Finance and Productivity Growth: The Role of Intangibles

Sudipto Karmakar[1] Marko Melolinna[2] Isabelle Roland[3] Philip Schnattinger[4]

[1]Bank of England

[2]Financial Conduct Authority

[3]Bank of England

[4]Bank of England

14th CompNet Annual Conference

25-26 September, 2025 in Vilnius, Lithuania

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- **Intangibles are an increasingly important driver of productivity growth**.
- **But they are hard to finance** because of
  - severe information asymmetries,
  - uncertain liquidation values.
- These frictions limit their use as loan collateral.

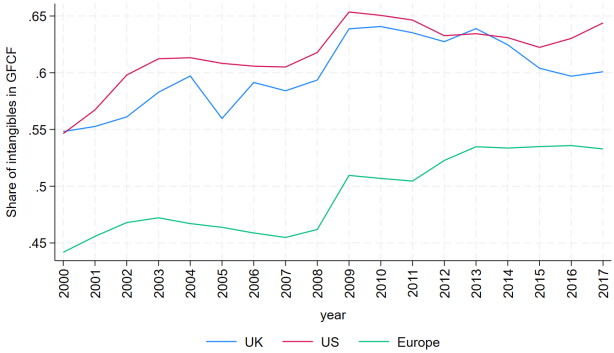
To what extent do financial frictions depress intangible investment and TFP growth?

UK case study

- **The UK's share of intangible investment in total investment is high** – similar to that of the US, and higher than in most EU-27 economies.
- **The UK has experienced a significant slowdown in TFP growth since the GFC**.
  - In 2022, TFP was approximately 17% below trend (average growth rate 1970-2007).
  - Slowdown in intangible investment is candidate explanation (e.g., Haskel et al., 2013).

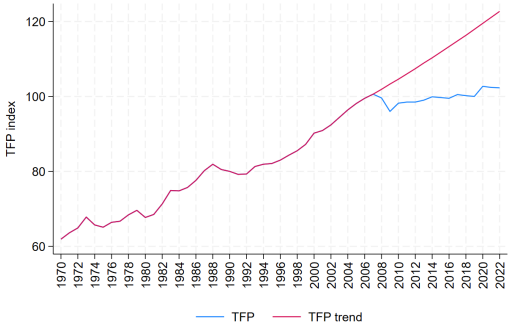
International comparison | UK productivity slowdown

Figure: Share of intangibles in total market economy investment



Source: EUKLEMS and ITANProd.

Figure: UK market sector TFP, 1970 to 2022 (2019 = 100)



Source: Office of National Statistics (ONS).

What we do

- **We develop a structural model** and estimate its parameters using data on large UK firms.
  - Model enables us to quantify the contribution of intangible investment to TFP growth.
- Standard firm profit-maximisation problem using dynamic optimization (Warusawitharana, 2015; Levine and Warusawitharana, 2021).
  - The firm maximizes its profits by choosing tangible and intangible investment.
  - Intangible capital is TFP-enhancing.
- **We augment the model with costly external debt financing**.
- Cost of debt is determined by:
  - Demand-side factors – leverage and intangible capital intensity.
  - Supply-side factors, which influence how these firm characteristics affect the debt premium.

What we do

- We introduce a novel method for estimating value function parameters that integrates a neural network with matching investment policy moments (similar to Cooper and Haltiwanger (2006) for capital adjustment cost). We estimate financial frictions for earnings-based leverage and the adjustment cost for tangible and intangible capital.

- We use the estimation results to perform two structural empirical exercises:
  - **Growth accounting analysis**: How has the contribution of intangible investment to aggregate TFP growth changed over time?
  - **Counterfactual analysis**: What is the loss of TFP growth resulting from a worsening of supply-side credit frictions affecting intangible investment in the post-GFC period relative to the pre-GFC period?

Preview of results

- **Sustained decline in contribution of intangible investment to TFP growth since GFC**.
- Primarily driven by demand-side factors:
  - Rise in earnings-based leverage, which reduces firms' debt capacity.
  - Elevated intangible capital intensity, which reduces collateral available to mitigate the impact of leverage.
- Decline amplified by post-GFC worsening of supply-side credit frictions.

