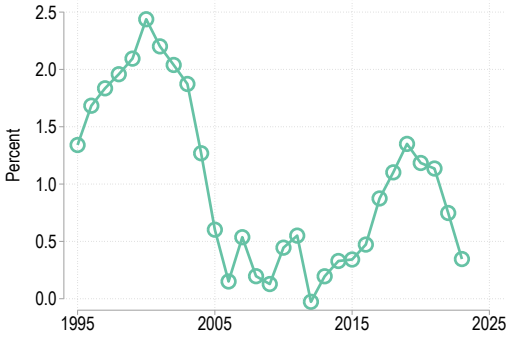
Emissions-Adjusted Total Factor Productivity

Maarten De Ridder and Lukasz Rachel

Motivation



(a) U.S. Productivity Growth

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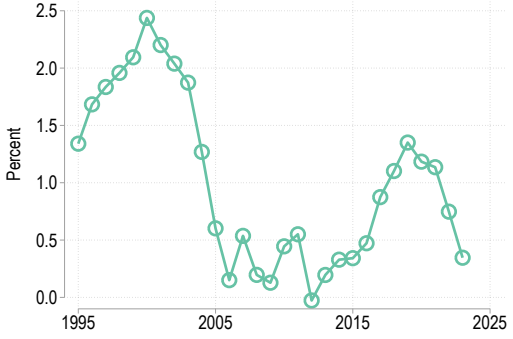

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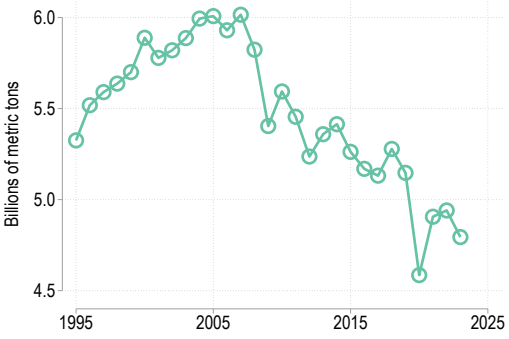

(b) U.S. Carbon Dioxide Emissions

This paper

Slowdown of productivity growth + reduction in carbon intensity in advanced economies

- Carbon emissions: by-product of production that reduces future economic activity
- Lower emissions are an increase in economy's efficiency not captured by today's TFP

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This paper: propose an emissions-adjusted measure of total factor productivity (TFPE)

- Marginal effect of inputs on present value of consumption taking climate damage into account
- Framework that nests seminal integrated assessment models (e.g. Nordhaus 1992)
- Straightforward. Relies on publicly available data + estimate of social cost of carbon

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Findings:

- For traditional, low estimates of social cost of carbon: TFPE $\approx$ TFP
- For recent, high estimates of social cost of carbon: U.S. TFPE growth is accelerating

Literature

- Macroeconomic effects of climate change

  e.g. Nordhaus (1977), Nordhaus (1992), Golosov et al. (2014), Hassler and Krussel (2018), Bilal and Kanzig (2024), Bilal and Stock (2025)

- Aggregate accounts that adjust for climate damages

  e.g. Weitzman and Lofgren (1997), Rodriguez et al. (2018), Nordhaus (2021), Agarwala and Martin (2022)

- Climate change, productivity and innovation

  e.g. Bovenberg and Smulders (1995), Acemoglu et al. (2012, 2016), Hemous and Olsen (2021), Hemous and Dechezlepetre (2022)

- Welfare relevance of aggregate output and total factor productivity

  e.g. Weitzman (1976), Jones and Klenow (2016), Rachel (2021), Basu et al. (2022), Adhami et al. (2024), Maideu-Morera (2024)

- Slowdown of (traditionally measured) productivity growth in advanced economies

  e.g. Adler et al. (2017), Goldin et al. (2024), Fernald et al. (2025)

Plan

Framework

Data

Results

Framework

Production and preferences

Standard setup of an integrated assessment model that nests Nordhaus (1992)

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$$Y_t = A_t F_t(\mathbb{K}_t) D(S_t)$$

