Debt and Equity Market Imperfections in a Production Economy

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ABSTRACT

We explore the consequences of a number of imperfections in financial intermediation for real economic activity in a production economy. Producers, financed by both debt and equity, face a mix of systemic and idiosyncratic uncertainty. Credit providers (banks) can either delegate interest rate setting to a loan manager with superior information about the borrower, or lend on the basis of public information alone (arm's length regime). We investigate the dependence of equilibrium on the lending regime choice, prior bias in public beliefs about aggregate productivity (business sentiment), and a prudential policy instrument (a convex dependence of bank capital requirements on the quantity of uncollateralized credit). We find (1) a dampening impact of delegation on aggregate credit to high-productivity firms (2) an adverse impact of incorrect sentiment on economic activity regardless of the sign of the prior bias (3) high sensitivity of economic activity to changes in macroprudential policies. So, introducing a macroprudential instrument has tangible welfare costs. The latter can be offset by fine-tuning capital charges as a function of corporate governance on the borrower side (specifically, by discouraging limited liability of borrowing firm managers).

JEL Classification: G21, G01, E22, D82

Keywords: bank, debt, agency, equity, capital, delegation

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1. Introduction

The latest global financial crisis has provided abundant examples of a sudden breakdown of credit relationships when poorly informed investors revised their previously held views. The aggregate magnitude of the ensuing negative financing shock to real economic activity was big enough to make the financial crisis go over into a severe worldwide recession. Although, initially, only a minority of financial institutions was affected by adverse balance sheet developments, businesses seemed to have difficulty finding a replacement for their original lender when the latter became either distressed or overcautious. In an ideal ("Modigliani-Miller") world of competitive and efficient financial intermediation often used as a convenient shortcut in macro models, there is no place for such effects. Although more recent DSGE-with-financial-frictions models assign a prominent position to the financial sector, they usually rely on a properly functioning financial intermediary as a *propagator of real shocks*. However, the latest global crisis, particularly the extent of credit decline at its peak, has uncovered a certain deficit of attention in macro modeling, to improperly functioning financial intermediaries as *a shock source*. The bulk of the existing macro literature is preoccupied with orderly market operation, conceding but a modest space to shortcomings, both on the capital provider and capital consumer sides.

On the other side, the theory of financial intermediation in its present state does not offer enough possibilities to compare relative strength of impact of its various phenomena of interest (such as agency, imperfect competition, institutional design, etc.) in a common setting relevant to macro theorists. Finally, asset pricing theory, once it has to depart from its well-fathomed Walrasian foundation, provides a lot of ambiguous messages about markets for producer liabilities, which are still to be integrated into the conventional macroeconomic paradigm.

All these problems could be alleviated if the workings of financial imperfections in macro theories were better understood. The present paper seeks to contribute to this objective by proposing a model of imperfect financial intermediaries in a production economy. To this end, we set up an environment in which firms seek both equity and debt financing under endogenous opacity. This means that some uncertainties in the producer performance are, in principle, resolvable in advance of the financing decision when the appropriate asset management regime is chosen, but incentives in financial institutions may work against the resolution. We are interested in consequences of this kind of imperfections for interest rates, capital formation and output in the affected real sector. In this paper, we discuss a two-period setup, mainly for reasons of space economy, although a multi-period generalization would constitute no conceptual problem under the chosen approach.

The firms have production functions with uncertain total factor productivity. This uncertainty has two components. The first is a systemic risk factor whose distribution function is known to everyone. In addition, there is a firm-specific component (firm's type) which is known to the firm management but cannot be precisely and credibly communicated to either equity investors or wholesale banks. The firm management can only send a public signal about the productivity level as a whole, in which systemic uncertainty contaminates the message about the idiosyncratic productivity component value. Only a loan manager with specific expertise (retail relationship banker) has the necessary non-transferrable skills to learn the borrowing firm's type.[2] Such a delegated manager can be hired by the wholesale bank for a fee to set the lending rate and collect the proceeds.