Literature

- **Intangibles are an important driver of firm-level and aggregate productivity**:
  - Doraszelski and Jaumandreu (2013), Crass and Peters (2014), Moran and Queralto (2018), Anzoategui et al. (2019).

- **Intangible investment is hard to finance, due to limited use as collateral**:
  - Williamson (1988), Shleifer and Vishny (1992), Almeida and Campello (2007), Brown et al. (2009, 2012), Chen (2014), Haskel and Westlake (2017), Lei et al. (2018) Demmou et al. (2019, 2020), Dell'Ariccia et al. (2021), Falato et al. (2022) Lee and Paluszynski (2022), Bøler et al. (2023).

- **Financial frictions affecting intangible investment have macroeconomic implications**:
  - Bøler et al. (2023) make use of a natural experiment to estimate the magnitude of collateral constraints and to quantify their impact on productivity.

Data

- **Worldscope financial account data on large UK firms**.
- Annual data for 1990-2017.
  - Need long panel for intangible capital stock estimation through PIM.
  - Structural analysis performed for 2000-2017.
- Exclude finance and oil sectors (ISIC 2-digit sectors 2, 19, 64–66).
- Standard cleaning, e.g. data winsorised at 1st-99th percentiles.
- $\approx$ 1000 observations per year.
- **Main measurement issues**:
  - Estimate firm-level TFP – control function estimation as in Ackerberg, Caves, and Frazer (2015).
  - Construct firm-level intangible capital stocks as in Peters and Taylor (2017).

Variable definitions | Descriptive stats | TFP estimation

Firm-level intangible capital stocks

- **We follow Peters and Taylor (2017) and accumulate intangible expenses into stocks**.
- Intangible capital stock (K_{it}^{int}) is defined as:

$$K_{it}^{int} = INT_{it} + A_{it} + B_{it}$$

- INT_{it} is balance sheet intangible assets for firm i at time t, A_{it} is accumulated R&D spending and B_{it} is accumulated sales, general and administrative (SG&A) spending.
- Accumulated R&D and SG&A spending are defined as:

$$A_{it} = (1 - d_{R\&D})A_{i,t-1} + R\&D_{it}$$

$$B_{it} = (1 - d_{SG\&A})B_{i,t-1} + 0.3SG\&A_{it}$$

Empirical methodology - Intangible intensity and TFP

- Document positive correlation between intangible intensity and firm-level TFP.

$$z_{it} = \alpha + \beta_1 I_{i,t-1} + \beta_2 X_{i,t-1} + c_i + f_t + e_{it}$$

- z_{it} is the natural logarithm of TFP or the unanticipated TFP component (Levine and Warusawitharana, 2021).
- $I_{i,t-1}$ is a dummy for high intangible investment intensity or high intangible capital intensity (top quartile).
- $X_{i,t-1}$ is a vector of firm characteristics, namely (industry-median corrected) leverage, age, size, profitability, and cash.
- c_i and f_t are firm and time fixed effects.

Empirical results - TFP

	TFP	TFP	TFP_{η}	TFP_{η}
Leverage (t-1)	0.0473	0.0520	0.0515	0.0590*
	(0.0347)	(0.0360)	(0.0315)	(0.0328)
Intangible capital intensity (t-1)	0.0664***		0.0805***	
	(0.0135)		(0.0106)	
Intangible investment intensity (t-1)		0.0277***		0.0159***
		(0.00683)		(0.00568)
Observations	24,302	23,293	24,302	23,293

- **Strong positive correlation between intangible capital/investment intensity and firm-level TFP**.
- Robust to using unanticipated TFP component to address reverse causality.
- Coefficients on the other control variables are mostly significant and intuitive.
- Larger positive TFP effects for intangible stocks rather than flows - time lags (Hall et al., 2010).