- Assume competitive relative price of investment goods. National product:

$$Y_t = C_t + p\mathbb{I}_t$$

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- Emissions are by-product of production:

$$E_t = \phi_t F_t(\mathbb{K}_t)$$

- Preferences:

$$\boldsymbol{V}_t = \mathbb{E}_t \sum_{s=0}^{\infty} \left(\prod_{k=t}^{t+s} \beta_k C_{t+s} \right)$$

Productivity

Total factor productivity:

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Emissions-adjusted total factor productivity:

$$TFPE_t \equiv \frac{d\boldsymbol{V}_t}{dF_t(\mathbb{K}_t)} = \frac{\partial \boldsymbol{V}_t}{\partial C_t}\frac{\partial C_t}{\partial F_t(\mathbb{K}_t)} + \frac{\partial \boldsymbol{V}_t}{\partial p'\mathbb{I}_t}\frac{\partial p'\mathbb{I}_t}{\partial F_t(\mathbb{K}_t)} + \frac{\partial \boldsymbol{V}_t}{\partial E_t}\frac{\partial E_t}{\partial F_t(\mathbb{K}_t)}$$

- First two terms equal traditionally-measured total factor productivity

Relationship between TFP and TFPE using the envelope theorem:

$$
\begin{aligned}
TFPE_t &= \underbrace{\frac{\partial \boldsymbol{V}_t}{\partial C_t}}_{=1} \frac{\partial C_t}{\partial F_t(\mathbb{K}_t)} + \underbrace{\frac{\partial \boldsymbol{V}_t}{\partial p' \mathbb{I}_t}}_{=1} \frac{\partial p' \mathbb{I}_t}{\partial F_t(\mathbb{K}_t)} + \frac{\partial \boldsymbol{V}_t}{\partial E_t} \frac{\partial E_t}{\partial F_t(\mathbb{K}_t)} \\
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Inserting standard productivity definition, the emissions function, and welfare function gives:

$$\begin{aligned} TFPE_t &= TFP_t + \frac{\partial \boldsymbol{V}_t}{\partial E_t} \frac{\partial E_t}{\partial F_t(\mathbb{K}_t)} \\ &= TFP_t + \phi_t \mathbb{E}_t \sum_{s=1}^{\infty} \left(\prod_{k=t}^{t+s} \beta_k \right) \frac{\partial C_{t+s}}{\partial E_t} \end{aligned}$$

- Additional term with the present value of marginal climate damages

Measuring TFPE

Present value of marginal damages is a complicated object

- Depends on damage function, production function, effect on inputs, depreciation rate of carbon
- However, it is equal to the textbook definition of the social cost of carbon (SCC)

Insert the derivative into TFPE and use that emissions intensity $\phi_t = E_t / F_t(\mathbb{K}_t)$

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Hence, emissions-adjusted TFP is a simple to calculate:

$$TFPE_t = TFP_t \cdot \left(1 - \frac{E_t}{Y_t} SCC_t \right)$$

TFPE at the country level

Conceptual Issue: Should TFPE at the country level consider the effect on global or local output?

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- Common approach in climate policy is to internalize global externality (Stern et al. 2022):

$$\begin{aligned} TFPE_{it} &= A_{it} D(S_t) + \mathbb{E}_t \sum_{s=1}^{\infty} \left(\prod_{k=t}^{t+s} \beta_k \right) \frac{\partial C_{t+s}}{\partial E_{it}} \frac{\partial E_{it}}{\partial F_{it}(\mathbb{K}_{it})} \\ &= TFP_{it} \quad \cdot \left(1 - \frac{E_{it}}{Y_{it}} SCC_t \right) \end{aligned}$$

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- TFPE can also be calculated when not internalizing any of global externality, or in-between cases:

$$\begin{aligned} TFPE_{it} &= A_{it} D(S_t) + \mathbb{E}_t \sum_{s=1}^{\infty} \left(\prod_{k=t}^{t+s} \beta_k \right) \frac{\partial C_{it+s}}{\partial E_{it}} \frac{\partial E_{it}}{\partial F_{it}(\mathbb{K}_{it})} \\ &= TFP_{it} \quad \cdot \left(1 - \frac{E_{it}}{Y_{it}} DCC_{it} \right) \end{aligned}$$

