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# Evaluating the German (New Keynesian) Phillips Curve*

## Abstract

This paper evaluates the New Keynesian Phillips Curve (NKPC) and its hybrid variant within a limited information framework for Germany. The main interest rests on the average frequency of price re-optimization of firms. We use the labor income share as the driving variable and consider a source of real rigidity by allowing for a fixed firm-specific capital stock. A GMM estimation strategy is employed as well as an identification robust method that is based upon the Anderson-Rubin statistic. We find out that the German Phillips Curve is purely forward looking. Moreover, our point estimates are consistent with the view that firms re-optimize prices every two to three quarters. While these estimates seem plausible from an economic point of view, the uncertainties around these estimates are very large and also consistent with perfect nominal price rigidity where firms never re-optimize prices. This analysis also offers some explanations why previous results for the German NKPC based on GMM differ considerably. First, standard GMM results are very sensitive to the way how orthogonality conditions are formulated. Additionally, model misspecifications may be left undetected by conventional $J$ tests. Taken together, this analysis points out the need for identification robust methods to get reliable estimates for the NKPC.

**Keywords:** Inflation dynamics, Phillips Curve, Weak Instruments, Optimal Instruments
**JEL-Codes:** E31; C13; C52

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# Evaluating the German (New Keynesian) Phillips Curve


## Zusammenfassung

Dieses Papier untersucht mit einem *Limited-Information*-Ansatz die Neu-Keynesianische Phillipskurve und ihre hybride Erweiterung für Deutschland. Das Hauptinteresse liegt dabei auf der durchschnittlichen Preissetzungsfrequenz der Firmen. Die Grenzkosten der Firmen werden durch die Lohnquote (Verhältnis von Lohnsumme am nominalen Bruttoinlandsprodukt) approximiert. Zusätzlich wird eine mögliche Quelle realer Rigiditäten durch einen fixen, firmenspezifischen Kapitalstock berücksichtigt. Eine GMM-Schätzung sowie ein identifikationsrobustes Verfahren, basierend auf der Anderson-Rubin Statistik, werden als Schätzverfahren verwendet. Die Schätzergebnisse zeigen, dass die deutsche Phillipskurve ausschließlich als vorausschauend zu charakterisieren ist. Die Punktschätzungen deuten darauf hin, dass Firmen ihre Preise alle zwei bis drei Quartale neu optimieren. Obwohl diese Schätzungen ökonomisch plausibel scheinen, sind sie mit großer Unsicherheit behaftet, so dass der Fall perfekter Preisrigidität statistisch nicht verworfen werden kann. Die vorliegende Analyse bietet zudem eine Erklärung für die Tatsache, dass vorhergehende Untersuchungen zur deutschen Phillipskurve zu sehr unterschiedlichen Ergebnissen gekommen sind. So hängen die GMM-Ergebnisse sehr stark davon ab, wie die Orthogonalitätsbedingungen formuliert werden. Zusätzlich scheinen Misspezifikationen des Modells durch $J$ Tests nicht erkannt zu werden. Darum unterstreicht diese Analyse die Verwendung von identifikationsrobusten Methoden um verlässliche Schätzungen für die Neu-Keynesianische Phillipskurve zu bekommen.


**Schlagwörter:** Inflationsprozess, Phillipskurve, Schwache Instrumente, Optimale Wahl der Instrumente
**JEL-Codes:** E31; C13; C52

# Evaluating the German (New Keynesian) Phillips Curve

## 1 Introduction

Explaining the evolution of aggregate prices is one of the most prominent issues in empirical macroeconomics. Nowadays, the canonical inflation model is the New Keynesian Phillips Curve (NKPC). Similar to earlier Phillips Curve specifications the NKPC relates price behavior to a measure of real economic activity. But, in contrast to traditional ones the NKPC can be derived directly from optimizing behavior of households and firms and thus builds on a suitable micro-foundation. The NKPC framework assumes monopolistically competitive firms that face nominal prices rigidities. The standard model of staggered price adjustment by Calvo (1983) has the attractive property that the coefficients of the NKPC directly depend on the average frequency with which prices are adjusted in the economy.

The aim of this paper is to determine the degree of nominal price rigidity in the German economy. Therefore we estimate the NKPC and allow for different specifications. A generalized version of the model proposed by Christiano, Eichenbaum and Evans (2005) is employed as our benchmark model specification that assumes a dynamic indexation scheme for those firms that do not re-optimize. Furthermore, we also consider a model with "rule-of-thumb" firms in the spirit of Galí and Gertler (1999) and Galí, Gertler and López-Salido (2001). We follow Galí et al. (2001) and Sbordone (2002) and allow for some real rigidities derived from the assumption of firm-specific capital.

Empirical studies that assess the degree of nominal price rigidity in the German economy through estimations of the Phillips curve are still rare. The primary evidence stems from cross country comparisons. Examples are Banerjee and Batini (2004), Benigno and López-Salido (2006), Leith and Malley (2007) or Rumler (2007). This evidence is in most cases based upon GMM estimation with additional aspects of an open economy. While the open economy aspect seems to be unimportant for German Phillips curve (at least according to Banerjee and Batini, 2004; Leith and Malley, 2007), their results vary considerably with respect to the degree of nominal price rigidity. The estimated average frequency of price re-optimization ranges from 2.5 quarters (Banerjee and Batini, 2004) to 13 quarters (Leith and Malley, 2007). Additionally, there is also disagreement on whether the inflation contains a lagged term (through backward looking behavior) or whether it is purely forward looking. A more rigorous treatment of nominal price rigidity in Germany is provided by Coenen, Levin and Christoffel (2007) that focuses on the interaction of real and nominal rigidities. Their estimation technique relies on indirect inference methods. They employ a generalized Calvo model, where their estimates point to a frequency of price re-optimization of roughly two quarters.

Our empirical strategy is as follows. We apply a standard GMM method to estimate the structural parameters of the Phillips Curve. Special attention is payed to the selection of relevant instruments. Since the choice of a particular instrument set can hardly justified by theory, we propose a statistical criterion to cull out relevant instruments. We then evaluate the robustness of our results with respect to several parameter restrictions, measures of real rigidity and additional lags of inflation. Next, we conduct an identification robust procedure based on a nonlinear Anderson-Rubin (AR) statistic (where we follow Ma, 2002; Mavroeidis, 2006) and compare these results with those obtained from standard GMM estimation. We contribute to this line of research by applying identification robust estimation techniques to estimate the German NKPC. As long as there are weak instrument problems present, the two procedures should display quite different results. In this case the GMM results are generally unreliable (see e.g. Stock and Wright, 2000).

For a given economically plausible degree of real rigidity, the estimates of the frequency of price re-optimization point to about 2.5 quarters. But this estimate is surrounded by a large degree of uncertainty, since the confidence intervals for this estimate are very large. Unless we do not restrict other parameter values, the estimated degree of nominal rigidity is both consistent with a very low degree of price stickiness and with a situation where prices are never re-optimized (perfect price rigidity). This also casts doubt concerning the proxy of marginal cost, the labor share, as driving variable of inflation (a finding that is also obtained by Mavroeidis, 2006; Kleibergen and Mavroeidis, 2008, for the US). Moreover, we find that backward looking behavior is unimportant for explaining the German inflation process and thus find that a purely forward looking specification is more appropriate. The identification robust procedure indicate some problems with the orthogonality conditions not detected by the conventional $J$ statistic.

This paper is organized as follows. We first present our basic model framework in Section 2. Then we turn to the econometric strategies for estimating and testing the different model specifications (Section 3). In Section 4 we discuss our data set and how we obtain the instrument set. Next, we present our econometric results (Section 5). Finally, we draw some conclusions in Section 6.

# 2 The Modeling Framework

This section presents the basic theoretical framework that includes monopolistically competitive goods markets and price stickiness. These are the two key elements in modern macroeconomic models that are used to analyze monetary policy. This model structure tries to ensure that it is consistent with the behavior of optimizing economic agents. Here, we are mainly interested in the price setting behavior of firms in order to derive an expression for aggregate inflation. Therefore, we assume random price contracts due to Calvo (1983) that is now standard in many macroeconomic models (e.g., Smets and Wouters, 2003; Christiano et al., 2005). However, we deviate from the standard Calvo model and assume that capital is firm-specific and is subject to a form of real rigidity, so that capital cannot be instantaneously reallocated and is thus a predetermined factor.[1]

## 2.1 The Market Structure

As is standard in New Keynesian models, we assume a monopolistic competitive environment with a continuum of firms indexed by $i \in [0, 1]$. Each firm $i$ produces a differentiated good $Y_t(i)$ according to a Cobb-Douglas technology

$$Y_t(i) = A_t \overline{K}_t(i)^{\alpha} N_t(i)^{1-\alpha}, \tag{1}$$

where $A_t$ is a common country wide technological factor, $\overline{K}_t(i)$ is the (fixed) firm-specific capital stock and $N_t(i)$ is the labor factor employed by firm $i$.

