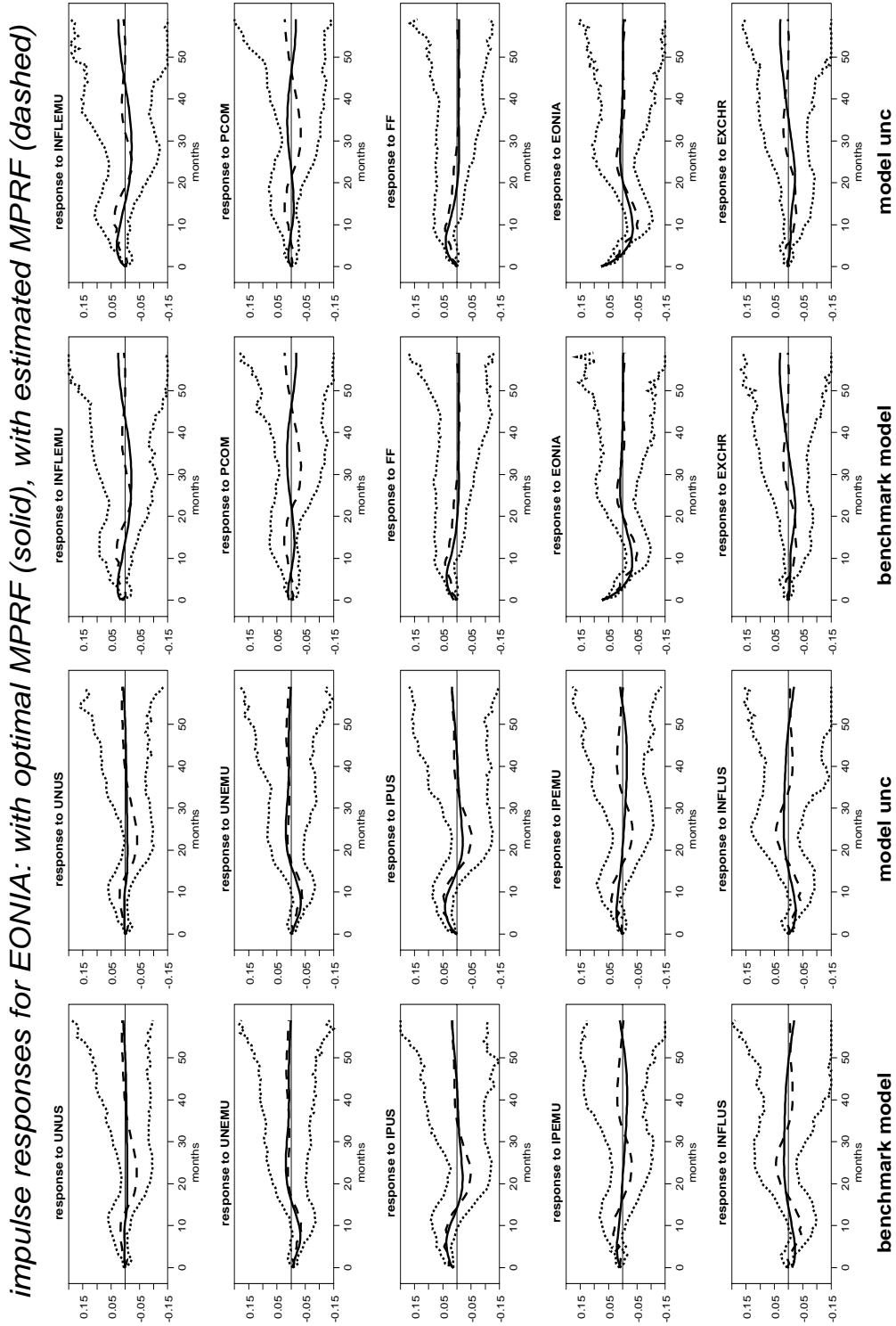


Figure 16: Impulse responses of EONIA for benchmark model and model with coefficient uncertainty

Table 5: Comparison of variance decompositions for benchmark model and model with coefficient uncertainty

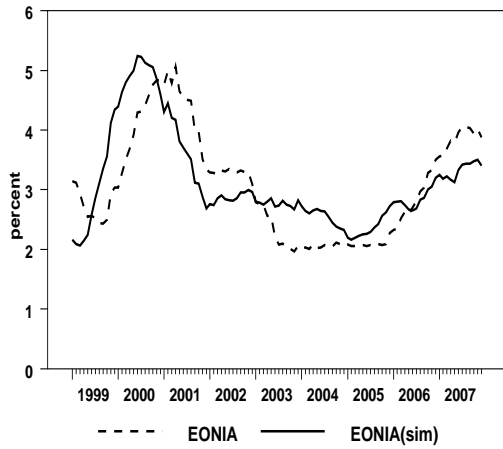
Percentage contribution of U.S.-Variables to k-month ahead EONIA forecast error variance						
	k=0	k=1	k=3	k=6	k=12	k=24
benchmark (/FF)	3.46 (5.59)	12.75 (12.26)	26.38 (22.43)	40.19 (27.99)	34.01 (25.98)	43.21 (26.36)
benchmark (FF)	0.02 (1.99)	0.57 (5.06)	6.22 (12.76)	10.39 (18.19)	12.26 (17.79)	9.45 (14.27)
benchmark (all)	3.48 (7.58)	13.32 (17.32)	32.60 (35.19)	50.58 (46.18)	46.27 (43.78)	52.66 (40.62)
model unc (/FF)	0.00 (0.00)	7.18 (4.43)	20.41 (18.06)	37.61 (27.39)	32.20 (27.14)	40.62 (27.24)
model unc (FF)	0.00 (0.00)	0.47 (2.09)	6.19 (11.41)	10.39 (18.02)	11.15 (18.06)	8.84 (15.19)
model unc (all)	0.00 (0.00)	7.64 (6.52)	26.60 (29.48)	48.04 (45.41)	43.36 (45.21)	49.46 (42.43)

