

LIQUIDITY AND SOLVENCY IN A MODEL OF EMERGING MARKET CRISES

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ABSTRACT. We investigate the relationship between short-term capital flows and emerging market banking crises. We argue that the likelihood of a self-fulfilling crisis is not determined by the amount of short-term funds per se (as is sometimes argued), but rather that if short-term capital flows heighten the probability to experiencing a crisis, this is to be viewed as a symptom of underlying unfavorable fundamentals. Raising reserve holdings to mitigate the economy's vulnerability may only sometimes be a sensible policy advice. Imposing capital inflow controls may sometimes be welfare enhancing. Furthermore, we show how a country can become trapped in a vicious circle where foreign creditors are unwilling to lend long-term because of a high default probability induced by large short term indebtedness. JEL Codes: G01, F32, D82

1. Introduction

A large build-up of short-term capital flows is widely perceived to be a crucial factor in the outbreak of financial sector or international banking crises.¹ However, the evidence for the harmful effects of short-term capital flows is mixed, if not inconclusive. On the one hand, Rodrik and Velasco (1999) and Radelet and Sachs (1998) provide evidence for short-term debt being a cause of the outbreak of emerging market crises. The former also uncover a positive relationship between the pre-crisis level of short-term debt and the severity of a crisis. Similarly, Detragiache and Spilimbergo (2004) detect a robust relationship between short-term indebtedness and the prevalence of a debt crisis, but they question the direction of causality.² On the other hand, Frankel and Rose (1996) dismiss short-term flows as a predictor for the outbreak of a crisis in those countries being hit by the Tequila effect in 1995. And Eichengreen and Rose (1998) even find a negative relationship between short-term debt and the probability of a crisis.

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¹See Montiel and Reinhart (1997) for a literature survey.

²Although Detragiache and Spilimbergo find that increased short-term borrowing raises the probability of a crisis, they note that it is very well possible that crisis-prone countries are forced to borrow short-term, thus reversing the causality. We provide some a theoretical underpinning for this view below.

In this paper we present a theoretical model that clarifies the connection between capital flows and the vulnerability of emerging market economies to financial turmoil and banking crises. Our model offers an explanation for when short-term capital flows become detrimental and when they do not. We are thus able to provide an explanation for the above mentioned conflicting empirical findings. Furthermore, we will discuss the case of harmful short-term flows in greater detail and analyze the possibility to mitigate its detrimental effects by means of capital controls or reserve holdings. Our analysis leads to the following four main results.

1. *Higher vulnerability due to short-term capital flows is a symptom of unfavorable fundamentals.* The theoretical approach that emphasizes the detrimental effects of short-term capital flows is the so-called “*panic view*” of international financial crises. It was put forward by Cole and Kehoe (1996), Sachs and Radelet (1998), Rodrik and Velasco (1999), Chang and Velasco (2000), or Chang and Velasco (2001), and reinterprets international financial crises as variants of banking panics. This perspective identifies the combination of illiquid investments with a short liability maturity structure and the resulting liquidity and maturity mismatches as a key factor behind international financial crises. Crises are then brought about by a panic on the side of short-term investors that causes them to withdraw their funds. The resulting sudden stop and the ensuing crisis vindicates the initial belief, which is why one speaks of *self-fulfilling panics*. Clearly, a sufficiently short maturity structure is a pre-condition for such a crisis to occur. The panic view leaves unsettled, however, *why* the panic occurs. After all, being vulnerable to a sudden stop (due to a short maturity structure) is still different from actually experiencing such a sudden stop. If the beliefs that give rise to the panic are not explicitly determined, it becomes impossible to identify those conditions which *ceteris paribus* induce a higher likelihood of a crisis.³ More specifically, a crucial problem of the “panic view” is its exclusive focus on the potential illiquidity of an otherwise solvent borrower, while the demur of Goodhart (1999, p. 345) that “[...] illiquidity implies at least a suspicion of insolvency” is disregarded. In reality it may be difficult, if not impossible, to disentangle default due to illiquidity from default due to insolvency. But the distinction is still theoretically instructive, since variations in the maturity structure or the ratio of reserve holdings to illiquid investments can have opposite effects on the liquidity and the solvency position of a borrower, hence on the creditor’s assessment thereof. Let us clarify the intuition behind this with a short example.

Consider a borrower who has to decide on the maturity structure of her debt, which she takes on to finance a long-term investment. Assume that the investment is illiquid, and suppose further that long-term debt comes at a premium. If the debtor would raise the average maturity of her debt, she might be less exposed to panic runs, and thus she would have a higher chance of surviving any roll-over date. But now (given that she manages to refinance any other short term debt as planned) she needs to fetch higher returns on her investments because long-term debt

³This point of criticism is equivalent to what macroeconomists usually refer to as the ‘Lucas critique’. Any comparative statics and policy recommendations based on such models may be misguided as they do not take into account how agents’ behavior depends on the structure of the economy.

is relatively more expensive than short-term debt. In sum, while long-term debt reduces the likelihood of becoming illiquid, the likelihood of becoming insolvent increases. The relevance of this trade-off is real for economies that are able to borrow long term only against a substantial term premium.

As the example shows, the effect of changes in the maturity structure on the total probability of default is not clear-cut. Rather, it depends on the weights that are attached to the probabilities of insolvency and of illiquidity in deriving the total probability of default. These weights are functions of structural parameters and they must be determined as the equilibrium outcome of a model which explicitly specifies the beliefs of short-term claimants. The beliefs in turn must account for the fact that the likelihood of future insolvency already influences every creditor's present decision to roll over or not. The probability of illiquidity is tied to the probability of insolvency and thereby to the fundamental return process which determines the value of bank's assets and its net worth. As the previous panic view models fail to explicate agents' beliefs, they are not suitable to model these aspects.

These considerations explain why we believe that if short-term capital flows are indeed heightening an economy's vulnerability, then this is to be viewed as a symptom rather than a cause. In such a case, the weight that creditors put on the probability of illiquidity is relatively large and exceeds the weight that is attached to the probability of becoming insolvent. But whether this occurs is not so much a question of the level of short-term exposure but rather is determined by fundamentals such as asset return volatility, risk and liquidity premia, and spreads, or seigniority of claims.

2. *Accumulating reserve holdings is only sometimes a sensible advise.* As observed by Detragiache and Spilimbergo (2004) the panic view has led to policy recommendations that propose to minimize the exposure of debtors to a self-fulfilling panic by hoarding international reserves or by restricting inflows of foreign capital. A prominent example is the Greenspan-Guidotti rule that states that the ratio of reserve holdings to short-term debt should equal unity. However, the accumulation of liquid reserve holdings is associated with a trade-off similar to the one described above for a short maturity structure. Liquid assets generally yield lower returns. An economy that invests its foreign debt into liquid assets may push down the risk of experiencing an illiquidity crisis. Yet, when the cost of foreign borrowing are relatively higher than the returns from liquid assets, the risk of becoming insolvent increases. Thus, whether or not the accumulation of reserve holdings is beneficial depends also on the underlying fundamental parameters. Theoretically, it is possible that parameter combinations exist under which short-term capital becomes detrimental while at the same time an increase in reserve holdings also induces a higher likelihood of a crisis. While already Greenspan (1999) or Eichengreen (2004a) make this point, it has so far not received further attention in the literature of self-fulfilling panics.

3. *Capital controls can sometimes be welfare-enhancing.* Is it a sensible policy to restrict the inflow of foreign capital by means of controls? As Eichengreen (2004a, p.290) explains, the problem is “(...) not whether or not to live with international capital flows; rather it is how to tame them”. Accordingly, an optimal capital control would balance the benefits of additional investments financed through higher foreign borrowing against the risk of higher susceptibility to financial crises. This view belongs to what Eichengreen (2004a) calls the ‘messy middle’.⁴ While it is rather hard to believe that foreign capital should have no effect whatsoever on the development of emerging markets, it is at least as difficult to believe that capital account liberalization is always benign (Eichengreen (2004b)). We show below that if short-term capital is detrimental, the imposition of short-term inflow controls is a rather sensible tool that enhances the welfare of the economy. This is somewhat in line with the conclusions by Ostry et al. (2010, p.15) that “(...) there may be circumstances in which capital controls are a legitimate component of the policy response to surges in capital inflows”.

4. *An economy may end up in a self-aggravating trap where it can only borrow short-term.* Our fourth result provides some theoretical underpinning to a point made by Detragiache and Spilimbergo (2004, p. 18), “that more crisis-prone countries are more likely to borrow short-term”. We show that if short-term capital flows contribute to an economy’s vulnerability, then this may lead in turn to self-aggravating situations where investors mainly invest short-term because of the high likelihood of a crisis that is then partly due to high short-term indebtedness. This does not stand in contrast to our first result, as such situations require a particular incidence of fundamental factors. Yet, causality goes both ways. Fortunately, in such situations, policy measures such as higher reserve holdings or capital controls may be associated with a multiplier or feedback effect. Once a policy measure lowers the risk of a crisis, investors’ incentives to lengthen their maturity structure rise, thereby further lowering this risk.

The remainder of the paper is structured as follows. In section 2 we present an international bank run model which is based on the global game bank run models by Morris and Shin (2009), Goldstein and Pauzner (2005), and Rochet and Vives (2004) and augment it with heterogeneous lenders by using results of Steiner and Sákovics (2010).⁵ We show in section 3 how the model allows for the separation of the effects of reserves and maturity structure on the equilibrium and thereby on the likelihood of a panic. In section 4 we discuss welfare aspects of two quantity control measures (a restriction on total inflows, and a restriction on short-term inflows only). In this section we fix the maturity preferences of foreign investors and do not allow for feedback between the likelihood of a crisis and the fraction of short-term debt. This assumption is

⁴The extreme positions are taken by: on the one hand the proponents of efficient markets who believe that international financial liberalization brings about an efficient allocation of scarce resources and always contributes to and enhances economic development (e.g. Lucas (1990)); on the other hand those who believe that capital flows do not exert any effect on growth and development but rather have a detrimental destabilizing effect on borrowing countries (e.g. Rodrik (1998)).