Each firm $i$ is faced with a demand function with a constant elasticity of substitution that is given by

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} Y_t, \tag{2}$$

where $Y_t$ is aggregate output (which equals aggregate demand), $P_t$ is the aggregate price level in the economy and $P_t(i)$ is the price that firm $i$ charges for good $Y_t(i)$. The price elasticity of demand for good $i$ is equal to $\epsilon$ (with $\epsilon > 1$).[2]

1 Here we follow Galí et al. (2001) and Sbordone (2002). See also Eichenbaum and Fisher (2007) for a more rigorous treatment of real rigidities in the Calvo price setting framework

2 According to Dixit and Stiglitz (1977) aggregate output $Y_t$ is a constant-elasticity-of-substitution aggregator $Y_t = \left[\int_0^1 Y_t(i)^{(\epsilon-1)/\epsilon} di\right]^{\epsilon/(\epsilon-1)}$. This expression abstracts from in-

Without any price frictions the price of the differentiated good is set as a constant mark-up over nominal marginal costs

$$P_t(i) = \mu \frac{W_t}{(1-\alpha)Y_t(i)/N_t(i)} = \mu MC_t(i), \tag{3}$$

with $\mu = \epsilon/(\epsilon - 1)$. In a symmetric equilibrium, all firms produce the same output, employ the same labor inputs and charge the same price. In this situation $p_t(i) = p_t$ (expressed in logs) and the optimal price under perfect price flexibility is equal to $p_t = \log(\mu) + mc_t$.

## 2.2 The Calvo Model

The second essential element of New Keynesian Macroeconomics are nominal rigidities. Sticky price models are now frequently employed to study the monetary transmission process. In the following analysis we concentrate solely on time-dependent models where we use in particular a Calvo (1983) style model.[3] This framework assumes that each firm optimizes its price only from time to time. This is motivated by costs associated with information gathering. The frequency of price re-optimization is thus a stochastic process with a constant probability that a firm sets its prices in an optimal way at each point in time. So, there is always a fraction of firms $1 - \theta$ in the economy that optimally adjust its prices. This arrival rate can be described by an exogenous stochastic process with the expected waiting time between price changes given by $1/(1-\theta)$.

A firm that reoptimizes, sets its price $P_t^*(i)$ in order to maximizes the expected discounted sum of profits

$$E_t \sum_{k=0}^{\infty} (\beta\theta)^k \upsilon_{t,t+k} \left[P_t^*(i) X_{t,t+k} - MC_{t,t+k}(i)\right] \frac{Y_{t+k}(i)}{P_{t+k}}, \tag{4}$$

vestment and foreign trade, so output $Y_t$ equals consumption $C_t$ and $P_t$ is the corresponding aggregate price index $P_t = \left[\int_0^1 P_t(i)^{1-\epsilon} di\right]^{1/(1-\epsilon)}$.

3 Another model class are state-dependent sticky prices models where the number of firms that changes prices in a given period is determined endogenously (e.g., Dotsey, King and Wolman, 1999). Another popular model besides the one of Calvo (1983) was developed by Taylor (1980).

subject to the demand constraints (2) and

$$X_{t,t+k} = \begin{cases} \prod_{l=0}^{k-1} \overline{\pi}^{1-\xi} \pi_{t+l}^{\xi} & \text{for } k > 0 \\ 1 & \text{for } k = 0. \end{cases} \tag{5}$$

with $\beta$ a constant discount factor, $\upsilon_{t,t+k} = U^{'}(C_t)/U^{'}(C_{t+k})$ the time-varying portion of the discount factor between $t$ and $t+k$; with $U^{'}(C_t)$ the marginal utility of consumption. $\overline{\pi}$ is the long-run average gross rate of inflation. When a firm does not re-optimize its price, it is assumed that it resets it according to some sort of indexation scheme. Our baseline specification is the partial indexation scheme used in Smets and Wouters's (2003) model and further discussed by Sahuc (2004) with $\xi \in [0, 1]$ that measures the degree of indexation to past inflation. This is a further generalization of Christiano et al.'s (2005) dynamic indexation scheme with $\xi = 1$, where prices are reset according to $P_t(i) = \pi_{t-1}P_{t-1}(i)$ during periods where firms do not re-optimize.

After solving the maximization problem in (4) and some further manipulations,[4] an expression for aggregate inflation can be derived of the form

$$\widehat{\pi}_t = \frac{\xi}{1+\beta\xi}\widehat{\pi}_{t-1} + \frac{\beta}{1+\beta\xi}E_t\widehat{\pi}_{t+1} + \frac{(1-\theta\beta)(1-\theta)}{(1-\beta\xi)\theta}A\widehat{s}_t, \tag{6}$$

where $\widehat{s}_t$ is the percentage deviation of average marginal cost $MC_t/P_t$ from its steady state. This type of equation is often referred to as the New Keynesian Phillips Curve.[5] Note that a particular feature of this inflation equation is its sound microeconomic foundation, i.e. it depends on structural parameters that have a direct economic interpretation. With $\xi = 0$ the expression reduces to the pure forward looking Phillips curve that coincides with a static indexation scheme.[6]

The parameter $A$ measures the degree to which inflation responds to changes in current and future values of real marginal costs. In contrast to a situation where all firms face the same marginal cost ($A = 1$), firm specific marginal cost may differ across firms due to differences in the output level. The differences in the output

4 See e.g. Sahuc (2004) or Walsh (2003, Ch. 5) for a derivation.

5 This expression is an augmented version of a specific relation that does not include the lagged inflation term. The version with an additional inflation lag is sometimes called "hybrid" Phillips Curve.

6 A static indexation scheme implies that firms set prices according to $P_{it} = \overline{\pi}P_{it-1}$ during periods where they do not reoptimize (e.g., Erceg, Henderson and Levin, 2000).

level are generated through the assumption of a fixed stock of firm-specific capital.[7] As shown by Sbordone (2002) and Galí et al. (2001) $A$ also depends on structural parameters with

$$A = \frac{1-\alpha}{1+\alpha(\epsilon-1)},$$

$\epsilon$ the elasticity of substitution among different goods from eq(2) and $\alpha$ the technology parameter from the Cobb-Douglas production function eq(1), whereas $\epsilon > 1$ and $0 < \alpha < 1$.

An additional way of modeling a smaller reaction of prices to marginal cost is proposed by Eichenbaum and Fisher (2007) and Coenen et al. (2007). They assume a varying elasticity of demand, but as shown by Coenen et al. (2007) this assumption does not lead to a substantial reduction of the sensitivity of prices to marginal cost for reasonable values of $\alpha$. In order to keep thinks simple we do not consider this type of additional friction.

## 2.3 A Variant with rule-of-thumb Firms

A variant of the above presented model (6) was presented by Galí and Gertler (1999). In this specification there are two types of firms; one fraction $1-\omega$ that re-optimizes prices according to the model of Calvo (as discussed in Section 2.2). In periods where firms cannot re-optimize they set prices according to a static indexation scheme. The other fraction $\omega$ of non-reoptimizing firms set prices according to a backward looking rule-of-thumb. With probability $\theta$ they set $P_{it} = \overline{\pi} P_{it-1}$. Otherwise, with probability $1-\theta$, they apply

$$P_t^{'} = \pi_{t-1}\overline{P}_t$$

with $\overline{P}_t = (1-\omega)P_t^* + \omega P_t^{'}$, where $P_t^*$ is the optimized price that is chosen by the fraction of firms that are forward looking.

In this setting an analog expression of (6) can be derived as

$$\widehat{\pi}_t = \frac{\omega}{\phi}\widehat{\pi}_{t-1} + \frac{\beta\theta}{\phi}E_t\widehat{\pi}_{t+1} + \frac{(1-\omega)(1-\theta\beta)(1-\theta)}{\phi}A\widehat{s}_t, \tag{7}$$

7 A more comprehensive discussion for the role of firm-specific capital is given by Eichenbaum and Fisher (2007) firms face convex capital adjustment costs. Our specification of $A$ can be seen as a special case of this framework where the adjustment costs are very high.

with $\phi = \theta + \omega\,[1 - \theta(1-\beta)]$. When $\omega = 0$ this expression is equivalent to the pure forward looking Phillips curve and thus equal to (6) as long as $\xi = 0$.

Finally, note that the explanatory variables are the same across the two Phillips curve specifications, the only difference is the way how the structural parameters appear in the two equations. While the interpretation of $\theta$ is the same, the parameters $\xi$ and $\omega$ have a different meaning depending on the particular model that both try to rationalize a lagged inflation term in the Phillips curve.