NOTES: Sample period is 1995:7-2007:12.

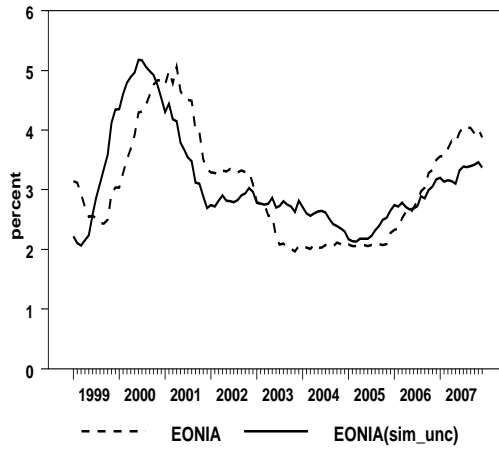
Numbers in brackets apply to the model including the optimal MPRF, number without brackets to the estimated VAR. (/FF) denotes the sum of the contributions of *UNUS*, *IPUS* and *INFLUS* to the EONIA forecast variance. (All) denotes the sum of these contributions plus the contribution of FF.

immediate reactions of the EONIA to all the shocks in a given period is zero.

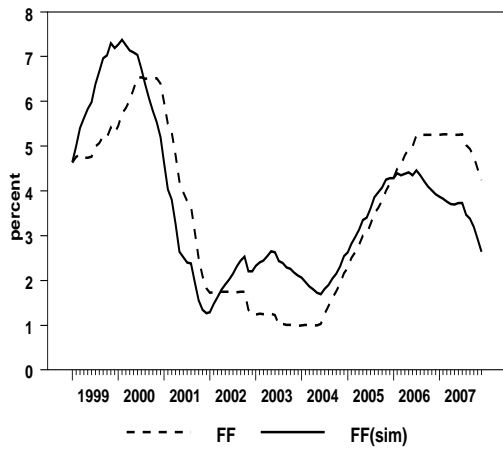
Table 5 provides some summary statistics on the variance decompositions of the benchmark model and the model with the optimal monetary policy reaction function assuming coefficient uncertainty. For each model the sum of the contributions to the EONIA forecast variance of shocks to all U.S. variables except for the Federal Funds Rate is shown first, the contribution of the Federal Funds Rate second, and finally, the sum of these two elements. The modification to the identification assumption leads naturally to a lower contribution of U.S. variables to the EONIA forecast variance in the first three months in the lower panel. The role of the Federal Funds Rate is similar in both panels although the optimal reaction function under uncertainty implies a more muted role for Federal Funds Rate shocks for  $k = 1$ .