- Firm maximizes PDV of future expected dividends by choosing tangible (i) and intangible investment (r).
- **Value function**:

$$V = \max_{i,r} E_0 \int_0^\infty e^{\rho t} \pi(z, k, s, b, i, r) dt$$

- **Laws of motion for state variables**:

$$\dot{z} = \mu_z - \rho_z(z - \mu_z) + \sigma_z dW \,,$$
$$\dot{s} = \delta_s s + r \,,\ \dot{k} = \delta_k k + i \,,$$
$$\dot{b} = \begin{cases} -\pi - bx & \text{if } \pi < 0 \\ -bx & \text{else} \end{cases}$$

- **Debt** is accumulated whenever current dividends are negative:
  - Debt accumulation is equal to $f = -\pi$ when $\pi < 0$ and $f = 0$ otherwise.
  - A fraction x of the stock of debt b is paid back in every period.

- **Current dividends** are the sum of income from production, minus the cost of tangible and intangible investment, minus the cost of debt.

$$\pi(z,k,s,b,i,r) = y(z,k,s) - i - r - \lambda_k \frac{k}{2}\left(\frac{i}{k} - \delta_k\right)^2 - \lambda_s \frac{s}{2}\left(\frac{r}{s} - \delta_s\right)^2 - b\left(x + B(z,k,s,b,\psi_1,\psi_2)\right)$$

  - Convex adjustment costs for both investment types (Eberly et al., 2008).

- **Output function** is defined as (assuming labour = 1 for simplicity):

$$y(z,s,k) = (z + s^{\gamma_1})(s+k)^{\alpha}$$

  - With α, $\gamma_1 \in (0,1)$ and $\gamma_1 < 1 - \alpha$ (DRS in both capital stocks).
  - Intangible capital enhances TFP (Klette & Kortum, 2004; Warusawitharana, 2015). Intangibles and TFP

- **Interest payments on debt**:

$$B(z, k, s, b, \psi_1, \psi_2) = \xi_0 + \frac{b}{y}\left(\psi_1 + \psi_2 \frac{s}{s+k}\right)$$

  - Base rate + parameterised leverage premium.

- **Firm-level conditions (demand side)**:
  - **Debt-to-earnings ratio** $\frac{b}{y}$ (Lian & Ma, 2021; Drechsel, 2023).
  - **Intangible intensity** $\frac{s}{s+k}$ - financial frictions bite more for intangible intensive firms (Demmou and Franco, 2021; Demmou, Franco, and Stefanescu, 2020; Falato et al., 2022).

- **Lending standards (supply side)**:
  - Parameters ψ_1 and $\psi_2 > 0$.
  - Determine to what extent firm-level conditions affect debt premium.

Solve with a neural network

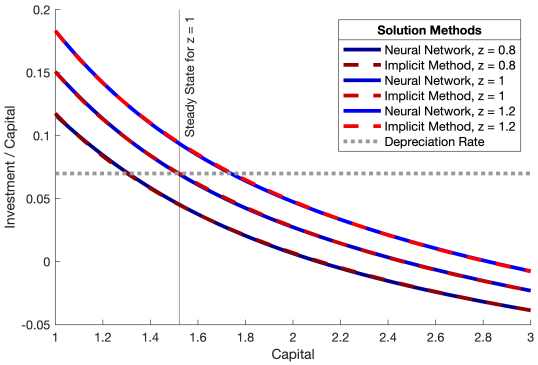
- **Use a NN to approximate the value function** as a nonlinear function of the state variables (see Fernández-Villaverde et al., 2023).
  - NN accurately captures non-linearities without specifying their structure ex ante.
- **Develop a new method to estimate a subset of value function parameters**.
  - Add parameters we want to estimate ($\Psi_X = [\gamma, \psi_1, \psi_2, \lambda_k, \lambda_s]$) to the state-space ($X = [z, k, s, b]$) for which we approximate the value function.
  - Calibrate remaining parameters $\bar{\Psi} = [\alpha, \delta_k, \delta_s, \rho, x, \xi, \rho_z, \mu_z, \sigma_z]$.
  - Let NN approximate value function for state space combining X and Ψ_X.
  - Estimate the parameters Ψ_X governing the value function by matching data moments.
- Approach leverages capacity of NN to efficiently handle value function approximation in high-dimensional state spaces.