Discussion

Three extensions:

1. Non-consumption utility costs of climate change
2. Concave utility instead of linear utility in preferences
3. Extending the framework to other inputs that affect future consumption

Relationship with green national accounts:

- Alternative to integrated assessment model: subtract climate damage from GDP
- If climate damage equals $E_{it} \times SSC_t$, TFPE is still productivity (TFPE $=$ Green GDP$/F(\mathbb{K})$)

▸ Green GDP

Extension: broader climate damages

Framework can be readily extended to broader definition of climate damages:

$$\widetilde{\boldsymbol{V}}_t = \mathbb{E}_t \sum_{s=0}^{\infty} \left(\prod_{k=t}^{t+s} \beta_k \left(C_{t+s} - \Omega(S_t) \right) \right)$$

Emissions-adjusted total factor productivity expression is unchanged:

$$TFPE_t \equiv \frac{d\widetilde{\boldsymbol{V}}_t}{dF_t(\mathbb{K}_t)}$$

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Hence:

$$TFPE_t = TFP_t \cdot \left(1 - \frac{E_t}{Y_t} \widetilde{SCC}_t \right)$$

- For example: Delgado et al. (QJE, 2022), old-age mortality adds 47$ to the cost of carbon

Extension: concave utility

Straightforward to calculate TFPE with a particular utility function:

$$\boldsymbol{U}_t = \mathbb{E}_t \left[\sum_{s=0}^{\infty} \beta^s u\left(C_{t+s}\right) \right] \quad , \quad TFPE_t \equiv \frac{\mathrm{d}\, \boldsymbol{U}_t}{\mathrm{d}\, F_t(\mathbb{K}_t)} \frac{1}{u'(C_t)}$$

- Divide TFPE by u' to measure productivity in consumption units (Adhami et al. 2024)

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Main result nests non-linear utility in consumption:

$$\frac{\partial \boldsymbol{U}_t}{\partial E_t} \frac{1}{u'(C_t)} = \mathbb{E}_t \left[\sum_{s=1}^{\infty} \beta^s \frac{u'(C_{t+s})}{u'(C_t)} \frac{\mathrm{d}\, C_{t+s}}{\mathrm{d}\, E_t} \right]$$

- Interpretation: utility function pins down the stochastic discount factor

Extension: other mispriced variables

Framework can be readily extended to variables other than carbon dioxide

- Other greenhouse gasses such as methane
- Other mispriced variables that affect future growth (e.g. research and development)

Welfare-relevant productivity: still the effect of inputs $F_t(\mathbb{K}_t)$ on $\boldsymbol{V}_t$:

$$\begin{aligned}\frac{\mathrm{d}\ \boldsymbol{V}_t}{\mathrm{d}\ F_t(\mathbb{K}_t)} &= \frac{\partial C_t}{\partial F_t(\mathbb{K}_t)} + \frac{\partial \mathbf{p}_t \mathbb{I}_t}{\partial F_t(\mathbb{K}_t)} + \sum_{X^j \in X} \left(\frac{\partial \boldsymbol{V}_t}{\partial X_t^j} \frac{\partial X_t^j}{\partial F_t(\mathbb{K}_t)} \right) \\ &= TFP_t \left(1 - \sum_{X^j \in X} -\frac{\partial X_t^j}{\partial Y_t} \frac{\partial \boldsymbol{V}_t}{\partial X_t^j} \right)\end{aligned}$$

Data

Data requirements

$$TFPE_{it} = TFP_{it}\left(1 - \frac{E_{it}}{Y_{it}} SCC_t\right)$$

Total factor productivity

- Total factor productivity at constant national prices from Penn World Table (Feenstra et al. 2015)