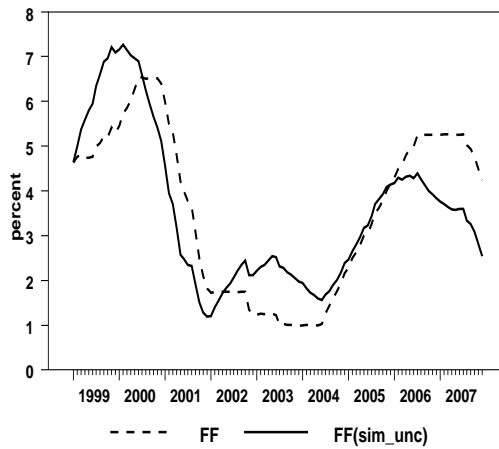
benchmark model



model unc



benchmark model



model unc

Figure 17: Simulation of EONIA and Federal Funds Rate with historical shocks

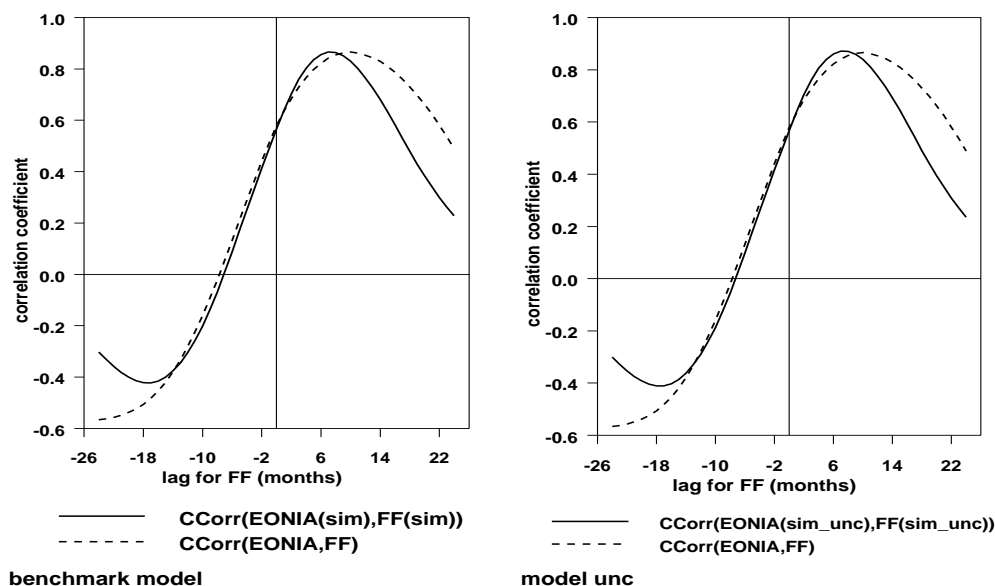


Figure 18: Correlation of simulated EONIA and simulated Federal Funds Rate

Figure 15 displays the time series for the EONIA and the Federal Funds Rate that result from simulating (3) with the optimal reaction function (8) (left column) and (11) with (13) (right column) using the historical series of structural shocks recovered from the estimated VAR. Again, the effect of the assumption of model uncertainty appears to be negligible. This result emerges as well from the comparison of the correlation of the simulated EONIA and Federal Funds Series in Figure 18.

## 5 The importance of the Federal Funds Rate

The various optimal monetary policy reaction functions studied so far all allow for an immediate or at least lagged response of the ECB to U.S. monetary policy as represented by the Federal Funds Rate. The impulse response analyses showed the optimal monetary policy reaction functions to imply an even stronger short-run response to Federal Funds Rate innovations than estimated in the VAR. Similarly, the variance decompositions indicated that the optimal monetary policy reaction functions attached increased relevance to Federal Funds Rate shocks compared to the estimated ones.

To provide a rough impression of how important the reaction of the ECB to the Fed-