Solve with a Neural Network

- Could solve with the implicit method described in Achdou et al. (2020).
- Quick to solve but requires extensive coding for four states.
- Short coding and longer solve with a neural network (similar to Fernández-Villaverde et al. (2023)).
- Easy to scale in states and performs similarly – shown for a simplified model with two states:

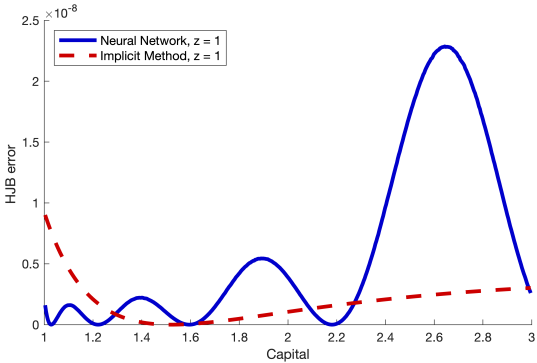
$$V = \max_{i,r} E_0 \int_0^\infty e^{\rho t} \pi(z,k) dt \qquad (1)$$

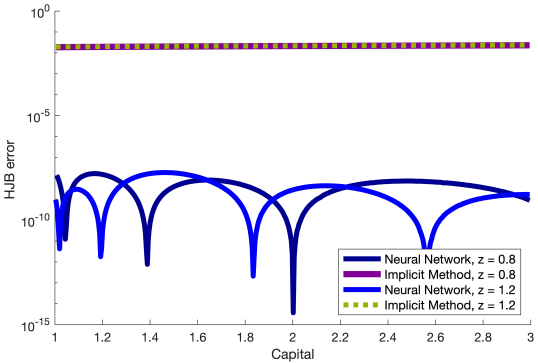
subject to: $\dot{z} = \mu_z - \rho_z(z - \mu_z) + \sigma_z dW$,
$\dot{k} = \delta_k k + i$



Neural Network solves the problem over the whole state space

HJB Errors for different productivity states.





1. Define the expanded state space $\widehat{X} = [X, \Psi_X]$, where X contains the original state variables and Ψ_X contains the parameters to be estimated.
2. Solve the value function $V(\widehat{X} \mid \bar{\Psi})$ using a neural network over the expanded state space $\widehat{X}$, holding fixed parameters $\bar{\Psi}$.
3. Estimate parameters by solving:

$$\hat{\Psi}_X = \arg\min_{\Psi_X} \left(\widehat{Y}(\Psi_X) - Y\right)' W^{-1} \left(\widehat{Y}(\Psi_X) - Y\right)$$

where $\widehat{Y}(\Psi_X)$ are model-generated moments, Y are observed data moments, and W^{-1} is the inverse variance-covariance matrix of the data moments.

Calibration NN structure NN training Estimation results

- We use our model to quantify the effect of intangible investment, and the financial frictions affecting it, on aggregate UK TFP growth.
- Given the state of firms z, k, s, b, what is the predicted intangible investment?
- **Growth accounting analysis**: How has the contribution of intangible investment to aggregate TFP growth changed over time?
- **Counterfactual analysis**: What is the loss of TFP growth resulting from a worsening of supply-side credit frictions affecting intangible investment in the post-GFC period relative to the pre-GFC period?

1. **State-space discretization**: Normalise stochastic productivity. Discretize annual state space using terciles for physical capital (k), intangible capital (s), and debt (b), yielding $J = 27$ combinations indexed by j.
2. **Averaging and normalization**: Compute average values of k, s, and b for each state-space combination within each year to ensure grid consistency with the model.
3. **Optimal investment policy**: Using estimated parameters $\hat{\Psi}_x = \{\psi_1, \psi_2, \lambda_k, \lambda_s, \gamma\}$, determine optimal intangible investment from solved value functions $V(z, k, s, b)$:

$$r = \frac{s}{\lambda_s} \left[\frac{\frac{\partial V(z,k,s,b,\hat{\Psi}_x)}{\partial s}}{1 - \frac{\partial V(z,k,s,b,\hat{\Psi}_x)}{\partial b} \mathbb{1}_{\pi<0}} \right] - 1 + s\delta_s \tag{2}$$