The first distinguishing feature of the model is that return on real investment is affected by a specific input required by the corporate governance mechanism in place in the firm. The input can be thought of as a separate strain of managerial human capital related to production (not to be mixed up with the knowledge about technology type, as in the previously mentioned relationship banker case). It is firm-specific and consequently, no individual is able to distribute its provision among all firms. This fact serves as an obstacle to full-fledged diversification of equity holdings by retail investors.[3] For simplicity, we concentrate on the extreme case by assuming that each of the retail investors can only observe the human capital level in a single firm. Then, by selecting the right parameters one can generate an economy in which holding shares in other firms is strictly dominated by only holding shares of the firm whose human capital level one knows. Thus, equity financing is possible but share demand only comes from a subset of knowledgeable investors.

Another key element of the model is a specific rationale for the existence of banks. Since, as agreed, the circle of possible equity holders of each firm is limited, firms also seek debt financing, whereas the retail investors look for opportunities to substitute for missing equity portfolio diversification by holding deposits. As opposed to retail investors, the bank (we use the term *wholesale bank*) can lend to any firm. That is why it can present itself as a diversifying intermediary but, at the same time, extract rents as an exclusive operator of the necessary financial technology.

Due to the mentioned technological exclusivity, the bank, or better said, financial services sector as a whole, disposes over a considerable market power. We have stylized the financial sector role in such a way that the outcome can replicate some prominent features of the recent turmoil. Namely, we take the common cause behind the financial crisis manifestations, as they were observed in different countries, to be the well-known agency problem of fund diversion. According to this view, financial institution sells claims to the public (here, collects deposits from retail investors) by declaring one investment pattern for the proceeds, whereas the actual management of borrowed funds follows a different pattern as far as it cannot be fully contracted and verified. The bank accepts deposits with the declared objective to invest them optimally in the whole spectrum of available firms, i.e. to diversify retail investors' funds for them. Naturally, returns on lending to every individual firm are higher if its type (the idiosyncratic component of its technology level) is observed, so that the wholesale bank is supposed to delegate to a relationship banker. However, the latter, being the exclusive holder of firm-specific knowledge, has a considerable bargaining power vis-à-vis the wholesale bank. Therefore, he can drive the required fee up to the level at which the wholesale bank becomes indifferent between employing his services and managing the loan itself based on public information alone. Then, it may happen that negotiations with the retail

[2] For simplicity, we only consider the case in which the loan manager finds out the type precisely, i.e. knows the same thing as the firm management. Generalizations allowing the relationship banker to learn the type with a noise, although with a higher precision than the public, are possible but do not add much to the qualitative insights of the model.

[3] One can imagine that every retail investor is somehow specialized in a particular economic sector represented by a set of identical firms. This is the sector to which the investor supplies own human capital (this time in a managerial capacity, so that specialization generates a learning-by-doing effect) and, as a result, disposes of sufficient expertise to make a qualified choice of stock to own within the sector, but not outside it. Alternatively, one could imagine a household of two, one member supplying equity financing and the other – human capital input, to a single familiar industry.

banker break down, delegation does not take place and the loan is managed by the wholesale bank "at arm's length". In the extreme, these arm's length loans can be packaged, tranched and sold to other banks in CDO form. And, at times, this can even be preferable both subjectively (the wholesale bank earns more on its loan portfolio) and socially (lower interest rates on average, more investment of debt-financed capital and hence higher output). The problem is that the outcome is sensitive, among other things, to the quality of public information.