# 3 Econometric Methodology

We now present our empirical model and discuss how we can conduct inference about the structural parameters of the Phillips curve model discussed above. In this analysis we take a limited information approach. This has the great advantage that we do not have to fully specify a hole general equilibrium model including the nature of the forcing variable. Instead, we can leave part of the model unspecified and only have to consider a single equation. As it is known from traditional simultaneous equation framework, full information methods may be more efficient, but may also be more sensitive to specification errors, since errors in one equation spread over to other equations as well.[8] We also present some shortcomings with standard GMM and present an identification robust variant to standard GMM estimation that is valid under much weaker assumptions.[9]

## 3.1 GMM

Our empirical model is given by

$$\widehat{\pi}_t = \gamma_b \widehat{\pi}_{t-1} + \gamma_f \widehat{\pi}_{t+1} + \lambda \widehat{s}_t + u_t, \tag{8}$$

where $u_t = \eta_t - \gamma_f \left(\widehat{\pi}_{t+1} - E_t \widehat{\pi}_{t+1}\right)$. Note that expected future inflation $E_t \widehat{\pi}_{t+1}$ has been replaced by its realization $\widehat{\pi}_{t+1}$, whereas the expectation error $(\widehat{\pi}_{t+1} - E_t \widehat{\pi}_{t+1})$ is part of the residual $u_t$. The coefficients $\gamma_b$, $\gamma_f$ and $\lambda$ depend in nonlinear form on the structural parameters $(\beta, \theta, \xi, \alpha, \epsilon)$ in the partial indexation model or on $(\beta, \theta, \omega, \alpha, \epsilon)$ in the model with rule-of-thumb firms.

Since the residual $u_t$ is correlated with $\widehat{\pi}_{t+1}$ (unless there exist forecast errors of future inflation) an instrumental variables estimator is needed in order to guarantee unbiased results. We employ a Generalized Method of Moments (GMM) estimator proposed by Hansen (1982) that is suited for dynamic non-linear models in order to estimate the structural model parameters. This approach is frequently applied to estimate inter-temporal asset pricing models.[10] Additionally, we use heteroskedas-

[8] Examples for ML techniques to estimate hybrid Phillips curve specifications include Fuhrer (1997), Lindé (2005) and Jondeau and Le Bihan (2006). See Jondeau and Le Bihan (2008) for discussion of properties of different estimators under misspecifications.

[9] Identification robust methods are currently unavailable for ML estimation. But as shown by Dufour, Khalaf and Kichian (2007a), these methods may also suffer from weak instrument problems in the context of the NKPC and may lead to wrong conclusions.

[10] See Hansen and Singleton (1982) for an early example.

tic and autocorrelation consistent (HAC) standard errors due to Newey and West (1987).[11]

The GMM approach is now also very frequently applied to estimate the parameters of the Calvo model. Examples include Galí and Gertler (1999), Galí et al. (2001) and Eichenbaum and Fisher (2007). First, we set up the orthogonality conditions for the partial indexation model (6). We use two different specifications that differ in the way how functions are normalized. These are given by

$$u_t^1 = \widehat{\pi}_t - \frac{\xi}{1+\beta\xi}\widehat{\pi}_{t-1} - \frac{\beta}{1+\beta\xi}\widehat{\pi}_{t+1} - \frac{(1-\theta\beta)(1-\theta)}{(1+\beta\xi)\theta}A\widehat{s}_t, \quad (9)$$

$$u_t^2 = (1+\beta\xi)\widehat{\pi}_t - \xi\widehat{\pi}_{t-1} - \beta\widehat{\pi}_{t+1} - \frac{(1-\theta\beta)(1-\theta)}{\theta}A\widehat{s}_t, \quad (10)$$

with the orthogonality conditions

$$E_{t-1}\left\{u_t^i\left(\beta,\theta,\xi\right)\mathbf{z_{t-1}}\right\} = 0 \quad (11)$$

for $i = 1, 2$. $\mathbf{z_{t-1}}$ is the vector of instruments that are assumed to be orthogonal to the error term $u_t^i$ (under the rationality assumption). Note that $\mathbf{z_{t-1}}$ does only include instruments dated $t-1$ or earlier in order to rule out simultaneity issues. This also guarantees that the information is already available at time $t$ due to a potential publication lag.

The standard two-step GMM estimates are obtained by minimizing

$$J(\vartheta^i) = \left[\frac{1}{T}\sum_{t=1}^{T}\phi_t^i(\vartheta^i)\right]' V(\vartheta_T^{i,1})^{-1}\left[\frac{1}{T}\sum_{t=1}^{T}\phi_t^i(\vartheta^i)\right], \quad (12)$$

where $\vartheta^i$ denotes the structural parameters of the model; $\phi_t(\vartheta^i) = u_t^i(\beta^i,\theta^i,\xi^i)\mathbf{z}_{t-1}$. This objective function is evaluated given an initial estimate $\vartheta_T^{i,1}$ for the weighting matrix. This initial estimator may be obtained by using the identity matrix or the instrument matrix as a weighting matrix for the first step. Since we consider two different transformations of the orthogonality conditions $u_t^1$ and $u_t^2$ the estimates for the structural parameter may differ across these specifications.[12]

[11] Throughout we use a lag length of 5 for the HAC estimator. In our case the estimated standard errors are not very sensitive to the particular choice of the lag length.

[12] It is well known that for finite samples the two-step GMM as well as the iterated GMM estimator may be sensitive to transformations of the orthogonality conditions (e.g., Hall,

From (9) and (10) it follows that $\theta$ and $A$ (and thus also $\alpha$ and $\epsilon$) cannot be separately identified. So we are only able to estimate $\theta$, the parameter that is of most interest, given reasonable values of $\alpha$ and $\epsilon$ which cannot be tested explicitly. To identify the remaining parameters $\beta$, $\theta$ and $\xi$ we need at least three valid instruments.

For the model with rule-of-thumb firms, similar orthogonality conditions can be formulated. They only differ with respect to the functional form of the parameters. The two normalizations are given by

$$u_t^{\prime 1} = \widehat{\pi}_t - \frac{\omega}{\phi}\widehat{\pi}_{t-1} - \frac{\beta\theta}{\phi}\widehat{\pi}_{t+1} - \frac{(1-\omega)(1-\theta\beta)(1-\theta)}{\phi}A\widehat{s}_t, \tag{13}$$

$$u_t^{\prime 2} = \phi\widehat{\pi}_t - \omega\widehat{\pi}_{t-1} - \beta\theta\widehat{\pi}_{t+1} - (1-\omega)(1-\theta\beta)(1-\theta)A\widehat{s}_t. \tag{14}$$

Again, the orthogonality conditions can be formulated as

$$E_{t-1}\left\{u_t^{\prime i}\left(\beta,\theta,\omega\right)\mathbf{z_{t-1}}\right\} = 0 \tag{15}$$

for $i = 1, 2$ with $\phi = \theta + \omega\left[1 - \theta(1-\beta)\right]$. Everything else is comparable with the partial indexation model.

## 3.2 An Identification Robust Alternative

So far our analysis implicitly rests on the assumption that our instrument set is sufficiently correlated with the endogenous variables under consideration. This means we have assumed that our regression analysis does not suffer from weak instrument problems. But as shown by a vast literature, the presence of weak instruments may cause serious distortions in standard IV point estimates, hypothesis tests and confidence intervals (see Stock, Wright and Yogo, 2002, for an overview of problems caused by weak instruments and some recommendations to deal with it.). Several authors, including Ma (2002), Dufour, Khalaf and Kichian (2006), Mavroeidis (2006), Martins and Gabriel (2006), Kleibergen and Mavroeidis (2008) and Nason and Smith (2008) provide evidence that weak instrument problems may be present in standard GMM estimations of the New Keynesian Phillips Curve.

That is why we have to highlight potential problems with weak instruments or weak identification in our estimation strategy as well. As shown by Mavroeidis (2005)

2005). Unless the model is not misspecified, the two different normalizations should lead to approximately similar results.

standard pre-tests of identification (or weak instrument problems) are inappropriate in this setting. So we reevaluate our GMM results with an identification robust method that is fully robust to problems induced by weak instruments and weak identification. Therefore, we stick to a non-linear variant of the Anderson-Rubin Statistic as suggested by Stock and Wright (2000). They show that identification robust confidence sets can be obtained from the continuous-updating GMM (CUE) objective function.[13] In the linear simultaneous equations model these so called $S$-sets are asymptotically equivalent to confidence sets constructed by inverting the Anderson-Rubin test statistic.[14]

As shown by Dufour (2003) the AR statistic is well suited for validating a structural model, since it is not only robust to the presence of weak instruments, but it is also robust to model misspecifications like overidentification and thus provides an alternative to the standard $J$ test. $S$-sets also share the characteristic of identification-robust procedures as described in Dufour (1997) which require that whenever parameters are not identified, the results should lead to uninformative and thus unbounded confidence sets. $S$-sets contain all parameter values for which the joint hypothesis $\vartheta = \vartheta_0$ and that the overidentifying conditions are valid. So, whenever the model is misspecified and the overidentifying conditions are invalid, the $S$-sets can be null. Contrary, with weak instruments (or irrelevant instruments), the $S$-sets can contain the entire parameter space. While this is a favored property of this test because it ensures robustness to many pitfalls, since the $J$ test has very low power for estimations of the NKPC (Mavroeidis, 2005). It also needs some caution in interpreting the results of the model. Particularly, when $S$-sets are small this can be because the model is correctly specified or because it is misspecified but does not lead to a full rejection.[15]

[13] The continuous-updating GMM estimator was invented by Hansen, Heaton and Yaron (1996). As opposed to the standard two-step GMM estimator, the CUE evaluates the weight matrix at the same parameter value as the orthogonality conditions.