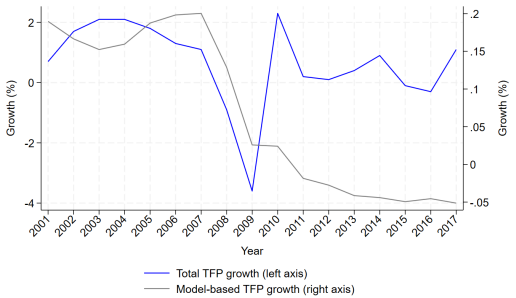
4. **Productivity calculation**: Aggregate implied TFP growth as a capital-weighted average across state spaces:

$$\text{Explained TFP growth}_t = \sum_{j=1}^{J} r_j^{\gamma} \frac{s_j + k_j}{\sum_{j=1}^{J}(s_j + k_j)} \tag{3}$$

5. **Benchmark comparison**: Compare model-implied aggregate productivity growth with actual ONS TFP data to quantify intangible capital's contribution to productivity growth.

Growth accounting: The contribution of intangible investment is declining

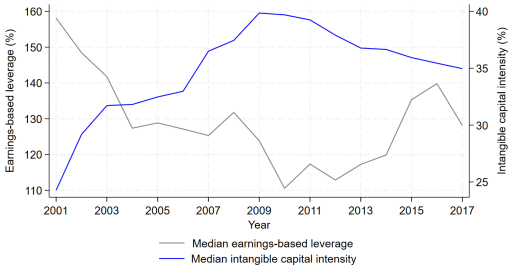
Figure: TFP growth explained by optimal intangible investment (%)



TFP growth explained by observed intangible investment

- Model-based TFP growth series accounts for only a small fraction of total TFP growth.
- Contribution of intangible investment to TFP growth has shown a persistent downward trend since the GFC.
  - Intangible investment contributed an average of 0.17pp to annual TFP growth between 2001 and 2008.
  - Contribution essentially vanished between 2009 and 2017.

Growth accounting: Demand-side explanation



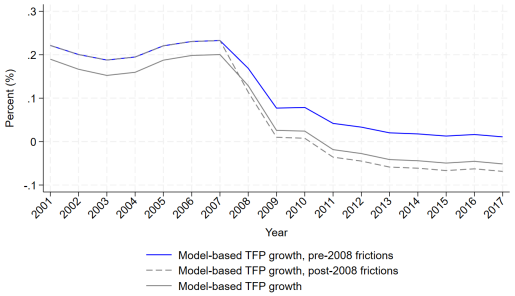
- Without a change in supply-side frictions (ψ_1 and ψ_2), this must result from changes in firm-level characteristics.
- Sharp increase in median earnings-based leverage from 2010 onwards.
- Median intangible capital intensity peaks in 2009 after a prolonged and sustained increase.
- Decline in intangible investment share translates into a decline in median intangible capital intensity from 2009 onward.

Counterfactual: Supply-side frictions have worsened

- Explore whether lending standards tightened post-GFC.
- Re-estimate ψ_1 and ψ_2 for pre- and post-GFC periods (2008 onward) - holding other parameters constant.

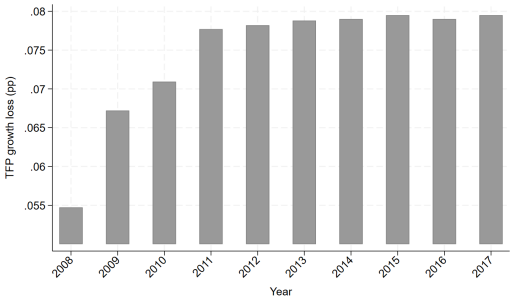
	ψ_1	ψ_2
Pre-GFC	Near 0	0.001029
Post-GFC	0.0073947	0.013903

Impact of tightening lending standards on aggregate TFP growth



- Model-based TFP growth scenarios: Pre-2008, post-2008, full-sample supply-side frictions.
- Post-GFC downward trend in contribution of intangible investment across all scenarios.
- Steepest decline when allow for post-GFC tightening of lending standards.
- Widening gap over time between pre-2008 and post-2008 supply-side frictions scenarios.