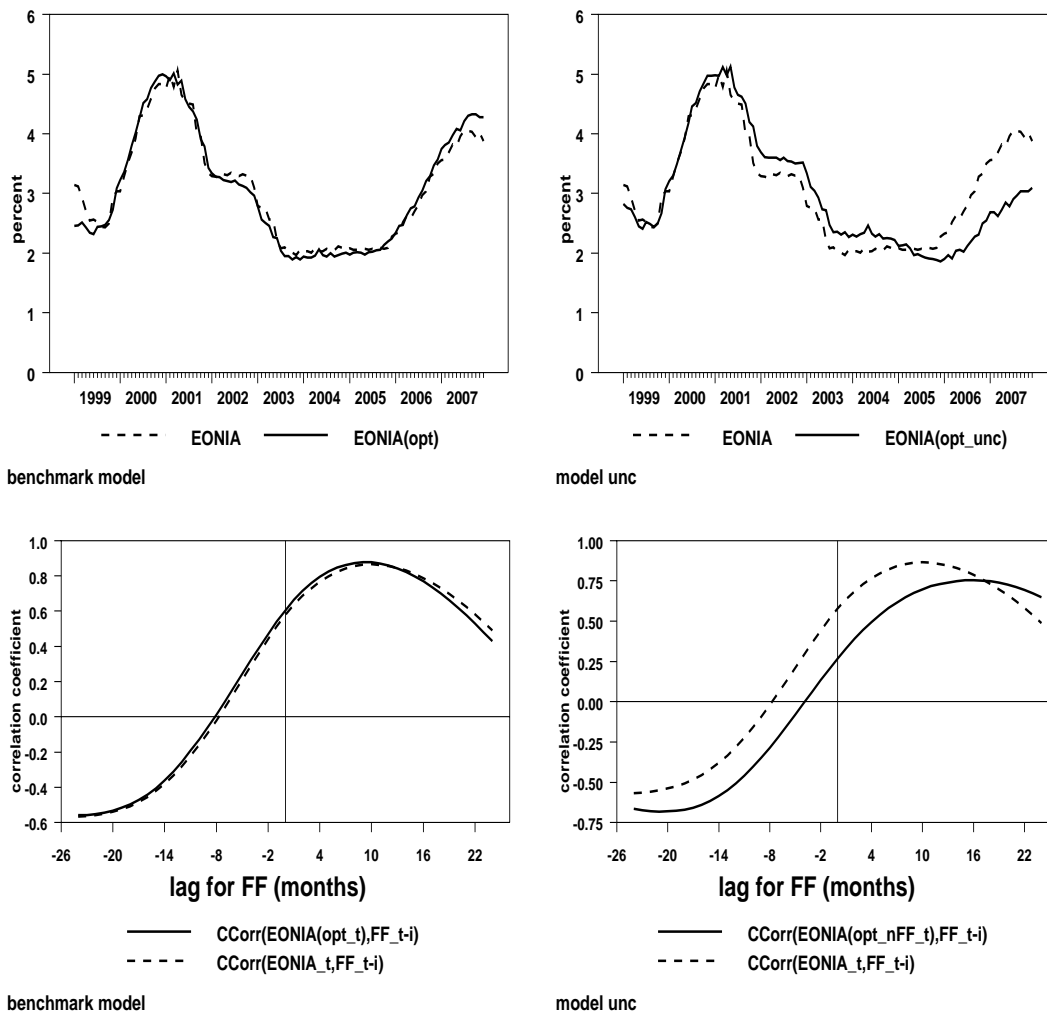


Figure 19: Actual and fitted optimal EONIA from benchmark model and model without FF

eral Funds Rate is for the ability of the optimal monetary policy reaction function to approximate the actually observed behavior of the EONIA we repeat the analysis with a deliberately misspecified model in which we estimated the VAR without the Federal Funds Rate equation. The effects of the systematic response of U.S. monetary policy to the other variables will to some extent be captured by changes in the other equations' coefficient estimates.<sup>21</sup> As shown in Figure 19 the fit of the model based optimal EONIA deteriorates strongly with a sum of squared deviations of the optimal

<sup>21</sup>Repeated substitution for the Federal Funds Rate from estimated Fed reaction will lead to an infinite VARMA representation of the model which is only approximated by the finite order VAR used in this exercise.

from the observed EONIA of 23.11. Furthermore, the fitting of the theoretical to the actual interest rate yields an implausibly high estimate of  $\lambda_u = 0.925$ . The optimal reaction function, however, is still able to follow the basic pattern of the evolution of the EONIA. Nevertheless, Figure 19 is strong evidence for the importance of a direct link of the EONIA to the Federal Funds Rate for obtaining a good approximating performance of the optimal ECB reaction function.

For the analysis of the impulse responses and variance decomposition we insert the optimal reaction function into the benchmark model, i.e. the model including the Federal Funds Rate and simulate the EONIA responses for the structural shocks identified in the benchmark model. Since the optimal ECB reaction function used in this exercise is constructed from the VAR that does not include the Federal Funds Rate the only effects of Federal Funds Rate shocks on the EONIA are those caused by the reactions of the other variables in the model to the Federal Funds Rate innovations and the responses of the EONIA to these movements. Remember that the reaction of the EONIA to the Federal Funds Rate derives from its predictive power for unemployment and inflation in the Euro area. If this forecasting ability works through the effects of changes in the Federal Funds Rate on the other variables in the model the construction of this reaction function without regard to the Federal Funds Rate should not result in an EONIA impulse response function to Federal Funds Rate shocks that differs much from the one in the benchmark model.<sup>22</sup> As Figure 20 shows, the EONIA impulse response to Federal Funds Rate shocks using the optimal monetary policy reaction function does not differ very strongly between the benchmark model and the restricted model with no direct reaction of the EONIA to the Federal Funds Rate. Relative to the estimated reaction function the restricted one lags somewhat behind while the unrestricted one leads it. In this respect, adjustments in the reactions of the EONIA to the other variables in the economy can at least partially compensate for the lack of direct response to the EONIA.

Furthermore, changes in the impulse responses in the right column can indicate the

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<sup>22</sup>The impact effect will differ, since in the benchmark model the EONIA responds immediately to the Federal Funds Rate shock. In the restricted model, however, the EONIA reacts only to the other variables in the model which, except for the exchange rate, are only affected by the Federal Funds Rate by a lag.

differences between the restricted and the unrestricted optimal reaction function. If the channel through which Federal Funds Rate shocks affect the ECB's goal variables is similar to that of one or more of the other structural shocks the impulse responses in the model with the restricted optimal reaction function to these shocks would be distorted since the restricted optimal reaction function would in part reflect the effects of Federal Funds Rate shocks and not the effects of 'pure' structural shocks to these variables. Interestingly, Figure 20 shows that the impulse responses the shocks to U.S. variables are not affected more strongly than those to shocks to Euro area variables. At least for periods up to two years after the shock these changes appear relatively moderate. Stronger changes can be found for the responses to the exchange rate and to commodity price inflation shocks. These results indicate that the channel through which Federal Funds Rate shocks affect the ECB's goal variables does not seem to be confined to the other U.S. variables.