[14] Dufour et al. (2006) evaluate the NKPC with the standard AR test which is closer related to 2SLS than GMM. They extended this framework in Dufour, Khalaf and Kichian (2007b) and in Dufour, Khalaf and Kichian (2008) to allow for heteroskedastic and autocorrelated residuals. Kleibergen and Mavroeidis (2008) consider not only Stock and Wright's (2000) approach, but also a Lagrange Multiplier statistic (as discussed in Kleibergen, 2007) and a Likelihood Ratio statistic (see Kleibergen, 2005) to construct identification robust confidence intervals for the NKPC. Martins and Gabriel (2006) evaluate the NKPC with generalized empirical likelihood (GEL) methods.

[15] This may become relevant when there are many instruments. In this case the power of the test might be too low to reject a potentially misspecified model.

The objective function of the CUE is given by

$$S(\vartheta) = \left[\frac{1}{T}\sum_{t=1}^{T}\phi_t(\vartheta)\right]' V(\vartheta)^{-1}\left[\frac{1}{T}\sum_{t=1}^{T}\phi_t(\vartheta)\right] \tag{16}$$

with $\vartheta$ the parameter vector of interest; $\phi_t(\vartheta) = u_t \mathbf{z_{t-1}}$ with $u_t = u_t(\beta, \theta, \xi)$ or $u_t = u_t(\beta, \theta, \omega)$ as defined in (9) and (13) and $\mathbf{z_{t-1}}$ the vector of instruments. Note that the CUE is invariant to transformations of the orthogonality condition, so we do not have to consider this differentiation. $V(\vartheta)$ is defined as a HAC estimator to allow for serial correlation as well as heteroskedasticity in the residuals. This coincides with the two-step estimator used above.

We now check whether our baseline GMM results hold when we use $S$-sets as suggested by Stock and Wright (2000). First, we can examine whether our GMM point estimates are also included the $S$-sets. This should be the case when the model is correctly specified and there are no weak instrument problems present. According to Stock and Wright (2000) $S(\vartheta_0) \xrightarrow{D} \chi^2_k$, where $S(\vartheta_0)$ is the objective as defined above evaluated at the true parameter values $(\beta_0, \theta_0, \xi_0)$ or $(\beta_0, \theta_0, \omega_0)$. Second, we can construct confidence intervals for the parameters of interest (so-called $S$-sets).[16] Here, we ask what parameter values are comparable with the model. Therefore all values which are not rejected form the confidence region.[17].

[16] We follow Stock and Wright and construct 90% $S$-set as they did in their paper.

[17] The paramter space that we consider involves all possible values in the range of 0 to 1. In the search process all values within this range are evaluated with increments of 0.01. For the measure of real rigidity $A$ we take as given the values for $\alpha$ and $\epsilon$ as calibrated in section (3.1)

# 4 Data and Empirical Implementation

Our sample period is 1973:1 - 2004:4. While data before 1973 are principally available, we take this date as starting point since it marks the end of the fixed exchange rate regime of the Bretton Woods system. This is also associated with a change in monetary policy that got more independent from external influences. Inflation is measured as the quarterly annualized change in the GDP deflator. From the production function (1) it follows that real marginal costs are proportional to the labor income share in national income. The labor share is defined as the total wage bill ($W_t N_t$) divided by nominal GDP ($P_t Y_t$). The variable $\widehat{s}_t$ is constructed as the percentage deviation of the labor share from its sample average (see Figure 1).[18]

Since in our Phillips curve specification the term $A$ cannot be separately identified, we have to calibrate $\alpha$ and $\epsilon$ in an economic reasonable way. We set $\alpha$, the output elasticity with respect to capital, equal to 0.3 how it is usually done for the German economy (e.g., Dreger and Schumacher, 2000). More controversial is the calibration of the elasticity of substitution among different goods. For the definition of the steady state mark-up $\mu$, it follows that the elasticity of substitution can be redefined as $\epsilon = \frac{\mu}{\mu-1}$. We consider a steady-state mark-up of 10% ($\mu = 1.1$) as our baseline value (as it was done by Galí et al., 2001; Eichenbaum and Fisher, 2007). This corresponse to $\epsilon = 11$.

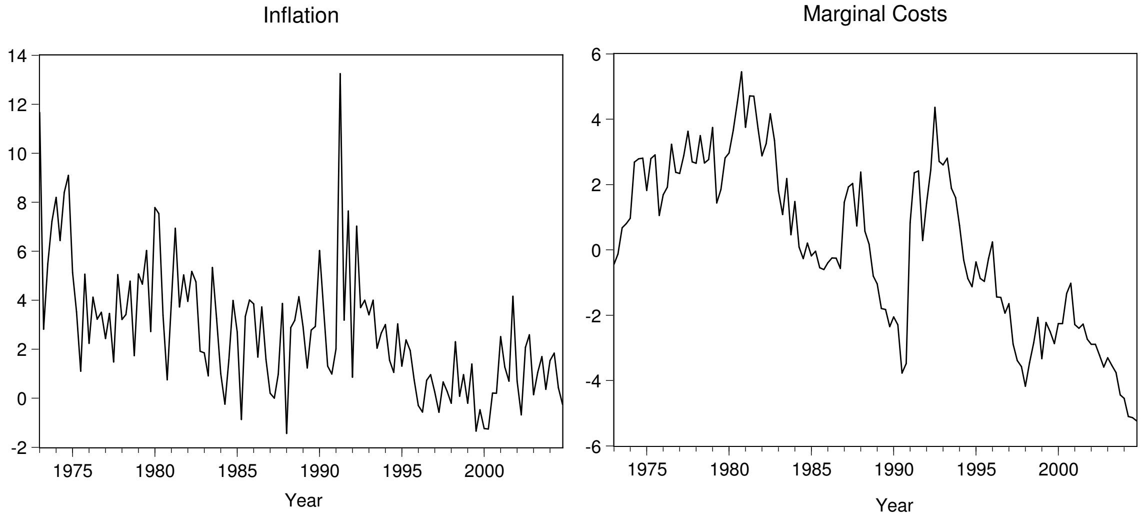
A next very crucial issue is concerned with the instrument vector $\mathbf{z_{t-1}}$. To be a valid instrument, variables have to fulfill two important characteristics. First, they have to be uncorrelated with the error term (which is the orthogonality condition). Second, they have to be correlated with the variable they have to instrument (that is the relevance condition). Both conditions have to be fulfilled to obtain reliable point estimates and confidence intervals of the model parameters. So the first practical challenge is to decide which variables should be included into the instrument set. In principle any variable dated $t-1$ and earlier may be considered as instrument since under rational expectations it fulfills the orthogonality conditions (so it fulfills the first condition of valid instrument). This leaves us with a potentially infinite set of possible variables that could be used as instruments. But as was early recognized by Tauchen (1986) and Kocherlakota (1990) instruments should be used quite parsimoniously.[19]

To deal with problems of redundant instruments we apply a unique two-step approach where we try to cull out those variables that are really relevant. The explicit

[18] This is the measure proposed by Galí and Gertler (1999), Galí et al. (2001) and Sbordone (2002).

[19] Tauchen (1986) finds in a simulation study that the inclusion of additional instruments that are not relevant or only marginally relevant leads to increasing bias of the parameter estimates.

Figure 1: Data series for Germany




Source: Statistisches Bundesamt; own calculations

treatment of instrument selection is rarely done for rational expectations models.[20] As a starting point we consider a wide range of possible instruments that include important macroeconomic indicators. This potential instrument list contains Galí and Gertler's (1999) instrument set with inflation, real marginal cost, real-time detrended GDP, wage inflation, commodity price inflation, and the long-short interest rate spread. Further we include as an additional potential instrument the short term interest rate (defined as the three month bill). For the variables $\widehat{\pi}_t$ and $\widehat{s}_t$ we allow for a potential lag length of five quarters; for the remaining candidate variables we use a maximal lag length of 2.[21] The first step of instrument selection contains a preselection of possible instruments within a VAR. Therefore, the two endogenous

[20] Many empirical work employs instrument sets that are used previously in other studies without checking whether they are really relevant. An early exception is Pesaran (1987) who emphasis pre-checking the conditions for identification in models with rational expectations. Other examples are Fuhrer and Olivei (2005) who use an instrument set that is derived from theory and Nason and Smith (2008) who check how many lags of the labor share can be used as instruments. More recently, there are some studies that employ factor analysis to construct an instrument set for rational expectation models in an environment with lots of potential instrument (e.g. see Beyer, Farmer, Henry and Marcellino, 2007; Bai and Ng, 2006)

[21] The choice of the potential lag length is orientated at previous experience. Galí et al. (2001) use a lag length for inflation of five quarters. The same lag length is also considered for the labor share as the driving variable. For the remaining variable we only consider the most recent lags since these are the variables that ought to be mostly correlated with those variables that they instrument.

variables $\widehat{\pi}_{t+1}$ and $\widehat{s}_t$ are regressed on all potential instruments. This specification can be formalized as

$$\begin{bmatrix} \widehat{\pi}_{t+1} \\ \widehat{s}_t \end{bmatrix} = \nu + \sum_{i=1}^{L_1} A_i y_{t-i} + \sum_{j=1}^{L_2} B_j x_{t-j} + u_t, \tag{17}$$

with $\nu$ a deterministic term, $y_{t-i} = [\widehat{\pi}_{t-i} \ \widehat{s}_{t-i}]'$ and $x_{t-j}$ the vector of all other predetermined variables with lag $j$. The maximal lag length is $L_1 = 5$ and $L_2 = 2$.