Impact of tightening lending standards on aggregate TFP growth



- Average annual TFP growth loss relative to pre-GFC benchmark: 0.074pp (2008-2017).
- $\approx$ 5% of decline in average market sector TFP growth between 2002-2007 and 2008-2017.

Conclusions

- **Sustained decline in contribution of intangible investment to TFP growth since GFC**.
- Primarily driven by demand-side factors:
  - Rise in earnings-based leverage, which reduces firms' debt capacity.
  - Elevated intangible capital intensity, which reduces collateral available to mitigate the impact of leverage.
- Decline amplified by post-GFC worsening of supply-side credit frictions.

Thank you!

References

Achdou, Yves, et al. (2022), “Income and wealth distribution in macroeconomics: A continuous-time approach.”, *The Review of Economic Studies*, Vol. 89.1, pp. 45-86

Ackerberg, DA., K. Caves and G. Frazer (2015). “Identification properties of recent production function estimators”, *Econometrica*, Vol. 83(6), pp. 2411–2451.

Buera, F., Karmakar, S. (2021), “Real Effects of Financial Distress: The Role of Heterogeneity”, *The Economic Journal*, Vol. 132, pp. 1309–1348

Cooper, R. W., and Haltiwanger, J. C. (2006). On the nature of capital adjustment costs. *The Review of Economic Studies*, 73(3), 611-633.

Duval R., Hong G. H. and Timmer Y. (2020). “Financial Frictions and the Great Productivity Slowdown”, *The Review of Financial Studies* (forthcoming).

Falato, A., Kadyrzhanova, D., Sim, J., and Steri, R. (2022), “Rising intangible capital, shrinking debt capacity, and the US corporate savings glut”, *The Journal of Finance*, 77(5), 2799-2852.

Fernández-Villaverde, J., Hurtado, S., and Nuno, G. (2023). “Financial frictions and the wealth distribution”, *Econometrica*, 91(3), 869-901.

Gopinath, G., Kalemli–Ozcan, S., Karabarbounis, L. and Villegas–Sanchez, C. (2017), “Capital Allocation and Productivity in South Europe”, *The Quarterly Journal of Economics*, pp. 1915–67

References

Hayashi, F. (1982). “Tobin’s marginal q and average q: A neoclassical interpretation”, *Econometrica*, 50(1), pp. 213-224.

Kalemli–Ozcan, S., Laeven, L. and Moreno, D. (2022), “Debt Overhang, Rollover Risk, and Corporate Investment: Evidence from the European Crisis”, *Journal of the European Economic Association*, 20(6), pp. 2353–2395

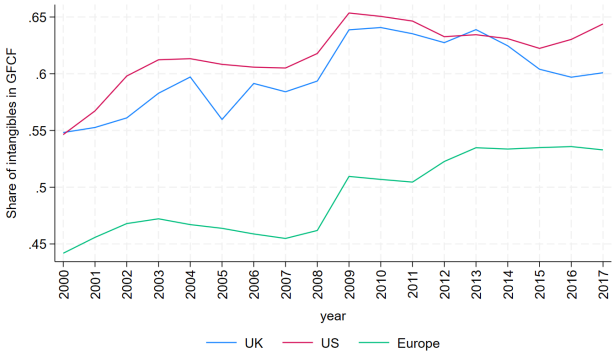
Levine, O. and Warusawitharana, M. (2021) “Finance and Productivity Growth: Firm–level Evidence”, *Journal of Monetary Economics*, Vol. 117, pp. 91-107

Maliar, L., Maliar, S. and Winant, P. (2014), “Deep learning for solving dynamic economic models”, *Journal of Monetary Economics*, 122, pp. 76–101

Peters, R. and Taylor, L. (2017), “Intangible capital and the investment–q relation”, *Journal of Financial Economics*, 123, pp. 251–272

Warusawitharana, M. (2015), “Research and development, profits, and firm value: A structural estimation", *Quantitative Economics*, 6, pp. 531–565

Figure: Share of intangibles in total market economy investment



Source: EUKLEMS and ITANProd.