⁵Takeda (2001) also considers a global game international bank run model, but he focuses on the investment and consumption decisions of domestic agents and neither on foreign creditors’ decisions to roll over debt, nor on the influence of the maturity structure of debt.

suitable for a small emerging market economy whose influence on world interest rates or investor maturity preferences is negligible. We relax this in section 5 where we endogenise the supply of short-term capital in order to derive a simple condition which the term structure of interest rates has to meet in order for a positive supply of long-term capital to exist. However, multiple equilibrium levels of short-term debt can arise which may lead to a self-aggravating trap: foreign creditors become increasingly unwilling to lend long-term because of the high likelihood of default induced by short-term indebtedness. This fact raises the probability of a crisis and vindicates the initial reluctance to lend long-term. Section 6 concludes.

2. The Model

We follow the trend in the literature and conduct our analysis of an emerging market crisis within an open-economy bank-run model. As pointed out by Chang and Velasco (2001), modeling the situation of an emerging market by means of a banking model is sensible because of two reasons. Firstly the banking sector is more important (compared to other credit mechanisms) in emerging economies than in mature economies. Secondly, illiquidity issues play a larger role for such countries because of their limited access to international financial markets and because of less developed domestic financial institutions. The openness of the economy is reflected in our model through the heterogeneity of the creditors. This is a useful approximation of an emerging market economy – e.g. China or Vietnam – where the domestic population has only access to limited financing arrangements, often under conditions dictated by the government, while additional foreign funding is to be obtained on international capital markets under conditions that are set by the market. Moreover, many emerging market crises of the recent past have been associated with banking crises that often predated the breakdown of a currency peg.⁶ From this “twin-crisis” perspective, the model can reflect the beginning of an emerging market crisis, that may end outside the scope of our considerations with the breakdown of a currency peg.

§2.1. Economic Environment

Consider a small open economy with three periods indexed by $t \in \{0, 1, 2\}$. There exist two groups of agents, domestic depositors (indexed with subscript d) and foreign investors (indexed with subscript f). We assume that domestic depositors are present in measure $\omega \in (0, 1)$, whereas foreign investors are present in measure $(1 - \omega)$. Agents in both groups are risk-neutral and they want to consume at either date 1 or date 2.

There exists a single good in the economy which can be used for consumption and for investment purposes. As our focus is on liquidity and maturity mismatches, we abstract from any exchange rate considerations and assume that at date 0 each agent in each group receives

⁶cf. Kaminsky and Reinhart (1999)

an initial endowment of 1 unit of the good. Agents receive no endowment at subsequent dates. Without an appropriate technology the endowments perish. We assume that neither domestic nor foreign agents have direct access to the economy's investment and storage technologies. In combination with the preference specifications and the endowment process this creates a need to invest.

At date 0, agents can place their endowments with a banking sector which then invests these into investment projects or stores it. Storing one unit of the good at date 0 provides the bank with immediate access to one unit at either date 1 or date 2. Thus, it creates safe liquidity. We denote the amount of stored funds by ϱ and refer to it as reserves.

The investment technology is risky and illiquid. Following Morris and Shin (2009), we assume that the investment returns are described by the stochastic process

$$\theta_1 = \theta_0 + \sigma_1 \varepsilon_1,$$

$$\theta_2 = \theta_1 + \sigma_2 \varepsilon_2,$$

where θ_0 is fixed and ε_1 and ε_2 are independently distributed random variables drawn from a standard normal distribution. We henceforth denote the c.d.f. of the standard normal distribution by $\Phi(\cdot)$ and its p.d.f. by $\Phi'(\cdot)$.⁷ We henceforth refer to the size of the scale parameter σ_2 as the degree of fundamental uncertainty.

While assets pay out θ_2 at date 2, they only yield $\psi\theta_1$ at date 1. The parameter $\psi \in (0, 1)$ reflects the illiquidity of the asset. One can think of $(1 - \psi)$ as the haircut that is applied in domestic repo markets or at a central bank's window when the asset is pledged as collateral. Furthermore, as we think in terms of a banking sector rather than in terms of an individual bank, the size of ψ also reflects the liquidity and thickness of the domestic money market. We henceforth refer to $\psi\theta_1$ as the collateral value of the asset. The date 0 price of the asset is normalized to one and the amount of investment into the asset is denoted by y .

§2.2. Debt Contracts

The bank issues demand deposit contracts to domestic agents and offers short- and long-term bonds to foreign agents.⁸

⁷The distributional assumptions entail that θ_2 , conditional on θ_0 , has mean θ_0 and standard deviation $\sqrt{\sigma_1^2 + \sigma_2^2}$. At date 1, conditional on the realization of ε_1 , its mean is given by θ_1 and its standard deviation by σ_2 .

⁸The assumption that domestic agents have only access to a reduced set of possible financing arrangements is in line with Chang and Velasco (2000, 2001). It takes its realism from the fact that in most emerging economies that have been haunted by financial crises in the past decades the domestic population's access to financial instruments was severely limited (reflected e.g. in high lending-deposit-rate spreads). The producer-biased strategy of growth in such countries often stunted the development of the financial sector (see Rajan (2010)) while the needs for funds exceeded domestic resources and savings. When these countries turned to international capital markets to make up for the lack of funding, domestic banks had to accept borrowing conditions set by a market rather than through administration of their government.

Demand deposit contracts have a date 2 face value of w_d^2 units and contain the option to prematurely withdraw at date 1. If the option is exercised, the depositor receives a *safe* payment of $w_1^d > 1$. The assumption that the date 1 payment is safe highlights the precaution motive of depositors who withdraw. The choice to withdraw or not is equivalent to trading off risky, higher returns at a later date against safer, but lower returns at an earlier date. We emphasize this trade-off by assuming the early payoff is always safe.

With respect to international borrowing, we follow Chang and Velasco (2001) and assume that foreign funds are in perfect elastic supply and that the bank faces a credit ceiling which we set equal to $(1 - \omega)$ without loss of generality. A fraction $\varphi \in (0, 1)$ of these funds is borrowed short-term. For now φ is treated as exogenous, but this will be relaxed below when we endogenize the foreign supply of short-term debt.

Foreign holders of short-term bonds can refuse to roll over at date 1. In that case claimants receive a safe payment of $w_1^f > 1$. If they decide to roll over the date 2 value of their claims is given by $w_f^2 > w_f^1$.

A long-term bond does not contain the option to prematurely roll off at date 1. The face value of long-term claims is given by $w_l > w_f^2$ which implies that long-term debt comes at a premium. We will rationalize this assumption when we endogenize the debt supply in section 5 below.

In case that the bank defaults, its investments are liquidated and the remaining claimants of group g only get hold of the liquidation value of the bank's assets. We denote group g 's claim on the legal estate by ℓ_g and assume that $\ell_g \leq \min\{w_d^1, w_f^1\}$ for $g \in \{d, f\}$.

§2.3. Bank Default

The bank's date 0 balance sheet constraint is given by

$$(1) \quad y + \varrho = \omega + (1 - \omega) = 1.$$

Its date 2 net worth, conditional on all domestic and foreign agents having rolled over at date 1, is given by

$$\varrho + \theta_2 y - \omega w_d^2 - (1 - \omega)[\varphi w_f^2 - (1 - \varphi)w_l].$$

If date 2's net worth becomes negative, the bank is said to be insolvent. The *solvency bound* is given by

$$\theta^s = \frac{\omega w_d^2 + (1 - \omega)[\varphi w_f^2 + (1 - \varphi)w_l] - \varrho}{y},$$

which, by substituting for y from equation (1), can be expressed as a function of reserves ϱ and foreign short-term debt φ ,

$$(2) \quad \theta^s(\varrho, \varphi) = \frac{\omega w_d^2 + (1 - \omega)[\varphi w_f^2 + (1 - \varphi)w_l] - \varrho}{1 - \varrho}.$$

Hence, the bank is declared insolvent whenever

$$\theta_2 < \theta^s(\varrho, \varphi).$$

Besides becoming insolvent at date 2, the bank can fail at date 1 due to (interim) illiquidity. This occurs when the liquidity at its disposal is not sufficient to meet aggregate withdrawals of its creditors. The *liquidity pool* of the bank at date 1 is given by

$$\varrho + \psi y \theta_1,$$

which we write as a function of reserves ϱ by using (1),

$$(3) \quad L(\varrho, \theta_1) = \varrho + \psi(1 - \varrho)\theta_1.$$

We will subsequently call $L(\varrho, \theta_1)$ the bank's *liquidity bound*.

The bank fails at date 1 when the value of interim roll-offs exceeds its liquidity bound. Let λ denote the fraction of domestic depositors who withdraw and let κ be the fraction of foreign short-term debt holders who refuse to roll over at date 1. Date 1 failure occurs whenever

$$(4) \quad \omega \lambda w_d^1 + (1 - \omega) \kappa \varphi w_f^1 > L(\varrho, \theta_1).$$

The solvency bound $\theta^s(\varrho, \varphi)$ and the liquidity bound $L(\varrho, \theta_1)$ are important quantities for our analysis.

The solvency bound is the minimal return that the bank needs to obtain at date 2. It is an increasing function of ϱ because reserve holdings do not earn the returns which the bank needs to pay off its liabilities. Furthermore, it is a decreasing function of φ because short-term debt is cheaper than long-term debt, and the bank's total liability burden is lower when the average maturity structure is shorter.

The liquidity bound is a measure for the bank's capacity to withstand interim withdrawals. It is a function of reserves. The liquidity bound increases with rising reserves when the collateral value falls below unity. We guarantee this property in equilibrium by imposing a regularity condition on the return structure (see lemma 2 below). Larger reserves then strengthen the bank's capacity to withstand interim outflows of funds.

§2.4. The Roll-Over Problem

The question about the likelihood of a liquidity crisis is tantamount to the question whether short-term creditors and depositors exercise their option to withdraw at date 1.

At date 1, the probability of date 2 insolvency (conditional on the realization of θ_1) is given by $\Phi\left(\frac{\theta^s - \theta_1}{\sigma_2}\right)$. Hence, for an agent of group $g \in \{d, f\}$ who compares the expected payoffs of rolling over or not, withdrawing becomes the dominant action whenever

$$\theta_1 < \theta^s - \sigma_2 \Phi^{-1}\left(\frac{w_g^2 - w_g^1}{w_g^2 - \ell_g}\right) =: \underline{\theta}_g.$$

All depositors and short-term creditors will roll-off whenever

$$\theta_1 < \min \{ \underline{\theta}_d, \underline{\theta}_f \} =: \underline{\theta}.$$

Henceforth we refer to the interval $(-\infty, \underline{\theta}]$ as the *lower dominance region*.