## 6 Summary and Conclusions

In this paper, we studied an explanation of the correlation between interest rates set by the ECB and by the Fed driven by economic fundamentals. We derived an optimal monetary policy reaction function for the ECB from a VAR model of the Euro area and the U.S. economies. This optimal reaction function makes the EONIA respond to the variables in the model based on their forecasting power concerning the ECB's goal variables. Our results showed that this optimal reaction function approximated the actually observed time series of the EONIA very closely and yielded plausible estimates for the parameters in the ECB's loss function. Furthermore, the optimal monetary policy reaction function implied impulse responses for the EONIA close to those from the estimated model and a similar relative importance of shocks to U.S. variables in explaining EONIA movements. In fact, the optimal response of the EONIA to Federal Funds Rate shocks and the explanatory power of these shocks for the EONIA forecast variance were somewhat stronger compared to the estimated monetary policy reaction function. The model that approximates the transmission mechanism of monetary policy is constructed from a structurally identified VAR. Since the estimated reaction function

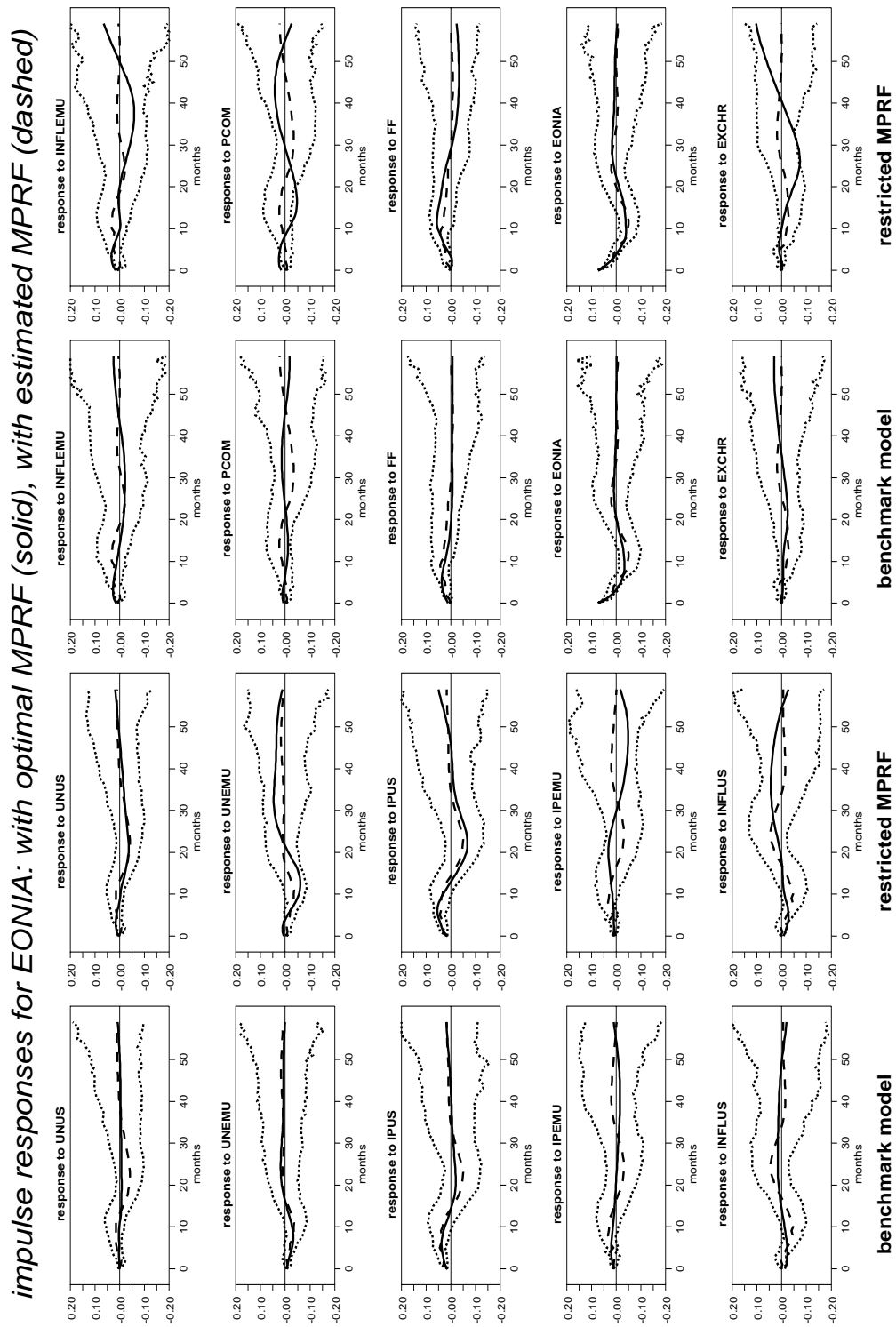


Figure 20: Impulse responses of EONIA for benchmark model and model with restricted optimal MPRF

is removed from this model before the construction of the optimal one leaving only the structural interrelationships between the non-policy variables and their dependence on the monetary policy interest rate behind, this result is not an artifact of the high predictive power of the Federal Funds Rate for the EONIA that is observed in reality.<sup>23</sup> Simulations using the optimal ECB reaction function showed the model's capability in capturing the observed U.S. and Euro area interest correlation for leads and lags up to about one year. For longer leads and lags the optimal reaction function underestimated the actual correlation. We investigated the robustness of these results with respect to the identification schemes employed in the estimation of the structural VAR and found the results to change almost not at all.