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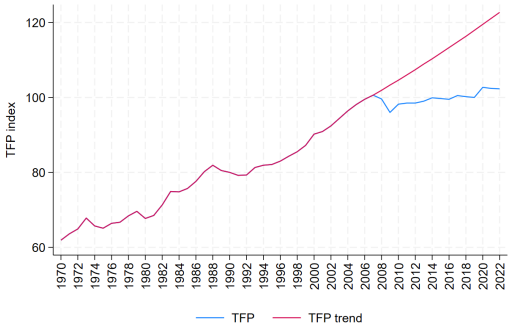
Figure: UK market sector TFP, 1970 to 2022 (2019 = 100)



Source: ONS.

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Data - main variables

Variable	Definition
TFP	Ackerberg-Caves-Frazer (2015) measure of TFP (log level)
Intangible capital intensity	Intangible capital stock over total assets
Intangible investment intensity	Year-on-year change in intangible capital stock over total assets
Leverage	Total debt over total assets (corrected for industry medians)
Age	Years since incorporation
Size	Real total assets (log level, deflated by aggregate GDP deflator)
Profitability	Profits (EBITDA) over total assets
Cash ratio	Cash and short-term investment over total assets

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Data - descriptive statistics

	Observations	Mean	Std. Dev.	Min	Max
TFP	27,712	1.057	0.538	-0.769	4.093
TFP growth	26,008	0.013	0.233	-1.585	1.942
Intangible capital intensity	27,712	0.504	0.562	0.000	4.618
Intangible investment intensity	26,677	0.042	0.219	-2.599	2.208
Leverage	27,712	0.201	0.192	0	1.365
Age	27,712	33.983	32.741	0	164
Size	27,712	4.660	2.088	-3.413	12.370
Profitability	27,566	0.061	0.243	-2.635	0.516
Cash ratio	27,705	0.134	0.163	0	0.906

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Data - TFP Estimation: The control function approach

- Estimating production functions is challenging due to simultaneity bias: unobserved TFP is correlated with input choices (K, L).
- This biases OLS coefficients, leading to inaccurate TFP estimates.
- We address this using the control function approach, expressing log TFP (ω_{it}) via observable proxies.
- We adopt Olley and Pakes (1996), and use intangible and tangible investment (r_{it}, i_{it}) as proxies for ω_{it}, conditional on capital (k_{it}).
- Log TFP is modeled as $\omega_{it} = h(i_{it}, r_{it}, k_{it})$, where $h(.)$ is an unknown, increasing function.

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Data - TFP Estimation: Olley and Pakes (1996)

- The production function becomes: $y_{it} = \beta_l l_{it} + \Phi(i_{it}, r_{it}, k_{it}) + \epsilon_{it}$, where $\Phi(i_{it}, k_{it}) = \beta_0 + \beta_k k_{it} + h(i_{it}, r_{it}, k_{it})$.
- Stage 1: β_l (labour elasticity) is consistently estimated by approximating $\Phi(.)$ with a polynomial. We apply the Ackerberg, Caves, and Frazer (2015) (ACF) correction to address potential collinearity.
- Stage 2: The capital elasticity (β_k) is identified by assuming TFP follows a first-order Markov process, making the TFP innovation uncorrelated with past capital choices. β_k is estimated using a moment condition.

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Decomposing TFP into expected and unexpected components

- Assume TFP follows a first-order Markov process:

$$\omega_{it} = E[\omega_{it}|\mathcal{I}_{it-1}] + \xi_{it}$$

- The expected (anticipated) TFP, $E[\omega_{it}|\mathcal{I}_{it-1}]$, is the part predictable by the firm based on past information.
- The unanticipated TFP (ξ_{it}) is the innovation, or unforeseen shock, to productivity.
- ξ_{it} is estimated as the residual from a regression of current TFP (ω_{it}) on its lagged values and other prior information.