Each agent's payoff from rolling over depends also on the number of others who roll over. Due to the illiquidity of the bank's assets, it can happen that already a small number of withdrawals is sufficient to exhaust the bank's liquidity pool and force it into default at date 1. In such cases every agent would clearly prefer to withdraw. Coordination between the agents is thus essential for the bank's interim survival - illiquidity is the result of a coordination failure. However, from equation (4) follows that illiquidity does not constitute a problem whenever

$$\theta_1 > \frac{\omega w_d^1 + (1 - \omega)\varphi w_f^1 - \varrho}{\psi(1 - \varrho)} =: \bar{\theta}.$$

The interesting and relevant⁹ case occurs when $\underline{\theta} < \bar{\theta}$. For any $\theta_1 \in (\underline{\theta}, \bar{\theta})$, agents face a situation of *strategic uncertainty* because the decisions of other agents affect the likelihood of the bank becoming illiquid, thus affect each agent's own payoff and hence her decision.

A sufficient condition for $\underline{\theta} < \bar{\theta}$ to hold is

$$(5) \quad \omega w_d^1 + (1 - \omega)\varphi w_f^1 > L(\varrho, \theta^s).$$

This condition says that the bank would be unable to fully cover its debt at date 1 when it is expected to become insolvent at date 2.¹⁰ In what follows we assume that condition (5) holds and we refer to the interval $[\bar{\theta}, \infty)$ as the *upper dominance region*.

Without an explicit formulation of how agents assess the likely behavior of others, the case $\theta_1 \in (\underline{\theta}, \bar{\theta})$ gives rise to a situation reminiscent of the results of Diamond and Dybvig (1983), or what has been labeled by Sachs and Radelet (1998) or Tirole (2002) the "panic-view" of financial crises. Different (arbitrarily imposed) sentiments lead to different collective actions, which in turn give rise to exactly those outcomes which were initially anticipated. Because multiple outcomes are consistent with any given balance sheet and payoff structure, this view cannot capture how the model's parameters might eventually shape the beliefs of agents.

In order to circumvent these problems we rely on the global game technique to specify short-term claimants' beliefs and to derive a unique equilibrium of the creditor coordination at date 1.

To this end, we assume that the shock realization ε_1 is not common knowledge among short-term claimants. Before making her decision to withdraw or not, each agent $i \in [0, \omega + \varphi(1 - \omega)]$ receives some precise information about the true state θ_1 . This information leads her to believe that the true value of θ_1 is distributed around some $x^i \in \mathbb{R}$. This x^i is henceforth called

⁹The characterization of the roll over decisions and the resulting outcome at date 1 were trivial if $\underline{\theta} > \bar{\theta}$. All claimants would roll over for $\theta_1 > \underline{\theta}$ because they could be certain that θ_1 would be so large that the bank would never face any liquidity shortage.

¹⁰Note that $\mathbb{E}(\theta_2 | \theta^s) = \theta^s$.

the signal of the agent. The signal errors, given by $x^i - \theta_1$, are independently and normally distributed with common mean zero and standard deviation τ_1 .

A strategy for typical agent i is defined as a decision rule $s^i : x^i \mapsto a^i$, that associates a decision $a^i \in \{\text{withdraw, roll over}\}$ with each possible signal x^i . Domestic and foreign agents use joint threshold strategies if domestic depositors withdraw if and only if their signal x^i falls below a threshold value x_d^* , and if foreign creditors refuse to roll over if and only if their signal x^i falls below some threshold value x_f^* . A joint strategy profile (x_d^*, x_f^*) constitutes a Bayes-Nash-equilibrium point of the model if no agent can improve her expected payoff by unilaterally deviating to a different strategy. We call the agents who receive a signal exactly equal to x_g^* the *critical* agents.

As the following proposition shows, the coordination game between the short-term creditors has a unique (Bayes-Nash-)equilibrium in threshold strategies if and only if the signals become sufficiently precise.

Proposition 1. *For sufficiently small τ_1 , there exists a unique equilibrium point in joint threshold strategies. Default at date 1 occurs for any $\theta_1 < \theta^*$. Otherwise liquidity is sufficient to continue until date 2. Since τ_1 is small,*

$$(6) \quad x_g^* \rightarrow \theta^*, \quad g \in \{d, f\}.$$

The equilibrium threshold θ^* is given by the solution to

$$(7) \quad \sum m_g w_{1,g} o_g = \Phi \left(\frac{\theta_1 - \theta^s}{\sigma_2} \right) (\varrho + \psi(1 - \varrho)\theta_1),$$

where $m_d := \omega$, $m_f := \varphi(1 - \omega)$, and $o_g := \frac{w_{1,g} - \ell_g}{w_{2,g} - \ell_g}$.

Proof. See Appendix. □

3. Comparative Statics

§3.1. Illiquidity versus Insolvency

We explained at the outset that variations in maturity composition and reserve holdings can either raise or lower the vulnerability of the economy. Vulnerability in this context is synonymous to the ex ante probability of a crisis.¹¹ The latter is, conditional on θ_0 , given by

$$p_{IL} := \mathbf{Pr}(\theta_1 < \theta^* | \theta_0) = \Phi \left(\frac{\theta^* - \theta_0}{\sigma_1} \right).$$

According to this definition, the probability of a liquidity crisis includes cases where funds are withdrawn because the bank is considered to be insolvent at the subsequent date. Morris and Shin (2009) exclude such cases from their definition of illiquidity. Thus, they refer to illiquidity

¹¹cf. Furman and Stiglitz (1998, p. 6).

only in case that the bank is fully solvent at date 2 and yet defaults at date 1 due to a pure mis-coordination of beliefs. We prefer the definition above because it is impossible in reality to ex post determine whether a bank is unable to roll over short-term debt because it would have been virtually insolvent, or whether it would have been solvent would it not have been denied credit. Agents might deny to roll over for different reasons: either because they fear that the bank becomes insolvent, or because they fear that too many other agents withdraw, or because of a combination of both; but the default at date 1 has only one cause, namely the bank's lack of liquidity.¹²

The impossibility to ex post determine whether a defaulted bank would have been solvent or insolvent does not imply that the creditors' ex ante assessment of the bank's solvency situation is unimportant. In contrast, everything that affects the bank's solvency also affects the bank's liquidity, because illiquidity is to some extent conditional on agents putting a sufficiently high probability on the bank being subsequently insolvent. Charles Goodhart's (1999) remark that illiquidity implies a suspicion of insolvency is reflected in our model insofar as the threshold θ^* is a function of the solvency bound θ^s (cf. equation (7)).

As already explained in the introduction, parameter variations can bring about entirely opposite effects on the solvency bound and on the liquidity bound. And as the probability of illiquidity is affected by both of these bounds, it is a priori not clear how such variations translate into a change of the probability of illiquidity. To highlight this issue, we introduce the following terminology. We say that a parameter causes a *solvency effect* when a change in this parameter alters the probability p_{IL} through a change in the solvency bound θ^s . Similarly, a parameter is said to cause a *liquidity effect* when its effect on p_{IL} is brought about by a change in the capacity to withstand a run.¹³ We call effects which decrease $(1 - p_{IL})$ negative, and those that increase $(1 - p_{IL})$ positive. Choosing the terminology in this way ensures that it corresponds to what would be intuitively a "positive" or "negative" outcome.

Since φ and ϱ affect p_{IL} only through the threshold θ^* , and since p_{IL} is a strictly monotone function of θ^* , it suffices for the comparative statics to examine the signs of the derivatives $\partial\theta^*/\partial\varrho$ and $\partial\theta^*/\partial\varphi$.

¹²The discussion that surrounded the Asian financial crisis can be used to illustrate this matter. It is a well-documented empirical fact that the East Asian countries that were hit by the crisis all suffered from a sudden reversal of capital flows. But there is no universal agreement on the reason for this sudden stop. For example, Corsetti et al (1999) emphasize the "moral hazard" perspective and question the solvency of the affected economies. In contrast, Sachs and Radelet (1998) attribute the withdrawal to a self-fulfilling panic, i.e. to a pure mis-coordination of creditors' beliefs.

¹³I.e. a liquidity effect is either caused by a change in the RHS of equation (3) for a given LHS, or by a change in the LHS, for a given RHS.

Hence, the effect of a change in the fraction of short-term financing on the equilibrium threshold (and thus on the probability of a liquidity crisis) is given by

$$(8) \quad \frac{\partial \theta^*}{\partial \varphi} \geq 0 \Leftrightarrow \underbrace{-\frac{1}{\sigma_2} \Phi' \left(\frac{\theta^* - \theta^s}{\sigma_2} \right) \frac{\partial \theta^s}{\partial \varphi}}_{\text{Positive Solvency Effect}} \leq \underbrace{\frac{m_f w_f^1 o_f}{L(\varrho, \theta^*)}}_{\text{Negative Liquidity Effect}}.$$

Short-term capital flows are associated with a positive solvency effect, which is displayed on the left-hand side of the last inequality. Short-term capital is cheaper than long-term capital and the probability of insolvency therefore decreases with a shorter maturity structure. By contrast, the liquidity effect on the right-hand side is negative. An increase in short-term debt increases, for a given liquidity pool, the amount of possible date 1 claims and thus raises the risk of illiquidity.

Similarly, for the effect of a change in reserve holdings we find

$$(9) \quad \frac{\partial \theta^*}{\partial \varrho} \leq 0 \Leftrightarrow \underbrace{\frac{1}{\sigma_2} \frac{\Phi' \left(\frac{\theta^* - \theta^s}{\sigma_2} \right)}{\Phi \left(\frac{\theta^* - \theta^s}{\sigma_2} \right)} \frac{\partial \theta^s}{\partial \varrho}}_{\text{Negative Solvency Effect}} \leq \underbrace{\frac{(1 - \psi \theta^*)}{L(\varrho, \theta^*)}}_{\text{Liquidity Effect}}.$$

In this case, the solvency effect is negative. As reserves do not yield positive net returns, an increase in reserves moves the solvency bound up, thus increasing the probability of becoming insolvent. The sign of the liquidity effect is a priori not clear. If the collateral value of the asset at the critical threshold exceeds unity—i.e. if $\psi \theta^* \geq 1$ —an increase in reserves would reduce the available liquidity, thus rendering the liquidity effect negative. This is intuitive, since at the critical margin, the bank would be better off not to hold any reserves at all. In this case, the only equilibria that can exist are either equilibria where agents always run, or equilibria where the asset is super-liquid and the bank would never hold any reserves. To obtain meaningful equilibria and to rule out the case of a “super-liquid” asset we are imposing some restrictions on the return structure.