After estimating the full model we apply a model reduction procedure that works through a sequential elimination of regressors in order to obtain a model that lead to the smallest value of the particular information criterion. We base the selection procedure on two selection criteria (AIC and SC), that are frequently used in time series analysis (e.g., Lütkepohl, 2005). Accordingly, we end up with restrictions on $A_i$ and $B_i$ that determine our instrument sets $\mathbf{z_{t-1}}(c^{AIC})$ and $\mathbf{z_{t-1}}(c^{SC})$, where $c^j$ denotes which elements of the candidate set are included in a particular moment condition. Besides the two instrument sets based on the information criteria, we also take Galí et al.'s (2001) set as a benchmark.

Thus, we have three candidate instrument sets with the following size:

- AIC based instrument set: that includes 14 of 21 potential instruments (see Figure 5),
- SC based instrument set: that includes 11 instruments (see Figure 5),
- Galí et al.'s (2001) instrument set: that includes inflation with lags $t-1$ to $t-5$, labor share, wage inflation and output gap from $t-1$ to $t-2$ (all together 11 instruments),

The sensitiveness of our results with respect to different instrument sets may also indicate whether there are problems with redundant instruments or weak instruments.

As a second step we also apply a moment selection check after we performed the GMM estimation to evaluate our preselection based on model reduction techniques. This strategy is based upon the relevance condition. Therefore we use a moment selection criterion proposed by Hall, Inoue, Jana and Shin (2007). This criteria is defined as

$$RMSC(c) = ln\left[\left|\hat{V}_{\theta,T}(c)\right|\right] + (|c| - p)\, ln(T^{1/3})/T^{1/3} \tag{18}$$

where $\hat{V}_{\theta,T}(c)$ is the covariance matrix of the model parameters conditional on the instrument set $c$. The second term is a BIC-type penalty term with $T$ the sample size and $p$ the number of parameters to be estimated. The idea is to select the instrument vector that minimizes this criterion. Since the relevance condition can be interpreted as statement about the asymptotic variance of the estimator, the sample analog is the natural basis to construct an information criterion. Hall et al. (2007) show that the natural logarithm of the determinant of the variance-covariance matrix can serve for this purpose. Note that this procedure only works when there are no weak instrument problems present. Meaning that it is necessary to have at least some variables that are considerably correlated with the endogenous variables they have to instrument (otherwise the selection criterion may produce strange results).

# 5 Estimation Results

In this section we present the results of the structural model and their robustness to several empirical aspects. First, we check for the sensitivity with respect to different instrument sets and with respect to different orthogonality conditions. As pointed out above the instrument relevance is essential for the reliability of GMM point estimates and confidence intervals. So, we report estimation results with the instrument set used by Galí et al. (2001) and compare that with those that are based on a preselection as discussed in section (4). As a further aspect we consider different degrees of real rigidity and the effects on the estimated Calvo parameter. We further show how results change when we augment the PC model by additional lags of inflation. Finally, we present results based on the AR statistic and confront them with the baseline GMM estimates.

## 5.1 GMM Results

We begin by presenting our baseline GMM results for the partial indexation model as well as for the model with rule-of-thumb firms. These estimates take as given the degree of real rigidity with calibrated values for $\alpha$ and $\epsilon$ (see Section 4). Further, the SC based instrument set serves as our benchmark instrument set. Since this is the instrument set that is associated with the smallest RMSC criterion (see Tables 6 and 6). Table 2 shows the results based on the partial indexation model. Point estimates for $\theta$ vary from 0.61 to 0.69. These are different form zero and different from one as well (the latter is necessary for the model to hold at least from an economic perspective). The estimates display reasonable values for $\theta$ which implies that firms re-optimize prices about every 3 quarters. In addition, the $J$ test of overidentification does not indicate any problems for this specification. The point estimates of the discount factor $\beta$ are somewhere around one which is also plausible from an economic point of view. We find little evidence for the full indexation scheme ($\xi = 1$) as proposed by Christiano et al. (2005) since the coefficient tests reject this hypothesis. Furthermore, we do not find much evidence for partial indexation in general implying that $\xi$ is close or equal to zero. This finding favors a pure forward looking specification without a lagged inflation term.

The evidence is more mixed when looking at the model with rule-of-thumb firms (Table 2). Here, the results differ considerably with respect to the way how the orthogonality condition is formulated. This is particularly true for point estimates of $\theta$ where the first orthogonality condition produces similar results as the model with partial indexation. But with orthogonality condition (2) the estimated values for $\theta$ are much smaller. Additionally, the $J$ test is significant for that specification. This casts doubt on the estimation results based on condition (2), but also on the

Table 1: Partial Indexation model (unrestricted)

| | $\beta$ | $\theta$ | $\xi$ | $J$ | Freq. |
|---|---|---|---|---|---|
| (1) | 1.030 | 0.611 | 0.248 | 8.454 | 2.6 |
| | (0.058) | (0.181) | (0.156) | | |
| | [0.000] | [0.001] | [0.112] | [0.390] | |
| (2) | 1.036 | 0.690 | -0.182 | 10.820 | 3.2 |
| | (0.038) | (0.270) | (0.097) | | |
| | [0.000] | [0.011] | [0.059] | [0.212] | |

*Notes*: Standard errors in round brackets and p-values in square brackets. Rows (1) and (2) correspond to the two specifications of the orthogonality conditions eqs (9) and (10) in the text, respectively. A 5-lag Newey-West HAC estimate was used. Sample period: 1973:1-2004:4. SC based instrument set.

Table 2: Rule-of-Thumb model (unrestricted)

| | $\beta$ | $\theta$ | $\omega$ | $J$ | Freq. |
|---|---|---|---|---|---|
| (1) | 1.026 | 0.590 | 0.146 | 8.454 | 2.4 |
| | (0.052) | (0.184) | (0.092) | | |
| | [0.000] | [0.001] | [0.111] | [0.390] | |
| (2) | 0.908 | 0.178 | -0.019 | 16.945 | 1.2 |
| | (0.104) | (0.035) | (0.020) | | |
| | [0.000] | [0.000] | [0.340] | [0.031] | |

*Notes*: Standard errors in round brackets and p-values in square brackets. Rows (1) and (2) correspond to the two specifications of the orthogonality conditions eqs (13) and (14) in the text, respectively. A 5-lag Newey-West HAC estimate was used. Sample period: 1973:1-2004:4. SC based instrument set.

model in general. This sensitivity to the normalization of the orthogonality condition may indicate some form of model misspecification. The estimates for the remaining parameters do not differ much from the ones obtained with the partial indexation scheme. Again, the discount factor is close to one and the backward looking inflation term ($\omega$ in this specification) seems to be unimportant.

## 5.2 Sensitivity Analysis

The general results are relatively robust to the particular instrument set used (see Table 6 and 7). However, differences between the two orthogonality conditions get more pronounced for the instrument set from GGL. We further check the sensitivity of our results for different assumptions about firm specific marginal cost. We first show how the estimates of $\theta$ change when we assume a markup of 25% ($\mu = 1.25$) instead of 10% as assumed in our baseline specification (Table 8). By decreasing the degree of real rigidity, the point estimates for $\theta$ rise slightly whereas the remaining parameters are in principal uneffected. But again, the estimates of $\theta$ stay in an economic meaningful range and cannot be rejected on empirical grounds. Next we give up the assumption of firm specific marginal costs and assume equal marginal cost accross firms ($A = 1$) as in the baseline model of Galí and Gertler (1999). This leads to a further rise of the estimated parameter $\theta$ to about 0.8 in the partial indexation model and to 0.6 and 0.8 in the model with rule-of-thumb firms (this implies an average frequency of price-re-optimization between 3 and 9 quarters). This specification still coincides with a sticky price framework which manifests in a higher degree of nominal price rigidity. From an empirical point of view we cannot favor one spefication over the other which only differ with respect to the way how firm specific marginal cost deviate from average marginal cost. Since the model is compatible with different assumptions about firm specific marginal cost it also introduces an additional source of uncertainty in estimating $\theta$ and the frequency of re-optimization.

The basic findings also hold for the case when we restrict the different model specifications to the pure forward looking specification and a discount factor of $\beta = 0.99$ (Table 3). Therefore we employ a likelihood ratio type test where we check whether the imposed restrictions can be rejected (Table 9 and 10). The tests indicate that the restrictions cannot be rejected and are thus imposed. With these restrictions both model specifications (the partial indexation model as well as the model with rule-of-thumb firms) are the same. This specification is purely forward-looking (does not include a lagged inflation term) where the coefficients are non-linear functions of the parameter $\theta$. Again we can construct two different orthogonality conditions that differ with respect to the particular normalization. As with the rule-of-thumb specifications the estimation results for $\theta$ differ considerably. But when we impose less real rigidity ($A \rightarrow 1$) the values for $\theta$ converge slightly, but the frequency of re-optimization of price changes of orthogonality condition (1) is always twice as high as compared to condition (2).