Since the optimal monetary policy reaction function suggested an increase in the explanatory power of Federal Funds Rate shocks for EONIA movements compared to the estimated reaction function we studied a model in which the ECB was not allowed to react directly to the Federal Funds Rate. The optimal monetary policy reaction function in this model could only partially compensate for this restriction in the central bank's information set by optimally re-adjusting the EONIA reactions to the other variables in the model. This restriction led to a strong deterioration in the approximation quality of the optimal to the actually observed EONIA indicating that the Federal Funds Rate contains information important for explaining the ECB's monetary policy. However, the adjustments in the optimal reaction function led to impulse responses reaction of the EONIA to Federal Funds Rate shocks if the restricted reaction function was inserted into the standard model that closely resembled those for the optimal reaction function that was allowed to respond immediately to the Federal Funds Rate. This result showed that the the optimal reaction function could compensate qualitatively for the lack of direct response to the Federal Funds Rate by adjustments in the reactions to the other variables.

These adjustments to the optimal reaction function which aimed to compensate for the lack of a direct reaction to the Federal Funds Rate, however, necessarily impaired the optimal reactions to the other shocks in the model. As an interesting result the

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<sup>23</sup>This result is subject to the qualification of the contemporaneous interaction coefficients in  $A_0$  and  $c_0$  being identified correctly. Our results are robust with respect to changes in the identification schemes we can have some confidence in this assumption.

impulse responses most strongly affected were not the those to the shocks to U.S. unemployment, production and inflation but to commodity price inflation and exchange rate shocks. A possible interpretation might be that Federal Funds Rate shocks contain elements of global monetary policy, financial or inflationary shocks. A detailed study of the information content of Federal Funds Rate shocks and their implications for optimal monetary policy in this model would require augmenting the model by global variables in addition to commodity price inflation. This would result in an even larger model than the one presented here in the paper and would require a careful consideration of various possible identification schemes and is beyond the scope of this paper.

## Appendix A: Data description

Data sources:

Federal Reserve Bank of St.Louis (FRED)

<http://research.stlouisfed.org/fred2/>

European Central Bank (Statistical Data Warehouse)

<http://sdw.ecb.europa.eu/>

Deutsche Bundesbank

<http://www.bundesbank.de/statistik/statistik.php>

Description of data series:

- *UNUS*: Deviation of U.S. unemployment rate from trend unemployment.  
Data: U.S. civilian unemployment rate, seasonally adjusted. Source: FRED.  
Series ID: UNRATE.  
Trend unemployment is estimated as a quadratic trend for 1993M1 to 2007M12 (Sack, 2000).
- *UNEMU*: Deviation of Euro area unemployment rate from trend unemployment.  
Data: Standardised unemployment rate, Euro area, all ages, male & female, seasonally adjusted, not working day adjusted. Source: Statistical Data Warehouse, European Central Bank.  
Series ID: STS.M.U2.S.UNEH.RTT000.4.000.  
Trend unemployment is estimated as a quadratic trend for 1993M1 to 2007M12 (Sack, 2000).
- *IPUS*: 12-months growth rate of U.S. industrial production index.  
Data: Industrial production index (2007=100), seasonally adjusted. Source: FRED  
Series ID: INDPRO.
- *IPEMU*: 12-months growth rate of Euro area industrial production index.  
Data: Industrial production index (2005=100), Euro area 15, total industry, sea-

sonally and working day adjusted. Source: Statistical Data Warehouse, European Central Bank

Series ID: STS.M.I4.Y.PROD.NS0010.4.000.

- *INFLUS*: 12-months growth rate of U.S. Consumer Price Index.  
Data: Consumer price index for all urban consumers: all Items (1982-84=100), not seasonally adjusted. Source: FRED  
Series ID: CPIAUCNS.
- *INFLEMU*: 12-months growth rate of Euro area Consumer Price Index.  
Data: Harmonized Index of Consumer Prices (2005=100), Euro area, neither seasonally nor working day adjusted. Source: Statistical Data Warehouse, European Central Bank  
Series ID: ICP.M.U2.N.000000.4.INX.
- *PCOM*: Smoothed 12-months growth rate of index of commodity prices.  
Data: IMF commodities index (2005=100). Source: International Financial Statistics, International Monetary Fund  
Series ID: 00176ACDZF.  
Smoothed growth rate as average of current and preceding 23 months.
- *FF*: Effective Federal Funds Rate, average of daily figures. Source: FRED  
Series ID: FEDFUNDS.
- *EONIA*: EONIA, monthly average. Source: Deutsche Bundesbank  
Series ID: SU0304.  
Before 1999: FIBOR, monthly average: Source: Deutsche Bundesbank  
Series ID: SU0101
- *EXCHR*: U.S. Dollar/Euro exchange rate. Dollars per Euro, averages of daily figures. Source: FRED  
Series ID: EXUSEU.  
Before 1999: U.S. Dollar/ECU exchange rate. Dollars per ECU, averages of daily figures. Source: FRED  
Series ID: EXUSEC.