The asset is illiquid if the collateral value is smaller than unity, which is equivalent to $\psi \theta \leq 1$. From equation (7), a necessary condition for this to occur *in equilibrium* is

$$(10) \quad \sum w_g^1 m_g o_g < 1.$$

Furthermore, for sufficiently large terminal uncertainty we can actually prove a necessary and *sufficient* condition for the asset to be illiquid relative to cash reserves.

Lemma 2. *For sufficiently large σ_2 , $\psi \theta^* \leq 1$ if and only if $\sum w_g^1 m_g o_g \leq \mu := \Phi(0)$; moreover, θ^* tends to*

$$\tilde{\theta}^* = \frac{\sum w_g^1 m_g o_g - \mu \varrho}{\psi \mu (1 - \varrho)}.$$

Proof. See Appendix. □

The weights

$$o_g = \frac{w_g^1 - \ell_g}{w_g^2 - \ell_g}, \quad g \in \{d, f\}$$

are measures for the relative contribution of group g to the interim illiquidity of the bank. The numerators contain the safe gain from withdrawing, i.e. the difference between the safe payoff from withdrawing and the safe payoff in case of default. The denominators contain the net gain from rolling over that an agent obtains in case the bank does not default. When, say, $o_f > o_g$, the cost of rolling over for a foreign agent are higher compared to the potential gains than is the case for a domestic agent. Therefore foreign agents are more likely to withdraw early. This in turn means that the potential claims of group f against the bank are more likely to become effective withdrawals. Hence, each unit of claims of group f receives a higher weight. The lemma essentially relies on a restriction on the impact-weighted sum of date 1 claims. We can interpret this as the requirement that for both groups of agents the relative costs of rolling over do not become too large. If this condition would fail to hold, the critical agents can become indifferent between rolling over and withdrawing only if they would consider the collateral value to be above unity (the case of a super-liquid asset).¹⁴

From equations (8) and (9) follows that an increase in short-term capital flows and/or a reduction in reserve holdings increase the vulnerability if and only if the liquidity effects outweigh the respective solvency effects. Our first two results follow immediately. As can be seen from equation (8), whether or not short-term capital has the potential to raise the probability of illiquidity depends not so much on the level of short-term debt itself but rather on parameters such as the haircut, foreign interest rates or the degree of terminal uncertainty. Moreover, comparing equations (8) and (9) reveals that the parameter regions where liquidity effects dominate differ among these two equations. Hence, even if short-term capital is detrimental, one may not be sure whether raising reserves has a mitigating impact on the ex ante probability of a crisis.

But under what conditions will liquidity effects dominate solvency effects? A key parameter that causes both liquidity effects to dominate is the degree of terminal uncertainty σ_2 . The reason is straightforward. From the perspective of date 1, solvency is crucially dependent on the eventual realization of the random variable ε_2 . The marginal impact of parameter variations on the solvency bound is weighted, in essence, by σ_2 . When σ_2 gets large, the conditional probability of being solvent becomes less sensitive with respect to a change in the solvency bound. Agents attach a fairly constant probability to solvency, independently of any event that would potentially alter this probability. As the liquidity effects are not affected by σ_2 , one can find (finite) bounds such that for a σ_2 exceeding it, the liquidity effects always dominate. As we

¹⁴Another way of interpreting the o_g s is as a determinant for the relative risk tolerance of group g . If the ex ante likelihood of withdrawing is higher for group g than for group g' , we may say that agents of group g have a lower risk tolerance. We show in proposition 9 and corollary 10 in the appendix that the critical signal of group f lies above (below) the critical signal of group d whenever o_f is larger (smaller) than o_d . This implies that group f is more (less) likely to withdraw and therefore has a lower (higher) risk tolerance than group d .

already stressed in the introduction: If higher short-term capital flows heighten the vulnerability of the economy, then this should be rather viewed as a symptom of a particular combination of underlying fundamentals. In particular it may be a sign of high return volatility or uncertainty. In the remainder of the paper we will in particular focus on the case of dominating liquidity effects in order to further analyze the detrimental impact of short-term capital flows and how this impact can be mitigated by means of capital controls. To obtain clean analytical results, we will mainly work with the threshold $\tilde{\theta}^*$ that emerges when $\sigma_s \rightarrow \infty$.

4. The Resource Effect and Quantity Capital Controls

The question that we tackle in this section is whether it can be welfare-improving to impose controls on capital inflows in a situation where liquidity effects dominate. The answer is not clear-cut since even short-term capital inflows can have beneficial effects by raising the overall level of investment and by helping to fund the economy's development. We term this beneficial effect of capital inflows the *resource effect*. The resource effect was not present in the previous section's analysis since we conducted the discussion under the presumption of a fixed investment y . Yet, if there are less capital inflows, the overall investment scale decreases,¹⁵ which implies, for a given payoff structure, that the probability of a crisis may increase. We study the case of large terminal uncertainty, dominating liquidity effects, and thus the potential of short-term capital flows exerting a detrimental impact. We keep foreign investors' maturity preferences fixed throughout this section. The economy then tries to mitigate the impact of sudden roll offs by putting a cap on capital inflows. We compare different control policies according to their impact on the economy's welfare. We use the thresholds that result from one or the other particular policy as welfare measures: when the threshold becomes large, the probability of a crisis increases, thereby decreasing expected domestic consumption.¹⁶ Moreover, we then compare the different thresholds against the benchmark thresholds under autarky and under full long-term financing.

For large uncertainty, we have from lemma 2,

$$\lim_{\sigma_2 \rightarrow \infty} \theta^* = \frac{\sum_g w_g^1 o_g m_g - \mu \varrho}{\psi \mu (1 - \varrho)} =: \tilde{\theta}^*.$$

Next, consider the extreme case of autarky when the country is fully cut off from international capital markets. The country would never face a sudden roll off of foreign funds, but it also can only invest domestic resources of size ω . The autarky threshold becomes

$$\check{\theta}^* = \frac{w_d^1 \omega o_d - \mu \varrho}{\psi \mu (\omega - \varrho)}.$$

¹⁵If the shortfall from, say, short-term inflows is not compensated by an equivalent inflow of longer-term debt or equity.

¹⁶As we study the case with infinitely large uncertainty, the insolvency probability converges to 1/2 and remains insensitive to changes in parameters. We can therefore ignore its impact on expected consumption.

As a second benchmark, suppose that the country takes on long-term debt on international markets. The borrowing limit was set at $(1 - \omega)$. By setting $\varphi = 0$ in the expression for $\tilde{\theta}^*$, we obtain

$$\hat{\theta}^* = \frac{w_d^1 \omega o_d - \mu \varrho}{\psi \mu (1 - \varrho)}.$$

Clearly, we have $\hat{\theta}^* < \tilde{\theta}^*$, the autarky case is associated with a higher threshold and therefore a higher probability of a crisis and lower expected consumption than is the case of full long-term financing. This is due to the resource effect. Liabilities at date 1 exist only to the domestic agents, whereas the maximal amount of resources is employed due to long-term funding. This drives down the probability of experiencing a default due to illiquidity while the probability of experiencing a solvency crisis is unchanged because of highly volatile asset returns. We interpret the situation of full long-term funding as the first best in case of dominating liquidity effects.

In practice one may seldom encounter a situation where foreign investors' preferences for a particular maturity structure (summarized by φ) match the economy's first best $\varphi = 0$. For $\varphi > 0$ the threshold is shifted away from the first best benchmark. However, if the amount of short-term debt is not too large, the country may still do better than under autarky. Concretely, if

$$\varphi \leq \varphi^{\text{crit}} := \frac{w_d^1 m_d o_d - \mu \varrho}{(m_d - \varrho) w_f^1 o_f},$$

then some short-term debt is welfare-enhancing compared to the autarky situation.¹⁷

We study the welfare properties of quantity controls that put a ceiling on (i) all types of inflows, and (ii) only short-term inflows. A control on all types of inflows is tantamount to a reduction of size Δ on foreign funds. The impact of such a restriction on the date 0 balance sheet can be expressed as

$$y + \varrho = \omega + (1 - \omega - \Delta) = 1 - \Delta.$$

As we leave unchanged the proportions of short- and long-term capital, i.e. investor preferences, the respective weights of domestic and foreign short-term claimants are given by $m_d^c = \omega$ and $m_f^c = \varphi(1 - \omega - \Delta)$. The threshold becomes

$$\theta^c(\Delta) = \frac{\sum w_g^1 o_g m_g^c - \mu \varrho}{\psi \mu (1 - \Delta - \varrho)}.$$

For the second control type, when only a fraction $(1 - \delta)$ of short-term flows are admitted while long-term inflows can still flow in unrestricted, the date 0 balance sheet becomes

$$y + \varrho = \omega + (1 - \omega)(1 - \varphi + (1 - \delta)\varphi) = 1 - (1 - \omega)\delta\varphi.$$

¹⁷Note that we assume $\omega > \varrho$ throughout this section. Under autarky this assumption must always be true. While it need not be true when some foreign capital flows into the country, we nevertheless make the assumption to keep the different thresholds comparable.

The short-term claimants' weights are given by $m_d^{cc} = \omega$ and $m_f^{cc} = \varphi(1 - \delta)(1 - \omega)$. The respective threshold is given by

$$\theta^{cc} = \frac{\sum w_g^1 m_g^{cc} o_g - \mu \varrho}{\psi \mu (1 - \varrho - \varphi \delta (1 - \omega))}.$$

The question becomes whether and how inflow restrictions Δ or δ can make the country better off compared to either the autarky threshold, and / or the threshold that obtains under freely flowing capital.