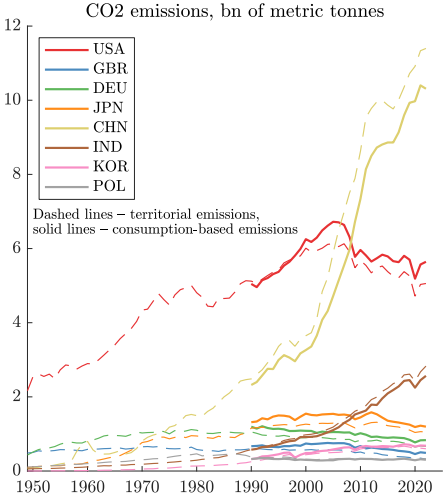
Carbon dioxide emissions

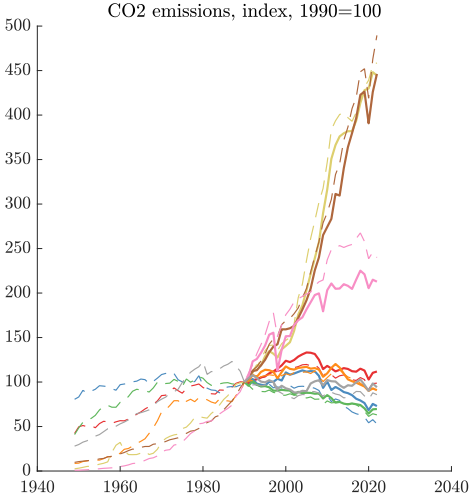
- *Consumption-based* carbon emissions from the Global Carbon Project (Andrew and Peters 2024)

Aggregate output

- Expenditure-side GDP at constant national prices from Penn World Table (Feenstra et al. 2015)

Emissions


CO2 emissions, bn of metric tonnes
USA
GBR
DEU
JPN
CHN
IND
KOR
POL
Dashed lines – territorial emissions,
solid lines – consumption-based emissions
12
10
8
6
4
2
0
1950
1960
1970
1980
1990
2000
2010
2020


CO2 emissions, index, 1990=100
500
450
400
350
300
250
200
150
100
50
0
1940
1960
1980
2000
2020
2040

Social cost of carbon

Two values - both assuming countries internalize global externality (equal SSC_t across countries)

1. Bilal and Kanzig (2025): \$923 (2017 USD) SCC from effect global temperature on global GDP
   - Upper bound of estimates in the literature. But: still focuses on narrow cost of climate change

2. Moore et al. (2024): \$253 (2017 USD) SCC from meta analysis

Assume SCC grows at 2% per year in line with World GDP (follows Nordhaus 2017, Prest et al. 2022)

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Assume SCC grows at 2% per year in line with World GDP (follows Nordhaus 2017, Prest et al. 2022)

For low values of the SCC, $TFPE \approx TFP$

- Implication: smallest productivity gains dwarf the costs of climate change
- Remainder of the analysis focuses on recent (higher) estimates

Results

Overview

Analysis consists of two main parts:

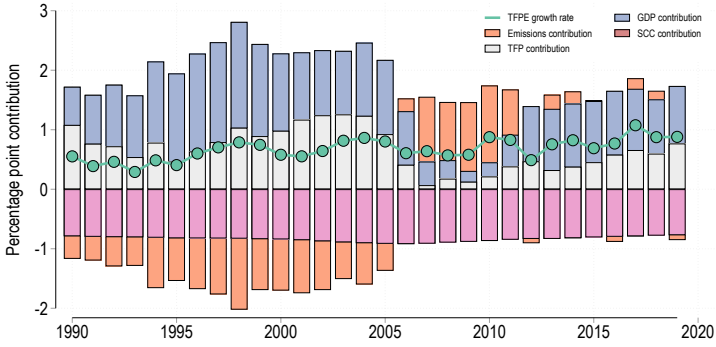
1. Growth rate of TFPE
   - Recent productivity growth in advanced economies is higher when adjusted for emissions
   - TFPE and TFP growth decouple in recent years