This brings us to the third key element of the model: public sentiment. Public information about firms' types emerges as a Bayesian update of a prior belief distribution by an unbiased signal coming from the firm. If the prior distribution is biased, we say that there exists a (prejudiced) public sentiment. For instance, assume that there are just two productivity types, high and low. A priori, both the retail investors and the wholesale bankers may believe that there are more low productivity firms (the mass of low type is bigger than ½, pessimistic sentiment), whereas in truth, high and low types both have mass ½. (Other possible combinations of truth and sentiment are discussed in Section 3.) Since each firm's public signal is noisy, the prior belief update, although able to reduce the bias, is unable to completely eliminate it. Accordingly, prior prejudice impacts on equilibrium equity prices, lending rates, investment volumes and output. It is also possible that, under a particular sentiment and other exogenous parameter values, there exists an equilibrium with delegated loan management but no equilibrium with arm's length management. So, in that case, if wholesale-retail banker bargaining about the fee to the latter is unsuccessful, there is a big group of firms (in the binary example above – all low type ones) that cease to operate because they cannot finance production with either equity or debt, and there is a considerable reduction in output.[4]

One of the model applications considered in the paper is the introduction of a policy tool motivated by financial stability considerations. Namely, we investigate the impact of additional (and convexly growing) regulatory capital charges on banks that lend to firms with a low relative size of own equity. Although the true advantages and disadvantages of such policy instruments can only become fully visible in a dynamic model (whereas ours is a two-period one), we are nevertheless able to gauge basic qualitative consequences of the said policy for economic fundamentals within each period. This only becomes possible when one unites features of a usual model of production with financial intermediation effects, so that in this regard, our contribution is novel to macroeconomic modeling literature.

Summarizing our main findings, we establish that

A. Under unbiased public sentiment, delegation somewhat suppresses economic activity, but the result can be reversed if prior sentiment is pessimistic
B. There are economic costs of incorrect sentiment regardless of the prior bias sign
C. Macroprudential capital surcharges on banks have a strong depressing effect on economic activity without any reduction of default rates
D. Limited liability of the borrowers is one of the main sources of equilibrium fragility. Introducing more downside risk at default for managers of borrower firms helps in reducing sensitivity to exogenous parameters. The resulting equilibria in a modified

[4] This effect generated by the model, as we believe, offers a plausible imitation of at least some instances of the transition from "purely financial" revision of beliefs and the corresponding turbulence in asset markets, to the real adverse impact on investment and GDP, as was observed during the latest crisis.

"proportional liability" regime also entail reduced default rates for the most risky borrowers, compared to the pure limited liability case.

1.1 Related literature

Since macroeconomics and finance theory have traditionally gone their own independent ways, investigations into the interplay of financial and real shocks on macro level have been few and far between. The concept of costly state verification (CSV) in contract theory (Townsend, 1979) has been, over the years, gradually finding its way into real business cycle models (first of all, by suggesting the appropriate way of modeling default on debt contracts). Inspired by CSV models, the financial accelerator construction of Bernanke et al. (1999) has been an influential example of feeding a financial sector factor into quantitative macroeconomic theory. However, the (fulfilled) objective of Bernanke et al. (1999) was to *codify*, not necessarily *explain*, the main realities of financial sector presence in the economy, as they strived to reflect empirically important business cycle phenomena related to financial frictions. In essence, Bernanke et al. (1999) and the succeeding DSGE-with-financial-frictions models (e.g. Christiano et al., 2008) accommodate plausible sources of financial shocks through an ingenious choice of free parameters in otherwise standard optimization problems of agents. As the very term "financial accelerator" suggests, the financial sector shapes the real shock propagation mechanism in the economy, but does not itself originate the events of interest in these models. This is because capital suppliers do not possess sufficient pre-requisites with regard to either standing in the market or informational endowments.[5] Therefore, they are unable to "misbehave" in a natural way (e.g. in terms of adverse selection, reputation, incomplete contracts, herding behavior, etc.) along the lines drawn by the financial intermediation theory. The latter, on the other hand, relies on toy models which provide but a very indirect, if any, empirical guidance. Another insufficiently developed link in the current state of financial accelerator literature is with the asset pricing theory. Naturally, the latter, to the degree it is trapped in the efficient market paradigm, does not make synergies any easier. With the outbreak of global crisis in 2007, a more in-depth synergetic analysis received an unprecedented impulse, but relevant contributions are naturally taking time to materialize. Therefore, most literature to the point is quite recent and many inspiring studies still exist in a preliminary form only.