It is intuitive that a control on all types of assets can never lead to an improvement above and beyond the autarky situation. Suppose that $0 < \varphi < \varphi^{\text{crit}}$. Unrestricted inflows constitute the second best, the economy is better off than under autarky, albeit some short-term debt pushes it away from the first best. Restricting total inflows in this case decreases total resources and must shift the threshold above $\tilde{\theta}^*$. Conversely, when $\varphi > \varphi^{\text{crit}}$, autarky constitutes the second best, short-term flows are highly detrimental and it might even be better to eliminate *all* inflows completely. From this perspective, controls on all types of inflows seem to be a rather crude measure. In contrast, short-term controls seem to be a more suitable measure as they attack the core problem of sudden stops and at the same time do not affect the resource effect of long-term funds. To get a grip on how controls on short-term debt work, suppose first that $\tilde{\theta}^* > \check{\theta}$. Is it possible to improve upon the autarky situation? Yes, as one can always set $\delta = 1$, thereby prohibiting all short-term inflows, and still secure long-term funds of $(1 - \omega)(1 - \varphi)$. Now consider a situation where $\tilde{\theta}^* < \check{\theta}$. A marginal increase in δ causes a marginal decrease in interim liabilities as well as a marginal decrease in total available resources. Hence, the impact on $\tilde{\theta}^*$ remains ambiguous. Yet, the respective impacts of these marginal changes on the threshold are dependent on φ , implying that there may exist a particular value φ^{cc} such that $\theta^{cc} > \tilde{\theta}^*$ for any $\varphi > \varphi^{cc}$. Interestingly, we find that the critical φ^{cc} lies outside the range of admissible values for φ , implying that the marginal benefit of decreasing interim liabilities always outweighs the marginal cost of reducing available resources. Quantity controls on short-term inflows can always improve the welfare relative to a situation of freely flowing capital.

The following lemma summarizes the discussion.

Proposition 3. Quantity control on total inflows: *For any $\varphi \in (0, \varphi^{\text{crit}})$, a situation of unrestricted capital inflows constitutes the second best since $\theta^c(\Delta) > \tilde{\theta}^*$ for any $\Delta \in (0, 1 - \omega)$.*

For any $\varphi \in (\varphi^{\text{crit}}, 1)$, the autarky level constitutes the second best, since $\check{\theta} \leq \theta^c(\Delta) < \tilde{\theta}^$ for any $\Delta \in (0, 1 - \omega)$. Restricting total inflows by setting $\Delta = (1 - \omega)$ restores the autarky level.*

Quantity control on short-term inflows: *For any $\varphi \in (0, 1)$, a control on short-term inflows leads to a welfare improvement compared to unrestricted capital flows since $\tilde{\theta}^* > \theta^{cc}(\delta)$ for any $\delta \in (0, 1)$.*

Moreover, a short-term restriction δ such that

$$\delta \geq \check{\delta} := \frac{\varphi - \varphi^{\text{crit}}}{\varphi(1 - \varphi^{\text{crit}})},$$

leads to a welfare improvement compared to the autarky level.

Proof. See appendix. □

The assumption that we employed in this section may approximate the situation of a small emerging market economy that takes prices and investor preferences as given without being able to exert any influence whatsoever on these variables. The next section discusses the relaxation of this assumption.

5. Endogenous Maturity Structure

§5.1. The Creditor Side

The maturity structure which eventually prevails in equilibrium depends crucially on the ex ante decision by foreign creditors to invest into short-term or long-term debt. In this section, we endogenize the maturity structure. Short-term debt contains the option to withdraw early. This option has a positive value since it makes lending less risky. To be in positive supply, long-term bonds must earn a sufficiently high premium. This reveals a potential trap: as the maturity of debt shortens, the probability of a crisis may increase so much that, from the perspective of foreign agents, a switch to less riskier, short-term debt becomes justified for a given term premium. Hence, there may be multiple consistent combinations of maturity structure and likelihood of a crisis. We will show below that if liquidity effects dominate solvency effects, it is indeed possible that multiple consistent maturity structures exist.

Chang and Velasco (2001) determine term and maturity structure simultaneously by taking the perspective of the borrower. By contrast, here, we consider the problem from the perspective of a lender who ultimately must be indifferent between the two forms of debt. Assume foreign investors have the choice between investing in long-term or short-term debt shortly prior to date 0. Short-term debt provides them with the option to prematurely withdraw at date 1, while long-term debt does not provide this option. We assume that when making their investment, foreign agents do not know the initial state of the economy, summarized by θ_0 . However, they receive a signal given by

$$x_0 = \theta_0 + \tau_0 \eta_i,$$

where the η_i are independently and standard normally distributed and τ_0 is a positive scale parameter. Given their signals, agents calculate the ex ante probabilities of illiquidity and insolvency and use these to compute the expected payoffs from long- and short-term debt respectively. We use the following abbreviations: $p_{IL}(x_0)$ denotes the ex ante probability of the bank becoming illiquid (conditional on signal x_0), $p_{L\wedge S}(x_0)$ is the ex ante probability of the bank being liquid *and* solvent (conditional on signal x_0), i.e. the probability of the bank surviving up and until the end of date 2, $p_{L\wedge IS}(x_0)$ is the ex ante probability of the bank being

liquid at date 1 and becoming insolvent at date 2, and $p_{IL\vee IS}(x_0)$ is the probability of either being illiquid *or* being insolvent (conditional on signal x_0). For notational simplicity we will omit explicit reference to the argument x_0 whenever possible. Using this notation, the expected payoff from investing into short-term debt can be written as

$$p_{IL}w_{1,f} + p_{L\wedge S}w_{2,f} + p_{L\wedge IS}\ell_f = p_{IL}(w_{1,f} - \ell_f) + p_{L\wedge S}w_{2,f} + p_{IL\vee IS}\ell_f.$$

The expected payoff from investing into long-term debt is given by

$$p_{L\wedge S}w_l + p_{IL\vee IS}\ell_f.$$

Investors who receive the signal x_0^* are indifferent between the two forms of debt, where x_0^* solves

$$p_{IL}(x_0)(w_{1,f} - \ell_f) + p_{L\wedge S}(x_0)w_{2,f} + p_{IL\vee IS}(x_0)\ell_f = p_{L\wedge S}(x_0)w_l + p_{IL\vee IS}(x_0)\ell_f.$$

Rewriting this gives the indifference condition

$$(11) \quad p_{IL}(x_0^*)(w_{1,f} - \ell_f) = p_{L\wedge S}(x_0^*)(w_l - w_{2,f}).$$

The probabilities are continuous functions of x_0 . And since the expected payoff from investing into short-term debt is decreasing in x_0 whereas the payoff from investing long-term is increasing in x_0 , there exists a unique intersection x_0^* that solves equation (11). Agents who receive a signal $x_0 < x_0^*$ will invest into short-term debt, while agents with a signal $x_0 > x_0^*$ will invest into long-term debt. Given that the critical signal x_0^* is a function of the parameters of the model and thus is a function of the amount of short-term debt φ , we can now write the foreign creditors' supply of short-term debt as

$$(12) \quad \varphi = \mathbf{Pr}(x_0 < x_0^*(\varphi) | \theta_0) = \Phi\left(\frac{x_0^*(\varphi) - \theta_0}{\tau_0}\right).$$

Any fixed point of equation (12) constitute an equilibrium supply of short-term debt.

§5.2. An Ex-Ante Trap

However, it is possible that multiple supply levels exist. A necessary and sufficient condition for the latter phenomenon to occur would be that the slope of $\mathbf{Pr}(x_0 < x_0^*(\varphi) | \theta_0)$, when evaluated at some φ^* such that $\theta_0 = x_0^*(\varphi^*)$, exceeds unity (see figure 1). The slope is given by

$$\frac{\partial \mathbf{Pr}(x_0 < x_0^*(\varphi) | \theta_0)}{\partial \varphi} = \Phi'\left(\frac{x_0^*(\varphi) - \theta_0}{\tau_0}\right) \frac{1}{\tau_0} \frac{\partial x_0^*(\varphi)}{\partial \varphi},$$

and the necessary and sufficient condition for multiple levels of short-term debt, consistent with the *unique* threshold θ^* , becomes

$$\left. \frac{\partial x_0^*(\varphi)}{\partial \varphi} \right|_{\varphi=\varphi^*(\theta_0)} > \tau_0 \sqrt{2\pi}.$$

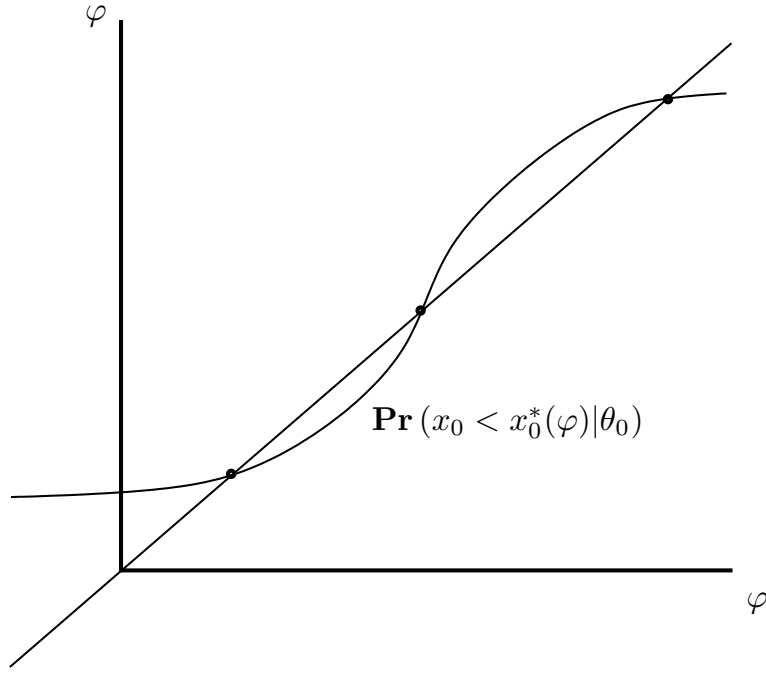


FIGURE 1. Multiple fixed points in the maturity structure of debt

Since the right-hand side is positive, the latter condition only holds if $\frac{\partial x_0^*(\varphi)}{\partial \varphi}$ is positive as well. The following lemma shows that this is true, if $\sigma_2 \gg 0$.