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Calibrated parameters

Parameter	Value	Description
		Calibrated Parameters
ρ	0.05	Annual discount factor of 0.9512
δ_k	0.07	Depreciation rate of physical capital
δ_s	0.2	Depreciation rate of intangible capital
α	0.17	Exponent on combined (Baseline) or physical (C-D) capital
x	0.1	Loan principal pay back rate
ξ	0.005	Bank rate
μ_z	1.83 (Baseline) 1.3 (C-D)	Otherwise unexplained TFP (for steady state normalisation)
ρ_z	0.859	Standard values for a stochastic firm productivity process
σ_z	0.0143	Standard values for a stochastic firm productivity process

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- **Input layer:** 9 nodes – 4 state variables (k, s, b, z) + 5 parameters $\Psi_X = \{\psi_1, \psi_2, \gamma, \lambda_k, \lambda_s\}$.
- **Hidden layers:** 7 fully connected layers ($256 \rightarrow 128 \rightarrow 64 \rightarrow 32 \rightarrow 16 \rightarrow 8 \rightarrow 4$ nodes).
- **Output layer:** 1 node (value function $V(\cdot)$).
- Each hidden node performs a "mini-regression" by taking a weighted sum of inputs from the previous layer, adding a bias, and then passing the result through a **non-linear activation function** (hyperbolic tangent: $\tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$) .

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- **Training objective:** NN is trained on simulated data to satisfy the theoretical model's optimality conditions (HJB equation).

$$\rho V(z,k,s,b) = \pi(z,k,s,b,i,r) + \frac{\partial V}{\partial k}(-\delta_k k + i) + \frac{\partial V}{\partial s}(-\delta_s s + r) + \frac{\partial V}{\partial b}(-xb + f) + \frac{\partial V}{\partial z}(\mu_z - \rho_z(z - \mu_z)) + \frac{\sigma_z^2}{2}\frac{\partial^2 V}{\partial^2 z}$$

- **Loss function components:**
  - HJB residual – Mean Squared Error (MSE) between LHS and RHS.
  - Monotonicity constraints: $\frac{\partial V}{\partial k}, \frac{\partial V}{\partial s}, \frac{\partial V}{\partial z} \geq 0$.
  - Concavity constraints (second-order conditions).
  - Parameter sign constraints (economic theory).
- **Training data** is simulated from scaled beta distributions across the state space.

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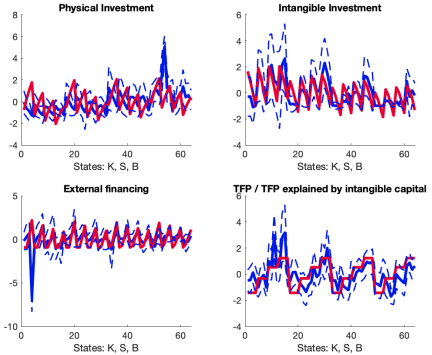
Estimation results and model fit

ψ_1	ψ_2	λ_k	λ_s	γ_1	γ_2
0.0072	0.0089	12.1070	9.1949	0.0844	0.1846

Figure: Model fit



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Estimation results

- Good model fit for predicted tangible and intangible investment and debt, conditional on states for the physical capital stock, the intangible capital stock, and debt (denoted K, S, B).
- Observed conditional moments are represented by the solid blue line, with their standard deviation intervals given by the blue dashed line.
- The red line shows the model-based predictions conditional on the states.

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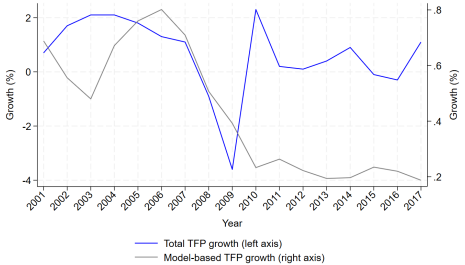
Figure: TFP growth explained by observed intangible investment (%)



- Contribution of intangible investment displays persistent downward trend since GFC.
- Contribution all but vanished between 2009 and 2017.

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