2. Level of TFPE
   - Countries with low E/Y: no room for TFPE growth $>$ TFP growth
     $\rightarrow$ TFPE growth $\neq$ strong climate performance
   - Ratio of TFPE and TFP: potential improvement in productivity from reaching net zero

TFP and TFPE growth for the United States

5-year moving averages

TFP and TFPE growth for the United States



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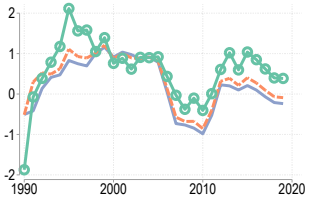
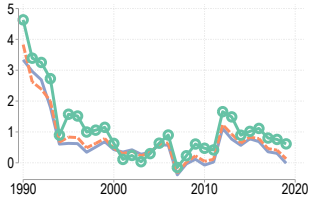
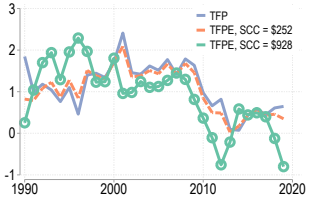


(c) United Kingdom

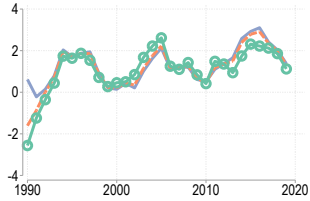


(d) Germany

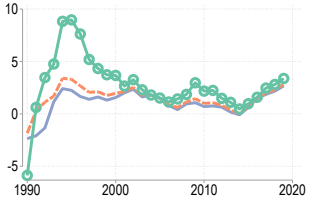


(e) Korea

(f) China



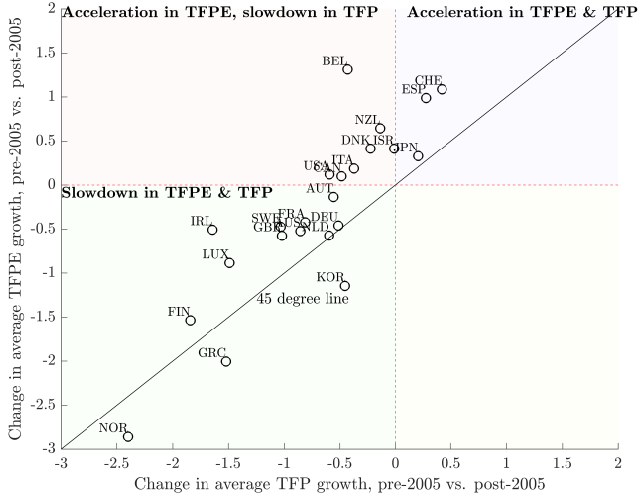
(g) Indonesia



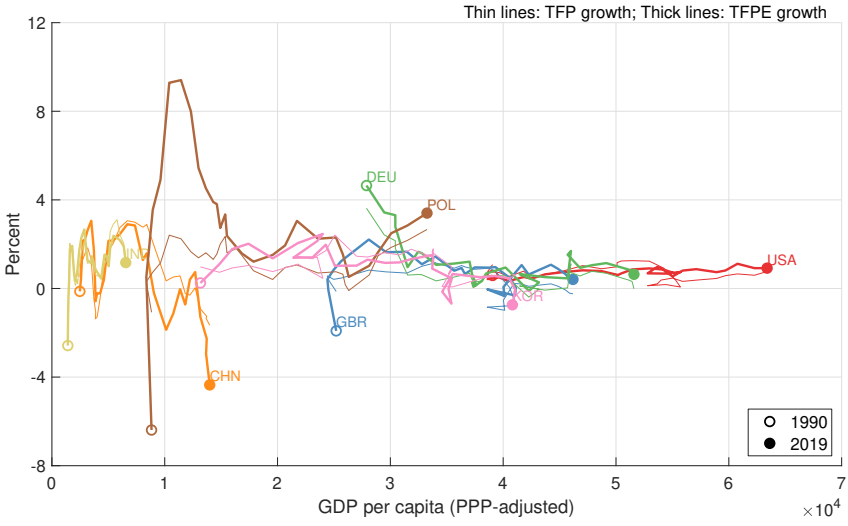
(h) Poland

5 year moving averages

Acceleration and slowdown in long averages of TFP and TFPE growth



TFP and TFPE growth rates along the development path



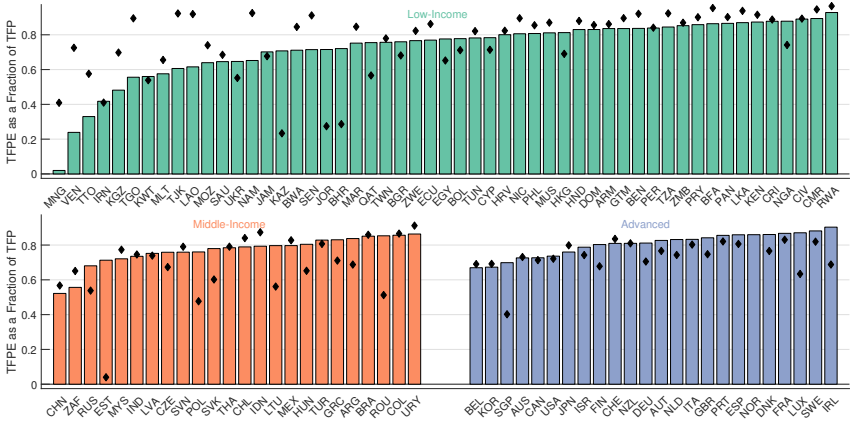