Lemma 4. *The derivative of the signal x_0^* with respect to φ is given by*

$$\frac{\partial x_0^*(\varphi)}{\partial \varphi} = (1 - \alpha(\sigma_2)) \frac{\partial \theta^*}{\partial \varphi} + \alpha(\sigma_2) \frac{\partial \theta^s}{\partial \varphi}.$$

For sufficiently large $\sigma_2 \gg 0$, we have $\alpha(\sigma_2) \approx 0$, and thus

$$\left. \frac{\partial x_0^*(\varphi)}{\partial \varphi} \right|_{\varphi=\varphi^*(\theta_0)} \approx \left. \frac{\partial \theta^*}{\partial \varphi} \right|_{\varphi=\varphi^*(\theta_0)} > 0.$$

Proof. See Appendix. □

Hence, the effect of a marginal increase in short-term capital flows on the probability with which an investor supplies short-term debt is given by a weighted average of the marginal effects of short-term debt on the threshold θ^* and the solvency bound θ^s . Whenever the terminal uncertainty becomes sufficiently large, so that the probability of insolvency becomes sufficiently insensitive with respect to variations in the solvency bound, multiple short-term debt levels are consistent with the unique equilibrium θ^* . The economy may become stuck in an equilibrium with a high level of short-term debt since foreign investors are unwilling to lend long-term because of the high default probability induced by the large short-term indebtedness. As already noted by Detragiache and Spilimbergo (2004) in their empirical analysis of the impact of short-term debt, it is very well possible that crisis-prone countries are forced to borrow short-term, thereby

reversing the causality between short-term inflows and the probability of a crisis. Our result is a special case of a country trapped in a crisis-prone equilibrium due to short-term inflows.

§5.3. Policy Measures Revisited

We now revisit the policy measures that we have already discussed above. While we have seen above that reserve holdings and controls have some potential to mitigate the vulnerability of the economy for given maturity preferences of foreign investors, the more important question is how they impact whenever the maturity choice becomes endogenous. Our model allows for a simple treatment of this question.

§5.3.1. Accumulation of Reserves. There may exist a feedback from higher reserve holdings to the perceived likelihood of a crisis to changes in the maturity structure. When higher reserve holdings attenuate the economy's exposure to sudden stops, then rational investors' incentives to invest into long-term debt may rise, thereby reducing the fraction of short-term debt and further reducing the likelihood of a crisis. The following lemma shows that this is indeed correct in our model when short-term flows are detrimental.

Proposition 5. *The impact of reserve holdings on the critical ex ante signal x_0^* is given by*

$$\frac{\partial x_0^*(\varrho, \varphi)}{\partial \varrho} = (1 - \alpha(\sigma_2)) \frac{\partial \theta^*}{\partial \varrho} + \alpha(\sigma_2) \frac{\partial \theta^s}{\partial \varrho},$$

with $\lim_{\sigma_2 \rightarrow \infty} \alpha(\sigma_2) = 0$. Therefore, when $\sigma_2 \gg 0$, an increase in reserve holdings raises the fraction of long-term debt in any unique / stable equilibrium. The derivative of the supply function $\varphi^*(\theta_0)$ is given by

$$\frac{\partial \varphi^*}{\partial \varrho} = \frac{\Phi' \left(\frac{x_0^* - \theta_0}{\tau_0} \right) \frac{\partial x_0^*(\varrho, \varphi)}{\partial \varrho} \frac{1}{\tau_0}}{1 - \Phi' \left(\frac{x_0^* - \theta_0}{\tau_0} \right) \frac{\partial x_0^*(\varrho, \varphi)}{\partial \varphi} \frac{1}{\tau_0}},$$

which is negative for $\sigma_2 \gg 0$.

Proof. See Appendix. □

§5.3.2. Capital Controls. We pick up the analysis of capital controls from section 4 where we kept investors' maturity preferences exogenous. We now show how controls actually feed back on the maturity choice. As in section 4, we consider the case of large terminal uncertainty. For the extreme case of large uncertainty when $\sigma_2 \rightarrow \infty$ we obtain the following closed form expression for the critical signal x_0^* that renders the investors indifferent between short- and long-term capital,

$$x_0^* = \tilde{\theta}^* - \hat{\sigma} \Phi^{-1} \left(\frac{1}{\hat{w} + (1 - \mu)} \right).$$

Agents with a signal less than x_0^* invest short-term. The fraction of short-term debt is given by the solution to

$$\varphi = \Phi \left(\frac{\tilde{\theta}^* - \hat{\sigma} \Phi^{-1} \left(\frac{1}{\hat{w} + (1-\mu)} \right) - \theta_0}{\tau_0} \right).$$

Imposing a bound on capital inflows of size Δ on all inflows, or of size δ only on short-term inflows, changes the threshold to $\theta^c(\Delta, \varphi)$, respectively to $\theta^{cc}(\delta, \varphi)$. This affects the probability of default and thereby affects investors' maturity choice. Our findings in this section complement the results from section 4. In fact, whenever capital controls are helpful in pushing down the threshold, they immediately raise investors' incentives to take on long-term debt. This implies the existence of a feedback or multiplier effect that increases the effectiveness of controls. The total impact of controls on the threshold can be written as

$$d\theta^i = \left(\frac{\partial \theta^i}{\partial j} + \frac{\partial \theta^i}{\partial \varphi} \frac{\partial \varphi}{\partial j} \right) dj,$$

where $i = c, cc$ and $j = \Delta, \delta$. Hence, the feedback effect that stems from the endogenous response of investors is given by the rightmost term. Its sign crucially depends on the sign of $\partial \varphi / \partial j$. The following lemma provides the conditions under which this term is negative and a multiplier effect exists.

Proposition 6. *Quantity control on total inflows: A reduction of total inflows of size Δ causes investors to shorten their maturity structure in any unique / stable equilibrium where the equilibrium level φ^c exceeds φ^{crit} , and conversely.*

Quantity control on short-term inflows: A bound on short-term inflows of size δ causes investors to shorten their maturity profile in any unique / stable equilibrium.

Proof. See Appendix. □

6. Conclusion

We have analyzed the hypothesis that large short-term capital flows to emerging market economies raise the vulnerability of these countries to banking crises. We focused on four main points that we illustrated by means of a global game bank run model. Firstly, if short-term capital flows contribute to a heightened vulnerability, then this should be viewed as a symptom of underlying fundamental conditions, in particular as a result of large fundamental uncertainty. Secondly, raising reserve holdings may only sometimes be a sensible policy advice, yet it is in the particular case if the fundamental uncertainty is large. Thirdly, in such a situation, one can additionally employ quantity controls on capital inflows to improve the welfare of the economy. Fourthly, the endogeneity of the creditor's maturity composition revealed a potential deadly trap. As the maturity profile of debt becomes shorter, the probability of a crisis may increase so much that, from the perspective of foreign agents, a switch to less riskier, short-term debt

becomes justified for a given term premium. Hence, there may be multiple equilibrium-consistent combinations of maturity structure and likelihood of a crisis. One combination is characterized by a low amount of short term debt, and a low probability of a crisis; in another situation, the probability of a crisis is high, and consequently, most foreign creditors refuse to invest in anything else than debt with short maturity.

7. Appendix

Proof of Proposition 1. The proposition is proved by applying proposition 7 from Steiner and Sákovics (2010). We first show that our model matches the assumptions required for applying their result. The proof of our proposition then follows immediately.

We normalize payoffs so that they are identical to the payoff difference:

	Bank Survives	Bank Fails
Rollover (action 1)	$w_g^2 - w_g^1$	$\ell_g - w_g^1$
Withdraw (action 0)	0	0

Note that normalizing payoffs in this way does not change the underlying game. We define the aggregate action as:

$$a = \int_0^{m_d+m_f} w_{g(i)}^1 a_i di, \quad a_i \in \{0, 1\}, \text{ where } g(i) \text{ denotes } i\text{'s group.}$$

The bank survives date 1 if:

$$(13) \quad a \geq \sum m_g w_g^1 - L(\varrho, \theta_1).$$

We set $u(a, \theta_1) = 1$ if (13) holds and $u(a, \theta_1) = 0$ if not.

By using the normalized payoffs, we can write the payoff from rolling over for a typical agent of group g as

$$u_g(a, \theta_1) = \begin{cases} \bar{u}_g(a, \theta_1) := (1 - p(\theta_1))(w_g^2 - \ell_g) + \ell_g - w_g^1 & \text{if } u(a, \theta_1) = 1 \\ \underline{u}_g(a, \theta_1) := \ell_g - w_g^1 & \text{otherwise.} \end{cases}$$

where $p(\theta_1)$ is the probability of insolvency conditional on θ_1 .

Note that if we set $\beta_g(\theta_1) = (1 - p(\theta_1))(w_g^2 - \ell_g)$ and $\gamma_g = w_g^1 - \ell_g$ we can further write

$$u_g(a, \theta_1) = \beta_g(\theta_1) \cdot u(a, \theta_1) - \gamma_g.$$

The model is notationally equivalent to Steiner and Sákovics (2010). We have the following

Lemma 7. $\bar{u}_g(a, \theta_1)$ and $\underline{u}_g(a, \theta_1)$ satisfy assumptions **A0** - **A5** in Steiner and Sákovics (2010, p. 33 ff).

Lemma 8. *The equilibrium threshold is given by the solution to*

$$(14) \quad \sum m_g w_g^1 \frac{\gamma_g}{\beta_g(\theta_1)} = \int_0^{\sum m_g w_g^1} u(a, \theta_1) da.$$

Proof. See Steiner and Sákovics (2010, Proposition 7, p. 41). \square

Using equation (13), we rewrite equation (14) as

$$\sum m_g w_g^1 \frac{\gamma_g}{\beta_g(\theta_1)} = L(\varrho, \theta_1).$$

By substituting out $\beta_g(\theta_1)$, γ_g and $L(\varrho, \theta_1)$, and by multiplying both sides by $(1 - p(\theta_1))$, we obtain

$$(15) \quad \sum m_g w_g^1 \frac{w_g^1 - \ell_g}{w_g^2 - \ell_g} = \Phi\left(\frac{\theta_1 - \theta^s}{\sigma_2}\right) (\varrho + \psi(1 - \varrho)\theta_1). \quad \square$$

Proof of Lemma 2. We claim that, if σ_2 becomes large, $\Phi((\theta^* - \theta^s)/\sigma_2)$ tends to μ . Hence equation (7) holds if and hold only if $\mu \cdot (\varrho + \psi(1 - \varrho)\theta^*)$ tends to $\sum w_g m_g o_g$, and thus for large σ_2 , $\sum w_g m_g o_g \leq \mu$ if and only if $(\varrho + \psi(1 - \varrho)\theta^*) \leq 1$ if and only if $\psi\theta^* \leq 1$. The expression for $\tilde{\theta}^*$ follows readily.