Logically, in the course of the current crisis, interest has turned to propagation of real effects of *financial shocks proper*, so that empirical evidence of such propagation will no doubt soon abound (see e.g. Campello et al., 2009, for an up-to-date contribution). At the same time, the new wave of attention to the monetary policy role in the run-up to financial crises has rekindled interest in formal modeling of macro-prudential policy tools that augment standard Taylor rule-based interest rate policies. Already, quantitative assessments, based on tentative synthetic techniques, are being conducted under the impression of the ongoing financial crisis and global recession, see, e.g. Chapter III of the IMF October 2009 World Economic Outlook. The exercise conducted there uses the approach inspired by i.a. Aoki et al. (2004), Iacovello (2005), and Monacelli (2009). Naturally, a proper quantitative analysis of the workings of

[5] One example is the full competitiveness assumption which imposes the zero-profit constraint on lenders. What may be a gain in analytic convenience (a reduction of the number of free parameters) is also a loss in flexibility, since the market power of the lender is a feature one would really want to be able to model. Besides, it is often overlooked that zero profit is a two-way "egalitarian" constraint: not just economic profit is prohibited, but also losses are ruled out. But, to model a bank without a downside risk would be nearly irrelevant for meaningful applications, for which claim at least the reality of the current crisis, even if nothing else, shall provide enough evidence.

those additional instruments requires a more explicit role of financial intermediation than was usual in earlier macro models. Our paper constitutes a step in this direction, as we propose a fairly general way of introducing macroprudential instruments in a production economy with a financial sector. Unlike some other recent contributions that, although taking both corporate and bank default into consideration, leave systemic driving factors of default outside the model (de Walque et al., 2009), we preserve the main features of the risky lending paradigm of the financial intermediation literature (see e.g. Stiglitz and Weiss, 1981), but connect this paradigm to the ones of neo-classic and neo-Keynesian production economies.

Our model output testifies of a high sensitivity of the economic activity in equilibrium to the incentives within the borrowing firms, delegation within financial intermediaries, and the marginal rate of prudential capital charges. As regards the first two factors, our results are akin to the body of knowledge within the existing strand of literature that describes the far-reaching implications of the manager incentive scheme choice under separation of ownership and control in otherwise standard DSGE models (cf. Donaldson et al., 2009). Awareness of the costs of the third (non-linear capital charge) factor has so far been widespread among practitioners. The present model complements this awareness with a micro-founded analysis.

The rest of the paper is organized as follows. Section 2 explains the construction of the model. Section 3 introduces a parametric version of the model which we solve numerically. In that section, outcomes of various numeric experiments are reproduced and interpreted. Then, in Section 4, we experiment with a macro-prudential policy instrument that generates non-linear charges on non-collateralized loans, and confront the outcome with the effect of modifying borrower liability in default. Section 5 concludes.

1. Model

The economy offers a set **L** of production capacities, or industries, that also serve as opportunities to invest. Each production capacity has its own c.r.s. production function to be described later, with inputs provided in period one generating stochastic revenue in period two. All inputs, investment and output are expressed in terms of a single unit of account. There are two periods and three groups of agents: retail investors, firm managers (or simply firms) and banks. The latter group has two tiers: wholesale banks and relationship banks. Next, we describe the objectives and choices of the named agents one by one.