The starting point of the estimation period is 1995M7 which is given by the availability of the *PCOM* series and the use of six lags in the VAR. This series is available from 1992M1. Computing the annual growth rate and smoothing the series with a 24-months moving average shifts the starting date to 1995M1.

## Appendix B: Construction of state equation

This section describes, how equation (3) can be constructed from the estimated structural VAR. The example given here is for the benchmark identification scheme. First, the estimated structural VAR (1) and (2) is written compactly as

$$\tilde{\mathbf{A}}_0 \tilde{Z}_t = \tilde{\mathbf{k}} + \sum_{i=1}^q \tilde{\mathbf{A}}_i \tilde{Z}_{t-i} + \tilde{\ell}t + \tilde{v}_t \quad (\text{B1})$$

with  $\tilde{Z}_t = (UNUS_t, UNEMU_t, IPUS_t, IPEMU_t, INFLUS_t, INFLEMU_t, PCOM_t, FF_t, EONIA_t, EXCHR_t)'$ .  $\tilde{\mathbf{A}}_0$ ,  $\tilde{\mathbf{A}}_i$ ,  $\tilde{\mathbf{k}}$  and  $\tilde{\ell}$  are the estimated coefficient matrices and vectors. and  $\tilde{v}_t$  is the vector of structural disturbances. The ninth row corresponds to the estimated monetary policy reaction function (2) while the other rows correspond the non-policy part of the VAR (1).

We can write down a state-equation similar to (3) as

$$\Phi_0 X_{t+1} = \Phi_1 X_t + \Theta R_t + \Psi + \xi_{t+1}. \quad (\text{B2})$$

$X_t$  is the vector of state variables of the model and is given by  $(UNUS_t, \dots, UNUS_{t-q}, UNEMU_t, \dots, UNEMU_{t-q}, IPUS_t, \dots, IPUS_{t-q}, IPEMU_t, \dots, IPEMU_{t-q}, INFLUS_t, \dots, INFLUS_{t-q}, INFLEMU_t, \dots, INFLEMU_{t-q}, PCOM_t, \dots, PCOM_{t-q}, FF_t, \dots, FF_{t-q}, EONIA_{t-1}, \dots, EONIA_{t-q}, EXCHR_{t-1}, \dots, EXCHR_{t-q}, t)'$ . The vector of disturbances  $\xi$  is constructed correspondingly and contains the time  $t$  structural shocks to all variables except for *EONIA* and *EXCHR* and the time  $t - 1$  disturbance to *EXCHR*. The coefficient matrices  $\Phi_0$  and  $\Phi_1$  and the vector  $\Theta$  which contains the reaction coefficients of the state variables to the policy interest rate can be constructed easily from (B1). Note that the variable ordered causally after

the policy variable (*EXCHR*) enters the state-vector with its lagged value.  $\Psi$  contains the constant terms and a '1' in the row of the time index. Finally, (3) results from pre-multiplying (B2) by  $\Phi_0^{-1}$ . with  $F = \Phi_0^{-1}\Phi_1$ ,  $\mathbf{H} = \Phi_0^{-1}\Theta$ ,  $\mathbf{J} = \Phi_0^{-1}\Psi$  and  $\mu_{t+1} = \Phi_0^{-1}\zeta_{t+1}$ .

## Appendix C: Optimal reaction function

The central bank's optimization problem is given by the minimization of the present value of the intra-period loss function

$$(X_{t+i} - X^*)' \mathbf{G} (X_{t+i} - X^*). \quad (\text{C1})$$

and the transition equation for the state variables

$$X_{t+1} = \mathbf{F}X_t + \mathbf{H}R_t + \mathbf{J} + \mu_{t+1}. \quad (\text{C2})$$

The Bellman equation for this dynamic programming problem is

$$V(X_t) = \max_{R_t} \left\{ -(X_t - X^*)' \mathbf{G} (X_t - X^*) + \beta E_t [V(X_{t+1})] \right\} \quad (\text{C3})$$

Assume for the value function the following solution

$$V(X) = X' \Lambda X + 2X' \omega + \rho. \quad (\text{C4})$$

Substituting this solution (C4) and the transition equation (4B.2) into the Bellman equation (C3) results in

$$\begin{aligned} V(X_t) &= \max_{R_t} -\{(X_t - X^*)' G(X_t - X^*) \\ &\quad + \beta \mathbf{E}_t [X'_{t+1} \Lambda X_{t+1} + 2X'_{t+1} \omega + \rho] \\ &= \max_{R_t} -\{(X_t - X^*)' G(X_t - X^*) \\ &\quad + \beta \mathbf{E}_t [(FX_t + HR_t + J + \mu_{t+1})' \Lambda (FX_t + HR_t + J + \mu_{t+1}) \\ &\quad + 2(FX_t + HR_t + J + \mu_{t+1})' \omega + \rho]. \end{aligned} \quad (\text{C5})$$

Expanding this expression and observing that  $\mathbf{E}_t(\mu_{t+1}|X_t) = 0$  yields

$$\begin{aligned} V(X_t) &= \max_{R_t} -\{(X_t - X^*)' G(X_t - X^*) + \beta [X'_t F' \Lambda F X_t \\ &\quad + 2X'_t F' \Lambda H R_t + 2X'_t F' \Lambda J + i_t H' \Lambda H R_t + J' \Lambda H R_t + J' \Lambda J \\ &\quad + \mathbf{E}_t(\mu'_{t+1} \Lambda \mu_{t+1}) + 2X'_t F' \omega + 2R_t H' \omega + 2J' \omega + \rho]. \end{aligned} \quad (\text{C6})$$

This expression was simplified by noting that all additive terms in (C5) are scalar expressions and, hence, are symmetric.