We also have a look at the sensitivity of inflation to our marginal cost variable. We denote the reduced form coefficient in front of the marginal cost variable with $\lambda$ (which is defined as $\lambda = \frac{(1-0.99\theta)(1-\theta)}{\theta}A$). To evaluate whether $\lambda$ is significant we use the point estimates for $\theta$ and its variance to construct standard errors for $\lambda$ with the delta method. The results are displayed in Table 11 and are quite heterogeneous with respect to parameter values as well as for their significance level. For the first specification we find small values of $\lambda$ that are not significant at conventional levels. The opposite is true for the second orthogonality condition. There we find larger

Table 3: Frequency of Re-optimization (Restrictions: $\beta = 0.99$, $\xi = 0$, $\omega = 0$)

| | $A = 0.1750$ | | $A = 0.3182$ | | $A = 1$ | |
|---|---|---|---|---|---|---|
| | $\theta$ | $\frac{1}{1-\theta}$ | $\theta$ | $\frac{1}{1-\theta}$ | $\theta$ | $\frac{1}{1-\theta}$ |
| (1) | 0.577 | 2.36 | 0.664 | 2.98 | 0.795 | 4.88 |
| | [0.34,0.81] | | [0.46,0.87] | | [0.65,0.94] | |
| (2) | 0.179 | 1.22 | 0.326 | 1.48 | 0.607 | 2.54 |
| | [0.11,0.24] | | [0.24,0.41] | | [0.52,0.69] | |

*Notes*: Confidence intervals in square brackets. Rows (1) and (2) correspond to the two specifications of the orthogonality conditions eqs (13) and (14) in the text given the imposed restrictions, respectively. A 5-lag Newey-West HAC estimate was used. Sample period: 1973:1-2004:4. SC based instrument set. $J$ test never rejects any model.

values for $\lambda$ that are always significant. These result cast doubt whether marginal cost is indeed the driving variable for inflation or whether the labor share is the correct measure of marginal cost.[22]

Finally, we consider consider additional variables in our structural model of the Phillips curve. Since it was sometimes argued that the New Keynesian Phillips curve omits further inflation lags (e.g., Jondeau and Le Bihan, 2006), we check whether our basic results hold when we put three more lags of inflation into our Phillips curve specifications. When the former specification is correct additional lags should not be a determinant of actual inflation (they should be solely a predictor of future inflation).

Tables 12 and 13 shows the results of these augmented specifications. Although the general interpretation continues to hold, we find that in either case the estimate of $\theta$ is higher than based on our baseline specification. The other parameters do not considerably change and still lie inside a plausible range. Another important feature, the differences between the orthogonality conditions (1) and (2) in the rule-of-thumb model, is still present and is not overcome by the inclusion of the additional variables. Some of these lags indeed turn out to be significant determinates of inflation (specifically the fourth lag). Similarly to Galí and Gertler (1999) we also test whether the sum of these coefficients are different from zero. We use a Wald test and find no evidence that the sum of additional lags are important.

Overall, the inclusion of additional lags does not lead to a complete rejection of our original specification. But it further shows how sensitive estimates of $\theta$ are to small changes of the model.

[22] Mavroeidis (2006) also shows for the US that the marginal cost variable does not turn out to be significant.

Table 4: AR type test of the estimated parameters

| Null Hypothesis | Test Statistic | p-value |
|---|---|---|
| $H_0 : \beta_0 = 0.99, \theta_0 = \hat{\theta}_{GMM1} = 0.58$ | 19.28 | 0.056 |
| $H_0 : \beta_0 = 0.99, \theta_0 = \hat{\theta}_{GMM2} = 0.18$ | 33.10 | 0.001 |

*Notes*: The test is evaluated with the CUE objective function. The SC based instrument set is used. A Newey-West HAC estimate with 5 lags was used. Sample period: 1973:1-2004:4.

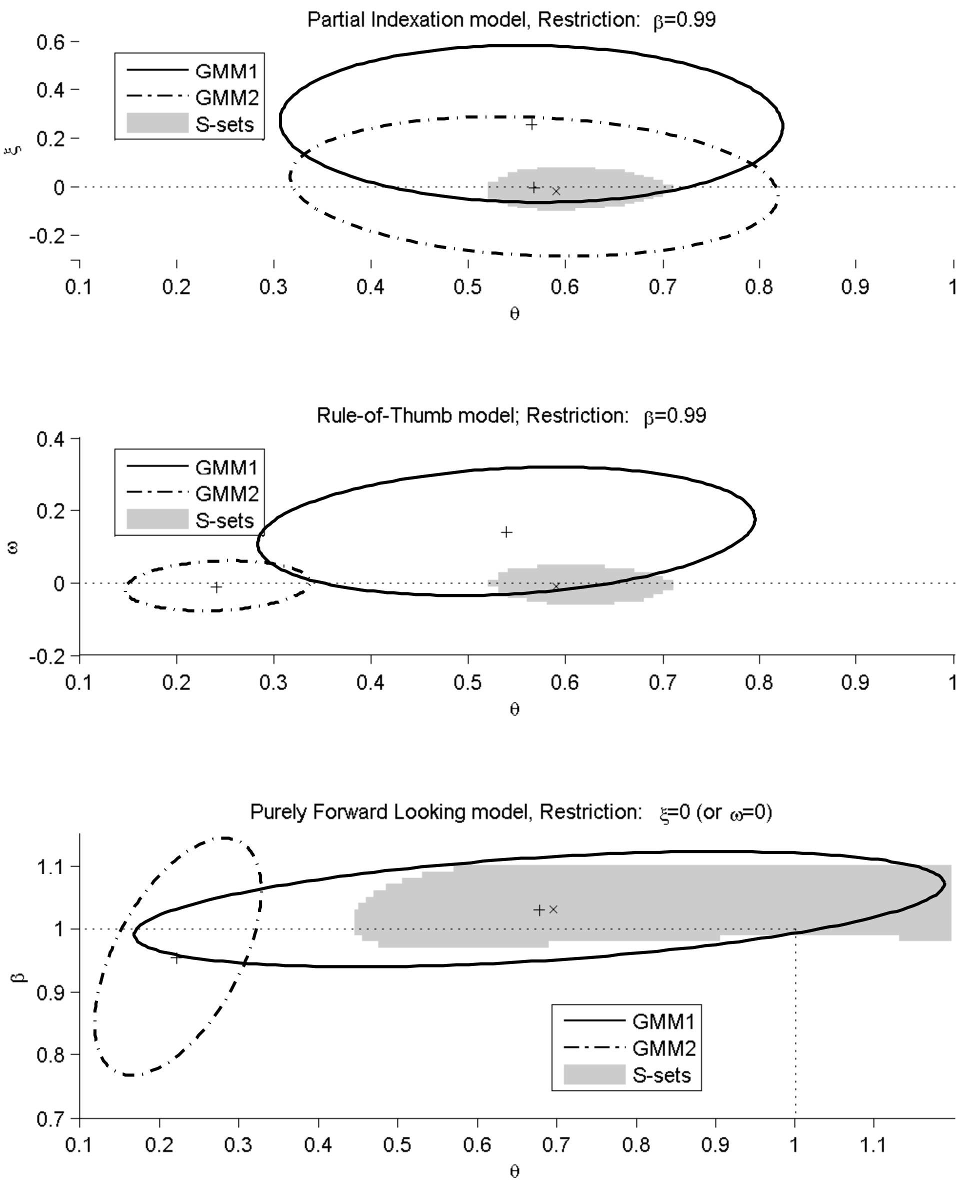
## 5.3 Results Based on the Identification Robust Procedure

So far our analysis rests on the assumption that our instrument set is sufficiently correlated with the endogenous variables under consideration. Whenever this assumption is violated and weak instrument problems occur, standard GMM estimates are unreliable. To examine whether those problems are present in this analysis we confront the baseline GMM estimates with the identification alternative as outlined in Section 3.2.

First, we check whether our GMM point estimates are also included the $S$-sets. This should be the case when the model is correctly specified and there are no weak instrument problems present. We start with the pure forward looking specification where we test the null hypothesis of whether $\beta$ and $\theta$ are $(0.99, 0.58)$ or $(0.99, 0.18)$ which corresponds to the GMM estimates of Table (3) with $A = 0.175$. According to Stock and Wright (2000) $S(\beta_0, \theta_0) \xrightarrow{D} \chi^2_k$, where $S(\beta_0, \theta_0)$ is the objective as defined above evaluated at the true parameter values. Figure (4) reports the results of this test type. The results indicate that problems with the orthogonality conditions may be present since the test rejects the hypothesis for both GMM point estimates, at least at the 10% level.[23] Now, we ask whether there exists a value of the parameter vector for which the model is not rejected. Given this particular instrument set (based on the SC) we find no parameter combination that lies inside the 90% $S$-set. That means that the confidence interval is empty and we have to reject the model. As mentioned above this indicates that the overidentifying conditions are invalid. So there may be one or more variables in our instrument set that do not fulfill the orthogonality condition. A natural candidate is a variable that is measured in $t-1$, so agents do not use this kind of information (due to a possible larger lag of publication). We exclude some of the instruments from period $t-1$ variable-by-variable and find out that wage inflation is the variable that causes the AR type test to reject the model. So we exclude that variable and redo the analysis.