2.1 Retail investors

Each retail investor disposes over a stock of initial wealth w_0 and a stock m_l of non-transferrable expertise in exactly one industry $l \in \mathbf{L}$. This human capital is sold to some firm from l (they are assumed identical) in period 1 at price z^l .For simplicity, we assume that human capital supply is inelastic, i.e. the whole stock m_l is sold regardless of the value of z^l. This same investor, or the second member of the same household, can use cash $w_0+z^l m_l$ available in period 1 to either buy shares in firms of the same industry l or put it in a bank account offering a fixed interest rate i. One share earns $y^l(A^l)$, where A^l is the total factor productivity parameter. Exact expressions will be given in the next subsection on firms. The important point is that, since another member of the same household supplies firm-specific human capital to l, the retail investor household knows the exact levels of inputs in the production function. Therefore, even though productivity realization in period 2 is uncertain, the degree of uncertainty is much lower than it would be if the investor decided to buy shares in another industry $n \in \mathbf{L}$. For an outsider, only return y^n without a breakdown into factor inputs and productivity would be known, which would combine uncertainties over A^n, physical capital k_n (see 2.2 on firms below) and m_n. Without going into technical details, we assume that the resulting uncertainty is so high that it is too risky and hence never optimal for any retail investor to reduce share holdings in "own" industry and buy shares in outside ones.

If the investor buys x_l shares in industry l at price p^l (which he takes as exogenous), his wealth in period 2 is equal to

$$w = x_l y^l + (1+i)\left(w_0 + z^l m_l - p^l x_l\right).$$

This final wealth, which is uncertain due to the uncertainty in A^l, enters the investor's utility function, whose conditional expectation in period 1 is maximized with respect to admissible choices of x_l. The interval of admissible choices is [0,1]. This means that the number of shares in each industry is normalized to unity and short-selling is not allowed.

Denote the investor utility by U and his subjective beliefs about the distribution of A^l-values by φ. We will only consider continuous non-atomic distributions so that φ is a well-defined density. Then the investor solves the problem

$$\sup_{0\le x_l \le 1} \int U\left(x_l y^l(A) + (1+i)\left(w_0 + z^l m_l - p^l x_l\right)\right)\varphi(A)dA\,. \qquad (1)$$

The outcome can be either an internal solution characterized by the first order condition

$$\int U'\left(x_l y^l(A) + (1+i)\left(w_0 + z^l m_l - p^l x_l\right)\right)\left[y^l(A) - (1+i)p^l\right]\varphi(A)dA = 0 \qquad (2)$$

or a corner solution in situations when the left hand side of (2) does not change sign for $x_l \in (0,1)$. We will exclude from consideration the trivial corner solution $x_l=0$ (corresponds to firms without any outside equity capital) and consider the remaining cases.

The internal solution is the one conventionally exploited by finance theory. In conjunction with the standard assumptions of identical investors (applied to our setting, this means a representative retail investor with special expertise in industry l, for each l separately) and market clearing (the representative investor holds $x_l=1$) it can be restated as

$$p^l = \frac{1}{1+i}\int U'\left(y^l(A) + (1+i)\left(w_0 + z^l m_l - p^l\right)\right)y^l(A)\varphi(A)dA\,. \qquad (3)$$

This expression can be interpreted as the expected payout on firm l stock discounted by the subjective stochastic discount factor. The latter is equal to $1/(1+i)$ times the investor's marginal utility of wealth U'. But, whereas standard asset pricing theories concentrate on the market pricing of risk that follows from the properties of the stochastic discount factor, we will keep in mind that the right hand side of (3) also depends on p^l, and look at (3) as an equation which determines this price implicitly.[6]

[6] Note that, being an equation which generalizes the conventional asset pricing formulae, (3) introduces equity market-based (co-)determination mechanism for physical capital. Such a mechanism is absent from extant financial accelerator models.

Additionally, we are interested in the corner solution x_l=1 which obtains when objective function (1) of the representative retail investor is increasing in x_l on the whole interval (0,1). Equivalently, the left hand side of (2) is everywhere positive in x_l and the investor actually gets to pay for the whole available stock the price below the expected discounted payout:

$$p^l < \frac{1}{1+i}\int U'\left(y^l(A)+(1+i)\left(w_0+z^l m_l - p^l\right)\right)y^l(A)\varphi(A)dA. \qquad (3C)$$

Naturally, there may be a whole continuum of prices satisfying this inequality. This situation is indeed possible and gives rise to multiple equilibria – an additional source of potential volatility not just in asset prices but also in interest rates, investment levels and output. Again, as was mentioned in the Introduction, a switch from a unique equilibrium implied by the internal price solution (3) to equilibrium multiplicity corresponding to a continuum of corner price solutions (3C) is possible by a mere shift of sentiment (a formal definition and extended discussion of the latter can be found in Section 3).