The first-order condition is

$$\begin{aligned} 0 &= 2\beta H'_t \Lambda F X_t + 2\beta H' \Lambda H R_t \\ &\quad + 2\beta H' \Lambda J + 2\beta H' \omega. \end{aligned} \quad (\text{C7})$$

This yields a solution for the policy instrument  $R_t$

$$R_t^* = - (H' \Lambda H)^{-1} (H' \Lambda F X_t + H' \Lambda J + H' \omega). \quad (\text{C8})$$

Substituting the optimal reaction function into (C6) results in

$$\begin{aligned} V(X_t) &= -(X_t - X^*)' G (X_t - X^*) + \beta [X'_t F' \Lambda F X_t \\ &\quad - 2X'_t F' \Lambda H (H' \Lambda H)^{-1} (H' \Lambda F X_t + H' \Lambda J + H' \omega) \\ &\quad + 2X'_t F' \Lambda J + (H' \Lambda H)^{-1} (H' \Lambda F X_t + H' \Lambda J + H' \omega)' H' \Lambda H \\ &\quad (H' \Lambda H)^{-1} (H' \Lambda F X_t + H' \Lambda J + H' \omega) \\ &\quad - J' \Lambda H (H' \Lambda H)^{-1} (H' \Lambda F X_t + H' \Lambda J + H' \omega) \\ &\quad + J' \Lambda J + \mathbf{E}_t(\mu'_{t+1} \Lambda \mu_{t+1}) + 2X'_t F' \omega + 2(H' \Lambda H)^{-1} \\ &\quad (H' \Lambda F X_t + H' \Lambda J + H' \omega) H' \omega + 2J' \omega + \rho]. \end{aligned} \quad (\text{C9})$$

This expression must be identical to (C4). Collecting all quadratic terms leads to the identity

$$\begin{aligned} X'_t \Lambda X_t &= -(X_t - X^*)' G (X_t - X^*) + \beta X'_t F' \Lambda F X_t \\ &\quad - 2\beta X'_t F' \Lambda H (H' \Lambda H)^{-1} H' \Lambda F X_t \\ &\quad + \beta X'_t F' \Lambda H (H' \Lambda H)^{-1} H' \Lambda H (H' \Lambda H)^{-1} H' \Lambda F X_t \\ &= -(X_t - X^*)' G (X_t - X^*) + \beta X'_t F' \Lambda F X_t \\ &\quad - \beta X'_t F' \Lambda H (H' \Lambda H)^{-1} H' \Lambda F X_t. \end{aligned} \quad (\text{C10})$$

This results in the Riccati equation

$$\Lambda = -G + \beta F' \Lambda F - \beta F' \Lambda H (H' \Lambda H)^{-1} H' \Lambda F. \quad (\text{C11})$$

$\omega$  can be obtained from collecting and equating all terms linear in  $X$ :

$$\begin{aligned}
2X_t'\omega &= -2\beta X_t'F'\Lambda H(H'\Lambda H)^{-1}(H'\Lambda J + H'\omega) + 2\beta X_t'F\Lambda J \\
&\quad + 2\beta(H'\Lambda H)^{-1}X_t'F'\Lambda H(H'\Lambda J + H'\omega) \\
&\quad - \beta J'\Lambda H(H'\Lambda H)^{-1}H'\Lambda F X_t + 2\beta X_t'F'\omega \\
&\quad + 2\beta(H'\Lambda H)^{-1}X_t'F'\Lambda H H'\omega + 2X_t'GX^*.
\end{aligned} \tag{C12}$$

Multiplication and collecting terms yields

$$\begin{aligned}
2X_t'\omega &= 2\beta X_t'F\Lambda J - 2\beta J'\Lambda H(H'\Lambda H)^{-1}H'\Lambda F X_t \\
&\quad + 2\beta X_t'F'\omega + 2\beta(H'\Lambda H)^{-1}X_t'F'\Lambda H H'\omega \\
&\quad + 2X_t'GX^*,
\end{aligned} \tag{C13}$$

where we again have made use of the symmetry of the scalar products. The result is

$$\begin{aligned}
\omega &= \left( I - \beta F' \left( I - \Lambda H (H'\Lambda H)^{-1} H \right) \right)^{-1} \\
&\quad \left( GX^* + \beta F' \Lambda \left( I - H (H'\Lambda H)^{-1} H' \Lambda \right) J \right).
\end{aligned} \tag{C14}$$

The solution for the constant  $\rho$  is irrelevant for the optimal policy reaction function.

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