[23] The test is more in favour of the first estimate, denoted by $\hat{\theta}_{GMM1}$. We follow Stock and Wright and take the 90% $S$-set as our final decision criterion.

Figure 2: Joint 90% $S$-sets and 90% GMM confidence ellipses for different specifications




*Notes:* SC based instrument set excluding wage inflation. Sample period: 1973:1-2004:4.

With the adjusted instrument set the $S$-sets are non-empty and can be used for inference of our model. Figure 2 shows the 90% confidence regions obtained with that method along with the standard GMM results and their 90% confidence ellipsis for different model specifications (both methods use the same instrument set). Generally, we find rather small $S$-sets irrespective which of the particular model used or restrictions imposed. For the partial indexation model, the computed $S$-set lies completly inside the two GMM ellipses (for the economic reasonable values). The regions all include the null of parameter $\xi$, implying that this value is not significantly different from zero. The results based upon the $S$-sets also imply a parameter value for $\theta$ of about 0.6 which translates into a frequency of price re-optimization of 2.5 quarters. The GMM results are similar. This estimate is in line with Coenen et al. (2007) who find an average frequency of price re-optimization of 2 quarters for the German economy, although with a different estimation strategy and a higher degree of real rigidities. The results based upon the rule-of-thumb model are in principle identical, even though the GMM estimates again differ quite substantially with respect to the transformation of the orthogonality condition. As mentioned above, $S$-sets are invariant to the normalization of the orthogonality conditions. From an empirical point of view, we cannot distinguish between the partial indexation model and the rule-of-thumb. But, as shown in the paper, the GMM estimates are sensitive to transformation of the orthogonality condition. That becomes very obvious in the rule-of-thumb model and the pure forward looking model where the differences between the two specifications are quite large.

Since the hybrid version of the Phillips curve is rejected we concentrate once more on the pure forward looking specification. While the $S$-set for this specification is again quite small, it already includes values for $\theta$ between 0.45 up to 1. This implies that the uncertainty about $\theta$ is quite high when no further restrictions on $\beta$ are imposed. This also translates into the sensitivity of inflation to marginal cost ($\theta = 1 \longrightarrow \lambda = 0$). When $\theta = 1$ prices are never re-optimized and thus do not respond to chances in marginal cost. As long as we cannot rule out the case that $\theta$ is equal to one, the model is economically meaningless and can also be seen as rejected.

Taken together, we show how different conclusions can be drawn depending on the particular estimation method. Interestingly, the identification robust procedure provides smaller confidence sets than conventional GMM (a standard finding for the US is the fact that identification robust procedures lead to larger confidence sets compared to standard GMM, see e.g. Ma, 2002; Mavroeidis, 2006). Notably, the GMM results are extremely sensitive to the way how the orthogonality conditions are formulated, a drawback not shared by our identification robust procedure. Additionally, identification robust inference with the nonlinear Anderson-Rubin Statistic may also help to detect model misspecifications not indicated by the standard $J$

test. These findings also offers an explanation for the large discrepancies of results between different studies of the German NKPC based on standard GMM. According to our analysis, a broad range of parameter values for $\theta$, the measure of nominal price rigidity, is compatible with our model.

# 6 Conclusion

This paper evaluates standard New Keynesian Phillips Curve specifications for Germany within a limited information framework. Besides the standard GMM estimation and test procedures, we also apply identification robust techniques. The presented evidence clearly favors a purely forward looking inflation equation for Germany which is in contrast to most other countries. The average frequency of price re-optimization of firms is estimated to be about two and three quarters, given plausible degree of real rigidity in the German economy. While these estimates seem plausible from an economic point of view, the uncertainty around these estimates are very large and also consistent with perfect nominal price rigidity where firms never re-optimize their prices. This also casts doubt concerning the labor share as driving variable for inflation.

In contrast to previous studies, confidence intervals from the identification robust procedure are smaller than results based on conventional GMM procedures. There is also some evidence of model misspecification that is not detected by the standard $J$ test of overidentifiying restrictions. These findings give an explanations why results for the German NKPC differ so much between existing studies based on GMM. Obviously, further work is needed to extend the basic framework for Germany. Empirical issues include to find a better proxy for the marginal cost measure (e.g. by the inclusion of real wage rigidities) and to deviate from the assumption of rational expectations through direct measures of inflation expectations.

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# Appendix

## A Data

The data are mainly taken from the national accounts database provided by the German Federal Statistical Office (known as Fachserie 18 Series 1.3). Additionally, data before reunification (prior 1991) are also available from the German Federal Statistical Office (Fachserie 18 Series S.28). In detail the series are defined as:

- Inflation: The inflation measure is constructed as the first difference of the quarterly log GDP deflator. The GDP deflator is defined as the ratio of nominal GDP and real chain-weighted GDP.
- Real marginal costs: The labor income share is computed as the total compensation of employees divided by nominal GDP.
- Instruments: Additional instruments that are considered are wage inflation ($\Delta w$) defined as the first difference of the log of compensation of employees; output gap ($y^{gap}$) constructed recursively as percentage deviation of real GDP from an Hodrick-Prescott filtered trend with $\lambda = 1600$; the three month money market rate ($r^s$); the long-short interest rate spread ($r^l - r^s$) between the ten year government bond yield and the three month money market rate; and commodity price inflation ($\Delta p^{comm}$) constructed as the first difference of the log of the HWWA commodity price index (defined in euro).

# B Tables

Table 5: Instrument selection based on Information Criteria

| | AIC | | SC | |
|---|---|---|---|---|
| | $\widehat{\pi}_{t+1}$ | $\widehat{s}_t$ | $\widehat{\pi}_{t+1}$ | $\widehat{s}_t$ |
| $\widehat{\pi}_{t-1}$ | -0.012 | 0.060 | -0.030 | 0.042 |
| | [-0.12] | [1.34] | [-0.29] | [0.94] |
| $\widehat{s}_{t-1}$ | -0.008 | 0.825 | -0.012 | 0.824 |
| | [-0.06] | [14.14] | [-0.09] | [13.71] |
| $\widehat{\pi}_{t-3}$ | 0.409 | 0.012 | 0.397 | 0.014 |
| | [5.09] | [0.36] | [4.86] | [0.41] |
| $\widehat{\pi}_{t-4}$ | 0.125 | 0.099 | 0.109 | 0.088 |
| | [1.53] | [2.82] | [1.33] | [2.50] |
| $\widehat{s}_{t-4}$ | 0.146 | 0.191 | 0.151 | 0.232 |
| | [0.76] | [2.32] | [0.78] | [2.80] |
| $\widehat{\pi}_{t-5}$ | 0.003 | 0.150 | -0.006 | 0.139 |
| | [0.04] | [4.12] | [-0.07] | [3.73] |
| $\widehat{s}_{t-5}$ | -0.120 | -0.144 | -0.102 | -0.177 |
| | [-0.70] | [-1.96] | [-0.59] | [-2.37] |
| $y^{gap}_{t-1}$ | 2.382 | 1.670 | 2.588 | 1.608 |
| | [2.62] | [4.30] | [3.13] | [4.52] |
| $(r^l - r^s)_{t-1}$ | 0.277 | -0.687 | -0.066 | -0.163 |
| | [0.54] | [-3.15] | [-0.64] | [-3.72] |
| $\Delta w_{t-1}$ | 0.124 | -0.031 | 0.137 | -0.021 |
| | [2.21] | [-1.30] | [2.46] | [-0.86] |
| $r^s_{t-1}$ | 0.518 | -0.479 | 0.147 | -0.076 |
| | [1.16] | [-2.52] | [2.18] | [-2.61] |
| $\Delta p^{comm}_{t-2}$ | 0.009 | 0.004 | | |
| | [1.85] | [1.97] | | |
| $(r^l - r^s)_{t-2}$ | -0.324 | 0.515 | | |
| | [-0.66] | [2.45] | | |
| $r^s_{t-2}$ | -0.397 | 0.391 | | |
| | [-0.90] | [2.09] | | |
| AIC | 1.1094 | | 1.1168 | |
| SC | 1.7429 | | 1.6146 | |

*Notes*: t-statistics in brackets.