2.2 Firms

Firms have c.r.s. production functions with uncertain productivity and transform physical capital *k* and human capital *m* into output. Internal funds of the firm are insufficient to cover production costs, so that it seeks external financing both in equity and debt form. The firm is a price-taker in both those markets. Recall that equity is sold to a subset of retail investors (those who observe the human capital input into the same firm), whereas debt financing is reserved to banks. Incorporating the experience of costly state verification modeling (initiated by Townsend, 1979), we assume that even delegated loan managers of relationship banks are unable to observe the human capital input with enough precision to support a state-contingent (equity) contract. This allows us to exclude from consideration the case of banks holding equity.

For the time being, unless this causes ambiguity, we will omit industry index *l* when discussing a firm's actions.

Human capital input *m* must be paid for up-front in period 1. For simplicity, we assume that firms do not have initial cash holdings to do this. So a firm using *m* units of human capital has to borrow from banks at least the amount *zm*. More borrowing may be needed to finance physical capital, for which the identity $k = k_0+p+b$ holds. Here, k_0 is the initial non-traded "foundation" stock, standing for e.g. the stock held by the company founders, *p* is the "market capitalization", i.e. the value of shares sold in the equity market (recall that we have normalized the number of shares to unity) and, finally, *b* is the physical capital financed by a bank loan.

In the second period, the firm produces $Af(k,m)$ units of output. We assume that the whole stock of physical capital is then released as a part of firm earnings so that, in total, they are equal to $Af(k,m)+k$. (Since this is a two-period model, it makes little sense to consider capital depreciation explicitly.) Recall that in period 2, *m*-input has already been paid for from bank credit.[7] So, the dividend to stockholders is equal to what remains of the output after total debt,

[7] In this way, we avoid the need to account for consequences of a possible firm default on the payment to *m*-suppliers. In principle, we could have defined a contract with *m*-suppliers receiving payment in period 2. Then, under default, these claimholders would have been pooled with the lending bank for the purpose of debt resolution. However, this would have meant unnecessary technical complications without a contribution to the

i.e. $zm+b$, is serviced. Default occurs if output is not enough to repay the debt, in which case the bank seizes all earnings. Let the lending rate be r (taken by the firm as given, see more in the next subsection). Formally, shareholder dividends are

$$y(A) = \max\{Af(k,m) + k - (1+r)(zm+b), 0\}.$$

If the firm does not default, this dividend can be also written as

$$Af(q+b,m)+q-(1+r)zm-rb, \quad (4)$$

where $q=k_0+p$ is total equity capital (traded and non-traded).[8]

Each firm is run by a risk-neutral manager. For simplicity, we assume that he acts in the best interest of the shareholders (i.e. we abstract from agency effects in the shareholder-firm manager relationship). That is, the manager's objective is to maximize the expected dividend. The important nuance is that the said expectations are formed on the basis of manager's superior knowledge of productivity.[9] Namely, we assume that productivity is a product of two components: $A=LS$, of which S is the systemic uncertainty, perceived by everybody in this economy as a random variable with known distribution (for simplicity, let it be the same distribution for all firms). On the other hand, L is the firm-specific component, whose exact realization is known to the manager (and also to the relationship banker, see Subsection 2.3 below) but not to either the retail investor or the wholesale bank.[10]

Let us assume that there are exactly as many firms (industries) as there are productivity types. Then, our use of the same letter to index the firm set **L** (lowercase l) and firm-specific productivity value (uppercase L, and lowercase l for its log) should not cause confusion.