To prove our claim, it suffices to show $(\theta^* - \theta^s)/\sigma_2 \rightarrow 0$ as $\sigma_2 \rightarrow \infty$. Towards a contradiction, let us suppose that $|\theta^* - \theta^s|/\sigma_2 > c > 0$ as $\sigma_2 \rightarrow \infty$ for some constant c . Since θ^s does not depend on σ_2 , we may choose c such that we have $|\theta^*|/\sigma_2 > c$, *viz.* $|\theta^*| > c\sigma_2$ as $\sigma_2 \rightarrow \infty$. First suppose θ^* is positive as $\sigma_2 \rightarrow \infty$. For sufficiently large σ_2 , we find

$$\Phi\left(\frac{\theta^* - \theta^s}{\sigma_2}\right) (\varrho + \psi(1 - \varrho)\theta^*) > \Phi(c) (\varrho + \psi(1 - \varrho)c\sigma_2) > \sum w_g m_g o_g,$$

and hence equation (7) cannot hold, contradicting that θ^* is a solution. So suppose θ^* is negative as $\sigma_2 \rightarrow \infty$. Since $\Phi(\cdot) > 0$, we find

$$\Phi\left(\frac{\theta^* - \theta^s}{\sigma_2}\right) (\varrho + \psi(1 - \varrho)\theta^*) < \Phi\left(\frac{\theta^* - \theta^s}{\sigma_2}\right) (\varrho - \psi(1 - \varrho)c\sigma_2) < 0,$$

hence equation (7) cannot hold, again contradicting that θ^* is a solution. \square

Proof of Lemma 4. From (11) we can define $x_0(\varphi)$ implicitly through

$$G(x_0, \varphi) := p_{IL}(x_0) \frac{w_{1,f} - \ell_f}{w_1 - w_{2,f}} - p_{L \wedge S}(x_0) = 0.$$

We have the following explicit expressions for the probabilities p_{IL} and $p_{L \wedge S}$.

For the probability of illiquidity,

$$p_{IL} = \mathbf{Pr}(\theta_1 \leq \theta^* | x_0) = \int_{-\infty}^{\frac{\theta^* - x_0}{\sigma}} \Phi'(z) dz,$$

where $\hat{\sigma} := \sqrt{\sigma_1^2 + \tau_0^2}$.

The probability of being liquid and solvent can be written as

$$p_{L\wedge S} = \mathbf{Pr}(\{\theta_1 > \theta^*\} \cap \{\theta_2 > \theta^s\} | x_0),$$

which, when the return process and the definition of the signal are used, can be written as

$$\mathbf{Pr}(\{\sigma_1\varepsilon_1 - \tau_0\eta_0 > \theta^* - x_0\} \cap \{\sigma_1\varepsilon_1 + \sigma_2\varepsilon_2 - \tau_0\eta_0 > \theta^s - x_0\})$$

Since all error terms are independently distributed, the joint density of $\sigma_1\varepsilon_1 - \tau_0\eta_0$ and ε_2 is given by

$$\Phi'(\sigma_1\varepsilon_1 - \tau_0\eta_0) \times \Phi'(\varepsilon_2).$$

We can thus write $p_{L\wedge S}$ as

$$\int_{-\infty}^{\frac{\theta^s - \theta^*}{\sigma_2}} \int_{\frac{\theta^s - x_0 - \sigma_2 t}{\hat{\sigma}}}^{\infty} \Phi'(z)\Phi'(t) dz dt + \int_{\frac{\theta^s - \theta^*}{\sigma_2}}^{\infty} \int_{\frac{\theta^* - x_0}{\hat{\sigma}}}^{\infty} \Phi'(z)\Phi'(t) dz dt$$

By the implicit function theorem¹⁸ the derivative of $\frac{\partial x_0}{\partial \varphi}$ is given by

$$-\frac{G_\varphi(x_0^*, \varphi)}{G_{x_0}(x_0^*, \varphi)} = -\frac{\frac{\partial p_{IL}}{\partial \varphi} \hat{w} - \frac{\partial p_{L\wedge S}}{\partial \varphi}}{\frac{\partial p_{IL}}{\partial x_0} \hat{w} - \frac{\partial p_{L\wedge S}}{\partial x_0}}.$$

For the derivatives of p_{IL} , we have

$$\frac{\partial p_{IL}}{\partial \varphi} = \frac{\partial \theta^*}{\partial \varphi} \Phi' \left(\frac{\theta^* - x_0}{\hat{\sigma}} \right),$$

and

$$\frac{\partial p_{IL}}{\partial x_0} = \frac{-1}{\hat{\sigma}} \Phi' \left(\frac{\theta^* - x_0}{\hat{\sigma}} \right).$$

The derivative of $p_{L\wedge S}$ with respect to x_0 becomes, by using Leibniz's rule,

$$\frac{\partial p_{L\wedge S}}{\partial x_0} = \frac{1}{\hat{\sigma}} \int_{-\infty}^{\frac{\theta^s - \theta^*}{\sigma_2}} \Phi' \left(\frac{\theta^s - x_0 - \sigma_2 t}{\hat{\sigma}} \right) \Phi'(t) dt + \frac{1}{\hat{\sigma}} \int_{\frac{\theta^s - \theta^*}{\sigma_2}}^{\infty} \Phi' \left(\frac{\theta^* - x_0}{\hat{\sigma}} \right) \Phi'(t) dt.$$

For the derivative of $p_{L\wedge S}$ with respect to φ , define

$$h(\varphi, t) := \int_{\frac{\theta^s - x_0 - \sigma_2 t}{\hat{\sigma}}}^{\infty} \Phi'(z)\Phi'(t) dz$$

and

$$k(\varphi, t) := \int_{\frac{\theta^* - x_0}{\hat{\sigma}}}^{\infty} \Phi'(z)\Phi'(t) dz.$$

From the definition of $p_{L\wedge S}$, we compute, by using Leibniz's rule,

$$\frac{\partial p_{L\wedge S}}{\partial \varphi} = \int_{-\infty}^{\frac{\theta^s - \theta^*}{\sigma_2}} h_\varphi(\varphi, t) dt + \int_{\frac{\theta^s - \theta^*}{\sigma_2}}^{\infty} k_\varphi(\varphi, t) dt,$$

¹⁸See e.g. Rudin (1976, p. 224) for a statement of the implicit function theorem.

where $h_\varphi(\varphi) = -\Phi' \left(\frac{\theta^s - x_0 - \sigma_2 t}{\hat{\sigma}} \right) \cdot \Phi'(t) \cdot \frac{\partial \theta^s}{\partial \varphi}$ and $k_\varphi(\varphi) = -\Phi' \left(\frac{\theta^* - x_0}{\hat{\sigma}} \right) \cdot \Phi'(t) \cdot \frac{\partial \theta^*}{\partial \varphi}$

Combining everything, we find, after some algebraic manipulations,

$$(16) \quad \frac{\partial x_0^*}{\partial \varphi} = -\frac{G_\varphi(x_0^*, \varphi)}{G_{x_0}(x_0^*, \varphi)} = (1 - \alpha(\sigma_2)) \frac{\partial \theta^*}{\partial \varphi} + \alpha(\sigma_2) \frac{\partial \theta^s}{\partial \varphi},$$

where

$$\alpha(\sigma_2) := \frac{\int_{-\infty}^{\frac{\theta^s - \theta^*}{\sigma_2}} \Phi' \left(\frac{\theta^s - x_0 - \sigma_2 t}{\hat{\sigma}} \right) \Phi'(t) dt}{\Phi' \left(\frac{\theta^* - x_0}{\hat{\sigma}} \right) \left[\hat{w} + \int_{\frac{\theta^s - \theta^*}{\sigma_2}}^{\infty} \Phi'(t) dt \right] + \int_{-\infty}^{\frac{\theta^s - \theta^*}{\sigma_2}} \Phi' \left(\frac{\theta^s - x_0 - \sigma_2 t}{\hat{\sigma}} \right) \Phi'(t) dt},$$

and $\hat{w} := \frac{w_{1,f} - \ell_f}{w_1 - w_{2,f}} > 0$. To complete the proof of the lemma, it remains to show that $\alpha(\sigma_2) \rightarrow 0$ for $\sigma_2 \rightarrow \infty$. Define $f_{\sigma_2}(t) := \Phi' \left(\frac{\theta^s - x_0 - \sigma_2 t}{\hat{\sigma}} \right) \Phi'(t)$. Note that

$$\lim_{\sigma_2 \rightarrow \infty} f_{\sigma_2}(t) = f \equiv 0,$$

almost everywhere. The function $g(t) := \Phi'(t)/\sqrt{2\pi}$ is integrable and dominates $f_{\sigma_2}(t)$ for all σ_2 . By the dominated convergence theorem¹⁹ we have

$$\lim_{\sigma_2 \rightarrow \infty} \int_{\mathbb{R}} f_{\sigma_2}(t) dt = \int_{\mathbb{R}} f dt \equiv 0.$$

The latter immediately implies that $\lim_{\sigma_2 \rightarrow \infty} \alpha(\sigma_2) = 0$. Moreover, as $\theta^s - \theta^* < 0$, the convergence is monotone.