Table 6: Partial Indexation model (unrestricted)

| Instruments | | $\beta$ | $\theta$ | $\xi$ | $J$ | RMSC |
|---|---|---|---|---|---|---|
| GGL's set | (1) | 0.996 | 0.632 | 0.309 | 8.877 | -9.76 |
| | | (0.084) | (0.216) | (0.153) | | |
| | | [0.000] | [0.004] | [0.044] | [0.353] | |
| | (2) | 1.047 | 0.980 | -0.333 | 11.095 | -1.63 |
| | | (0.040) | (54.43) | (0.072) | | |
| | | [0.000] | [0.986] | [0.000] | [0.196] | |
| AIC based | (1) | 1.035 | 0.646 | 0.294 | 9.787 | -9.18 |
| | | (0.062) | (0.217) | (0.147) | | |
| | | [0.000] | [0.003] | [0.045] | [0.550] | |
| | (2) | 1.039 | 0.743 | -0.178 | 12.125 | -10.14 |
| | | (0.038) | (0.367) | (0.094) | | |
| | | [0.000] | [0.043] | [0.059] | [0.354] | |
| SC based | (1) | 1.030 | 0.611 | 0.248 | 8.454 | -10.51 |
| | | (0.058) | (0.181) | (0.156) | | |
| | | [0.000] | [0.001] | [0.112] | [0.390] | |
| | (2) | 1.036 | 0.690 | -0.182 | 10.820 | -11.63 |
| | | (0.038) | (0.270) | (0.097) | | |
| | | [0.000] | [0.011] | [0.059] | [0.212] | |

*Notes*: Standard errors in round brackets and p-values in square brackets. Rows (1) and (2) correspond to the two specifications of the orthogonality conditions eqs (9) and (10) in the text, respectively. A 5-lag Newey-West HAC estimate was used. Sample period: 1973:1-2004:4.

Table 7: Rule-of-Thumb model (unrestricted)

| Instruments | | $\beta$ | $\theta$ | $\omega$ | $J$ | RMSC |
|---|---|---|---|---|---|---|
| GGL's set | (1) | 0.997 | 0.601 | 0.186 | 8.877 | -11.11 |
| | | (0.072) | (0.224) | (0.097) | | |
| | | [0.000] | [0.007] | [0.056] | [0.353] | |
| | (2) | 0.836 | 0.121 | -0.019 | 16.050 | -17.05 |
| | | (0.160) | (0.027) | (0.015) | | |
| | | [0.000] | [0.000] | [0.214] | [0.042] | |
| AIC based | (1) | 1.030 | 0.621 | 0.182 | 9.787 | -10.47 |
| | | (0.055) | (0.223) | (0.095) | | |
| | | [0.000] | [0.005] | [0.057] | [0.550] | |
| | (2) | 0.883 | 0.181 | -0.018 | 17.971 | -16.20 |
| | | (0.102) | (0.033) | (0.020) | | |
| | | [0.000] | [0.000] | [0.353] | [0.082] | |
| SC based | (1) | 1.026 | 0.590 | 0.146 | 8.454 | -11.87 |
| | | (0.052) | (0.184) | (0.092) | | |
| | | [0.000] | [0.001] | [0.111] | [0.390] | |
| | (2) | 0.908 | 0.178 | -0.019 | 16.945 | -16.83 |
| | | (0.104) | (0.035) | (0.020) | | |
| | | [0.000] | [0.000] | [0.340] | [0.031] | |

*Notes*: Standard errors in round brackets and p-values in square brackets. Rows (1) and (2) correspond to the two specifications of the orthogonality conditions eqs (13) and (14) in the text, respectively. A 5-lag Newey-West HAC estimate was used. Sample period: 1973:1-2004:4.

Table 8: Sensitivity to different values of $A$

| | Partial indexation model | | | | Model with Rule-of-Thumb firms | | | |
|---|---|---|---|---|---|---|---|---|
| | $\beta$ | $\theta$ | $\xi$ | $J$ | $\beta$ | $\theta$ | $\omega$ | $J$ |
| $\alpha = 0.3,\ \mu = 1.25 \longrightarrow \epsilon = 5,\ A = 0.3182$ | | | | | | | | |
| (1) | 1.030 | 0.690 | 0.248 | 8.454 | 1.027 | 0.669 | 0.165 | 8.454 |
| | (0.058) | (0.151) | (0.156) | | (0.053) | (0.158) | (0.101) | |
| | [0.000] | [0.000] | [0.112] | [0.390] | [0.000] | [0.000] | [0.101] | [0.390] |
| (2) | 1.036 | 0.755 | -0.182 | 10.820 | 0.970 | 0.335 | -0.033 | 15.695 |
| | (0.038) | (0.151) | (0.156) | | (0.080) | (0.050) | (0.037) | |
| | [0.000] | [0.001] | [0.097] | [0.212] | [0.000] | [0.000] | [0.377] | [0.047] |
| $A = 1$ | | | | | | | | |
| (1) | 1.030 | 0.805 | 0.248 | 8.454 | 1.028 | 0.788 | 0.195 | 8.454 |
| | (0.058) | (0.096) | (0.156) | | (0.055) | (0.106) | (0.118) | |
| | [0.000] | [0.000] | [0.112] | [0.390] | [0.000] | [0.000] | [0.112] | [0.390] |
| (2) | 1.036 | 0.846 | -0.182 | 10.820 | 1.030 | 0.635 | -0.078 | 13.410 |
| | (0.038) | (0.135) | (0.156) | | (0.053) | (0.051) | (0.069) | |
| | [0.000] | [0.000] | [0.097] | [0.212] | [0.000] | [0.000] | [0.255] | [0.099] |

*Notes*: see above. SC based instrument set.

Table 9: Restrictions in the Partial Indexation model

| $H_0 : \beta = 0.99,\ \xi = 0$ | LR-Test | p-value |
|---|---|---|
| (1) | 0.0581 | 0.9714 |
| (2) | 4.3360 | 0.1144 |

*Notes*: SC instrument set.

Table 10: Restrictions in the Rule-of-Thumb model

| $H_0 : \beta = 0.99,\ \omega = 0$ | LR-Test | p-value |
|---|---|---|
| (1) | 5.9409 | 0.0513 |
| (2) | 2.1815 | 0.3360 |

*Notes*: SC instrument set.

Table 11: Sensitivity to marginal cost (Restrictions: $\beta = 0.99$, $\xi = 0$, $\omega = 0$)

$$\lambda = \frac{(1-0.99\theta)(1-\theta)}{\theta} A$$

| | $A = 0.1750$ | | $A = 0.3182$ | | $A = 1$ | |
|---|---|---|---|---|---|---|
| | $\lambda$ | $J$ | $\lambda$ | $J$ | $\lambda$ | $J$ |
| (1) | 0.055 | 11.236 | 0.055 | 11.236 | 0.055 | 11.236 |
| | (0.042) | | (0.042) | | (0.042) | |
| | [0.195] | [0.339] | [0.195] | [0.339] | [0.195] | [0.390] |
| (2) | 0.663 | 14.873 | 0.445 | 14.136 | 0.259 | 13.054 |
| | (0.178) | | (0.117) | | (0.072) | |
| | [0.000] | [0.137] | [0.000] | [0.137] | [0.001] | [0.221] |

*Notes*: Standard errors are computed with the delta method.

Table 12: Partial Indexation model with additional Lags

| | $\beta$ | $\theta$ | $\xi$ | $\phi_2$ | $\phi_3$ | $\phi_4$ | $H_0: \phi_2 + \phi_3 + \phi_4 = 0$ | $J$ |
|---|---|---|---|---|---|---|---|---|
| (1) | 0.793 | 0.846 | 0.141 | 0.0911 | -0.196 | 0.275 | 2.227 | 5.609 |
| | (0.146) | (0.428) | (0.109) | (0.093) | (0.081) | (0.069) | | |
| | [0.000] | [0.048] | [0.193] | [0.327] | [0.015] | [0.000] | [0.527] | [0.468] |
| (2) | 0.825 | 0.868 | 0.046 | 0.113 | -0.217 | 0.289 | 2.568 | 6.367 |
| | (0.131) | (0.507) | (0.099) | (0.093) | (0.089) | (0.074) | | |
| | [0.000] | [0.087] | [0.641] | [0.226] | [0.015] | [0.000] | [0.463] | [0.383] |

*Notes*: SC based instrument set (plus inflation at the second lag).

Table 13: Rule-of-Thumb model with additional Lags

| | $\beta$ | $\theta$ | $\xi$ | $\phi_2$ | $\phi_3$ | $\phi_4$ | $H_0: \phi_2 + \phi_3 + \phi_4 = 0$ | $J$ |
|---|---|---|---|---|---|---|---|---|
| (1) | 0.798 | 0.831 | 0.118 | 0.0911 | -0.196 | 0.275 | 2.227 | 5.609 |
| | (0.146) | (0.454) | (0.094) | (0.093) | (0.081) | (0.069) | | |
| | [0.000] | [0.067] | [0.211] | [0.327] | [0.015] | [0.000] | [0.527] | [0.468] |
| (2) | 1.081 | 0.273 | 0.016 | 0.0120 | -0.105 | 0.059 | 0.398 | 13.180 |
| | (0.186) | (0.082) | (0.039) | (0.031) | (0.033) | (0.044) | | |
| | [0.000] | [0.001] | [0.690] | [0.526] | [0.001] | [0.177] | [0.941] | [0.059] |

*Notes*: SC based instrument set (plus inflation at the second lag).