TFPE in levels

Next: compare the level of TFPE to the level of TFP

$$\frac{TFPE_{it}}{TFP_{it}} = \left(1 - \frac{E_{it}}{Y_{it}} SCC_t\right)$$

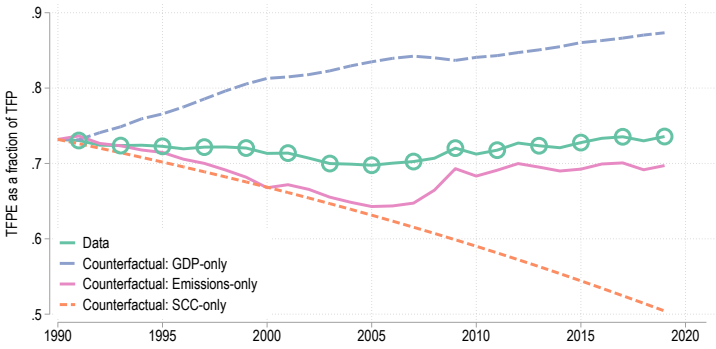
- Ratio is at most 1 (for $E_{it} = 0$) and can be negative if climate damage exceeds current output
- Reminder: country with $E_{it} = 0$ has no room for TFPE growth beyond growth in TFP
- Convenient interpretation: $\frac{TFPE_{it}}{TFP_{it}}$ is potential improvement in productivity from reaching $E_{it} = 0$

The level gap between TFP and TFPE



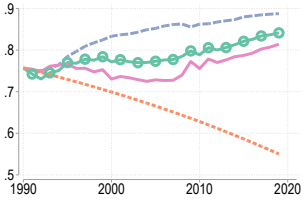
Diamonds: TFPE/TFP ratio in 1990. Bars: TFPE/TFP ratio in 2019.

Wedge between TFPE and TFP and counterfactuals in the United States

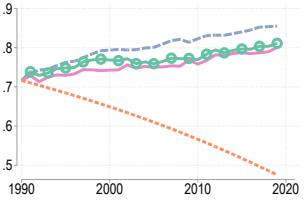


Lines: counterfactual path of TFPE holding other factors constant

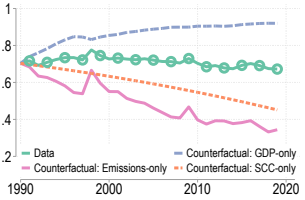
Wedge between TFPE and TFP and counterfactuals in Selected Economies