For finite $\sigma_2 \gg 0$, the term

$$\int_{-\infty}^{\frac{\theta^s - \theta^*}{\sigma_2}} \Phi' \left(\frac{\theta^s - x_0 - \sigma_2 t}{\hat{\sigma}} \right) \Phi'(t) dt$$

becomes negligibly small, so that we have

$$\frac{\partial x_0(\varphi)}{\partial \varphi} \approx \frac{\partial \theta^*}{\partial \varphi}.$$

It follows from the discussion in section 3 that this is positive for $\sigma_2 \gg 0$. □

Proof of proposition 3. Quantity control on total inflows: A simple calculation shows

$$\theta^c(\Delta) > \tilde{\theta}^* \Leftrightarrow \varphi < \varphi^{\text{crit}}.$$

Taking the derivative,

$$\frac{d\theta^c}{d\Delta} > 0 \Leftrightarrow \varphi < \varphi^{\text{crit}},$$

shows that any control $\Delta \in (0, 1 - \omega)$ can decrease the threshold below $\tilde{\theta}^*$ if and only if $\tilde{\theta}^* > \tilde{\theta}$.

This proves the first part of the proposition.

Quantity controls on short-term inflows: Define

$$\varphi^{cc} := \frac{1 - \varrho}{1 - \omega} - \frac{(\omega - \varrho)\varphi^{\text{crit}}}{1 - \omega}.$$

¹⁹See Rudin (1976) for a statement of the dominated convergence theorem.

A simple calculation shows

$$\theta^{cc}(\delta) > \tilde{\theta}^* \Leftrightarrow \varphi > \varphi^{cc}.$$

Taking the derivative,

$$\frac{d\theta^{cc}}{d\delta} > 0 \Leftrightarrow \varphi > \varphi^{cc},$$

shows that any $\delta \in (0, 1)$ can decrease the threshold below $\tilde{\theta}^*$ if and only if φ does not exceed φ^{cc} . Since by assumption $\omega - \varrho > 0$, we have

$$\varphi^{cc} > 1 \Leftrightarrow \omega > \varrho.$$

So for any $\varphi \in (0, 1)$, we can rule out the case $\varphi > \varphi^{cc}$.

For the autarky case, observe that

$$\theta^{cc}(\delta) \leq \check{\theta} \Leftrightarrow \varphi \leq \frac{\varphi^{\text{crit}}}{(1 - \delta) + \delta\varphi^{\text{crit}}}.$$

The latter can be rewritten as

$$\varphi \leq \frac{\varphi^{\text{crit}}}{(1 - \delta) + \delta\varphi^{\text{crit}}} \Leftrightarrow \delta \geq \frac{\varphi - \varphi^{\text{crit}}}{\varphi(1 - \varphi^{\text{crit}})}. \quad \square$$

Proof of proposition 5. It can be show that (in the same way as lemma 4 was shown),

$$\frac{\partial x_0^*(\varrho, \varphi)}{\partial \varrho} = (1 - \alpha(\sigma_2)) \frac{\partial \theta^*}{\partial \varrho} + \alpha(\sigma_2) \frac{\partial \theta^s}{\partial \varrho},$$

where from lemma 4 follows that $\lim_{\sigma_2 \rightarrow \infty} \alpha(\sigma_2) = 0$.

Using the implicit function theorem,

$$\frac{\partial \varphi}{\partial \varrho} = -\frac{G_\varrho(\varphi, \varrho)}{G_\varphi(\varphi, \varrho)} = \frac{\frac{1}{\tau_0} \Phi' \left(\frac{x_0^*(\varphi, \varrho) - \theta_0}{\tau_0} \right) \frac{\partial x_0^*}{\partial \varrho}}{1 - \frac{1}{\tau_0} \Phi' \left(\frac{x_0^*(\varphi, \varrho) - \theta_0}{\tau_0} \right) \frac{\partial x_0^*}{\partial \varphi}}.$$

For $\sigma_2 \gg 0$ we have

$$\left. \frac{\partial \varphi}{\partial \varrho} \right|_{\varphi=\varphi^*(\theta_0)} = \frac{\frac{1}{\tau_0} \Phi' \left(\frac{x_0^*(\varphi, \varrho) - \theta_0}{\tau_0} \right) \frac{\partial \theta^*}{\partial \varrho}}{1 - \frac{1}{\tau_0} \Phi' \left(\frac{x_0^*(\varphi, \varrho) - \theta_0}{\tau_0} \right) \frac{\partial \theta^*}{\partial \varphi}}.$$

Given that the regularity condition holds, the numerator is negative for $\sigma \gg 0$. The sign of the denominator is negative in any unique / stable equilibrium. \square

Proof of proposition 6. (1) Using the expressions for $p_{L \wedge S}$ and p_{IL} from above, we calculate,

$$\lim_{\sigma_2 \rightarrow \infty} p_{L \wedge S}(x_0) = 1 - \Phi \left(\frac{\tilde{\theta}^* - x_0}{\hat{\sigma}} \right) (1 - \mu),$$

and

$$\lim_{\sigma_2 \rightarrow \infty} p_{IL}(x_0) = \Phi \left(\frac{\tilde{\theta}^* - x_0}{\hat{\sigma}} \right).$$

Combining these expressions with the indifference condition yields

$$x_0^* = \tilde{\theta}^* - \hat{\sigma}\Phi^{-1}\left(\frac{1}{\hat{w} + (1-\mu)}\right).$$

Agents with a signal less than x_0^* invest short-term. The fraction of short-term debt is then given by the solution to

$$(17) \quad \varphi - \Phi\left(\frac{\tilde{\theta}^*(\varphi) - \hat{\sigma}\Phi^{-1}\left(\frac{1}{\hat{w} + (1-\mu)}\right) - \theta_0}{\tau_0}\right) = 0.$$

When a control on *total inflows* of size Δ is imposed such that the threshold becomes θ^c , the level of short-term debt will be given by the solution to

$$\varphi^c - \Phi\left(\frac{\theta^c(\varphi^c, \Delta) - \hat{\sigma}\Phi^{-1}\left(\frac{1}{\hat{w} + (1-\mu)}\right) - \theta_0}{\tau_0}\right) = 0.$$

Application of the implicit function theorem yields

$$\frac{\partial\varphi^c}{\partial\Delta} = \frac{\Phi'\left(\frac{\theta^c - \hat{\sigma}\Phi^{-1}\left(\frac{1}{\hat{w} + (1-\mu)}\right) - \theta_0}{\tau_0}\right) \frac{\partial\theta^c}{\partial\Delta} \frac{1}{\tau_0}}{1 - \Phi'\left(\frac{\theta^c - \hat{\sigma}\Phi^{-1}\left(\frac{1}{\hat{w} + (1-\mu)}\right) - \theta_0}{\tau_0}\right) \frac{\partial\theta^c}{\partial\varphi} \frac{1}{\tau_0}}$$

The denominator is positive in any stable / unique equilibrium. It follows,

$$\text{sgn}\left(\frac{\partial\varphi^c}{\partial\Delta}\right) = \text{sgn}\left(\frac{\partial\theta^c}{\partial\Delta}\right)$$

It follows from proposition 5 that an increase in controls increases the threshold and therefore the fraction of short-term debt if and only if $\varphi \in (0, \varphi^{\text{crit}})$. Conversely, whenever $\varphi \in (\varphi^{\text{crit}}, 1)$, then limiting total inflows actually decreases the fraction of short-term debt and the probability of a crisis decreases.

(2) The same logic as under (1) can be applied with θ^{cc} substituted for θ^c . Yet, as the derivative $\partial\theta^{cc}/\partial\delta$ is always negative for $\varphi \in (0, 1)$, we must have that $\partial\varphi^{cc}/\partial\delta$ is always negative. \square

Proposition 9. *Define the relative distance between the signals by*

$$\xi := \frac{x_d - x_f}{\tau_1}.$$

In equilibrium, we have

$$(18) \quad \xi^*(\varrho, \varphi) = \Phi^{-1}\left(\frac{o_d L(\varrho, \theta^*)}{\sum m_g w_{1,g} o_g}\right) - \Phi^{-1}\left(\frac{o_f L(\varrho, \theta^*)}{\sum m_g w_{1,g} o_g}\right) \in (-1, 1).$$

Proof of Proposition 9. From the proof of proposition 1 follows that a critical agent of group g is indifferent if and only if

$$\beta_g(\theta^*)q_g(\xi) - \gamma_g = 0,$$

where $q_g(\xi) := \mathbf{Pr}(\theta_1 > \theta^* | x_g^*) = \Phi\left(\frac{x_g^* - \theta^*}{\tau_1}\right)$. As $\Phi(\cdot)$ has a well-defined inverse, we have for $g \in \{d, f\}$

$$(19) \quad \frac{x_g^* - \theta^*}{\tau_1} = \Phi^{-1}\left(\frac{\gamma_g}{\beta_g(\theta^*)}\right).$$

Note that ξ remains well-defined if $\tau_1 \rightarrow 0$ as its numerator and its denominator shrink at the same rate. Using the definition of ξ , we can write $x_f^* = x_d^* - \tau_1 \xi^*$. We rewrite equation (19) for $g = f$ in terms of x_d^* and ξ^* as

$$\frac{x_d^* - \theta^*}{\tau_1} - \xi^* = \Phi^{-1}\left(\frac{\gamma_f}{\beta_f(\theta^*)}\right).$$

Equating with equation (19) for $g = d$ gives

$$\Phi^{-1}\left(\frac{\gamma_d}{\beta_d(\theta^*)}\right) - \xi^* = \Phi^{-1}\left(\frac{\gamma_f}{\beta_f(\theta^*)}\right).$$

Substituting for $\beta_g(\theta^*)$ and γ_g ,

$$\Phi^{-1}\left(\frac{w_d^1 - \ell_d}{(1 - p(\theta_1^*))(w_d^2 - \ell_d)}\right) - \xi^* = \Phi^{-1}\left(\frac{w_f^1 - \ell_f}{(1 - p(\theta_1^*))(w_f^2 - \ell_f)}\right).$$

Using the fact that equation (7) implies

$$\frac{\sum m_g w_g^1 o_g}{L(\varrho, \theta^*)} = \Phi\left(\frac{\theta_1 - \theta^s}{\sigma_2}\right)$$

and substituting into the former equation gives (18). \square

We say that a group of agents has a lower *risk-tolerance* compared to the other group, if they are more likely to withdraw early, i.e. if their critical signal is higher than the critical signal of the other group.

The following corollary follows immediately from proposition 9.

Corollary 10. *Whenever $o_g > o_{g'}$, then $x_g^* > x_{g'}^*$ for $g, g' \in \{d, f\}$ and $g \neq g'$. Consequently agents of group g have a lower risk tolerance.*

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