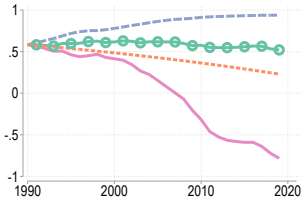
(i) United Kingdom



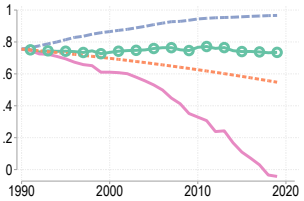
(j) Germany



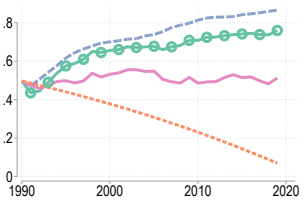
(k) Korea



(l) China



(m) Indonesia



(n) Poland

5-year moving averages

Conclusion

Traditional productivity statistics do not quantify the effect of emissions on future consumption

This paper: proposes a measure of productivity that adjusts for damage from carbon emissions (TFPE)

- Requires data on emissions, aggregate output, productivity, and estimate of social cost of carbon

TFPE changes recent patterns of productivity growth

- For high values of SCC: U.S. productivity growth accelerates after 2005
- Increasing emissions in China, South Korea imply that TFPE growth is negative

Appendix

Review of Weitzman (1976)

Net national product is:

$$Y(L, K, p) \equiv \max_{C, I \in \mathcal{S}(L,K)} C + pI$$

- C: consumption, K: stock of capital, I: net investment, p: relative price investment goods
- Law of motion for capital in continuous time: $\frac{\partial K(t)}{\partial t} = -I(t)$.
- $\mathcal{S}(L, K)$: production possibilities
- Assume constant interest rate r and constant labor L

Weitzman shows that in a competitive equilibrium (denoted by $*$), NNP is annuity value of welfare:

$$W^*(t) \equiv \int_t^\infty e^{-r(s-t)} C^*(s) ds = \int_t^\infty e^{-r(s-t)} NNP(t) ds = \frac{Y^*(t)}{r}$$

$$Y^*(t) = r \int_t^\infty e^{-r(s-t)} C^*(s) ds = rW^*(t).$$

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Review of Weitzman (1976) - Derivation

Two equations must hold in the competitive equilibrium:

$$Y^* \equiv Y(L, K^*, p) = C^* + pI^* \quad , \quad \frac{\partial Y}{\partial K} \mid_* = rp - \dot{p}$$

Totally differentiate $Y(L, K^*, p)$; assuming that L is constant:

$$\frac{dY^*}{dt} = \frac{\partial Y^*}{\partial K^*}\frac{dK^*}{dt} + \frac{\partial Y^*}{\partial p}\frac{dp}{dt}$$

Insert competitive equilibrium equations and $\frac{dK^*}{dt} = I^*$ and $\frac{\partial Y^*}{\partial p} = I^*$ (envelope theorem):

$$\frac{dY^*(t)}{dt} = (rp - \dot{p})I^* + I^*\dot{p} = r \cdot p \cdot I^* = r\left(Y^*(t) - C^*(t)\right)$$

Solving this differential equation yields

$$Y^*(t) = r\int_t^\infty e^{-r(s-t)}C^*(s)ds.$$

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Green GDP and TFPE

Weitzman & Lofgren (1997): welfare-relevant national accounts adjust for "natural capital" depreciation

Nordhaus (2021): calculates "green GDP" by subtracting $E_{it} \times SCC_t$ from aggregate output

- Productivity index of "green GDP" overlaps with TFPE:

$$
\begin{aligned}
TFP_{it}^{green} &= \frac{A_{it} D(S_t) F_{it}(\mathbb{K}_{it}) - E_{it} SCC_t}{F_{it}(\mathbb{K}_{it})} \\
&= TFP_{it} - TFP_{it} \frac{E_{it}}{Y_{it}} SCC_t \\
&= TFP_{it} \left(1 - \frac{E_{it}}{Y_{it}} SCC_t\right) = TFPE_{it}
\end{aligned}
$$

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