The firm manager takes the offered level of lending rate r and the m-price z as given. It is natural to assume that the equity price p and the overall level of equity capital q are also exogenous to him. He decides on optimal levels of m and b knowing that in default, the dividend he strives to maximize is zero. The critical level of systemic production uncertainty above/below which the firms survives/defaults is[11]

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main task of the present analysis, which is to explore real consequences of interactions between firms and banks. In addition, the used cash-in-advance constraint for m-supply allows us to simultaneously equip the model with both a liquidity constraint on the borrower side and a source of leverage. The latter emerges because (Section 2.1) the sum of zm across retail investors performs both as the cash deposited by them in banks (in excess of the initial wealth) and the lower bound of credit volume granted by banks to firms.

[8] Mind the difference of our q-variable and the net worth variable of Bernanke et al. (1999) and successors: since the latter (financial frictions) models do not have explicit equity markets, their net worth value is monolithic, whereas ours is naturally split into foundation and traded stock.

[9] One can compare this feature with Bernanke et al. (1999) and successor models: these, too, contain both aggregate and firm-specific uncertainty but the role of the former is played down, at a fairly high cost for the interpretation of results. Indeed, when systemic uncertainty is present, Bernanke et al. (1999) do not even have a proper debt contract in the model, and the state-contingent hybrid they have to use instead is quite difficult to rationalize. On the contrary, our model faces systemic uncertainty as a key fundamental factor and lets it play due role in both equity and debt pricing.

[10] The exact L-knowledge by both the firm manager and the delegated loan manager (relationship banker) is a useful technical simplification which, on the other hand, is not central for the qualitative results. What is important is that the degree of knowledge on the firm and the relationship bank side, even if different, be higher than that of the retail investor and the wholesale bank.

[11] Although this cutoff value is formally analogous to similar parameters used by Bernanke et al. (1999), Christiano et al. (2008) and related models (the usual notation there is $\overline{\omega}$), one should keep in mind that our

$$S^d = \frac{(1+r)zm + rb - q}{Lf(q+b,m)} . \tag{5}$$

Therefore, his dividend expectation is calculated over realizations of S exceeding S^d. Let us denote the cumulative distribution function of S by X and the corresponding density – by χ. The survival probability is then $X^+(S^d)$=1-$X(S^d)$, and we will also need the notation

$$\Psi^+\left(S^d\right) = \int_{S^s}^{+\infty} S\chi(S)\,,\ \bar{S} = \int_0^{+\infty} S\chi(S)\,,\ \Psi^-\left(S^d\right) = \bar{S} - \Psi^+\left(S^d\right),\ \theta\left(S^d\right) = \frac{\Psi^+\left(S^d\right)}{X^+\left(S^d\right)}.$$

Note that $\theta\left(S^d\right)$ is the expected systemic productivity component conditioned on survival.

Lemma 1 *Given the equity capital level q, human capital price z and lending rate r, the optimal decisions of a firm of productivity type L on m and b are characterized by the first order conditions*

$$\theta\left(S^d\right)Lf_m(q+b,m) = (1+r)z\,, \tag{6a}$$
$$\theta\left(S^d\right)Lf_k(q+b,m) = r\,. \tag{6b}$$

(In (6), subscripts denote partial derivatives.) The proof is straightforward given that, when calculating expected dividends, the firm manager integrates only over realizations of S that exceed S^d. As a consequence, the marginal products enter the first order condition with the tail expectation multiplier $\Psi^+\left(S^d\right)$, whereas the remaining part of the partial derivative of the dividend expression (5) – with the survival probability multiplier $X^+(S^d)$.

Since we assume fixed supply of m, (6a) shall be interpreted as a market-clearing condition on z, i.e. characterization of the human capital price that equalizes fixed supply with the demand determined by the marginal product of m. The second optimality condition, (6b), is an implicit characterization of the credit demand b=$B(r)$ as a – decreasing - function of charged lending rate. This is the firm manager's reaction function in the game it plays with the bank (see 2.3 and the subsequent section). Naturally, B also depends on q, z and the parameters of the model, but we omit them for simplicity in the notation.

Remark Since the production function is c.r.s., by combining (5), (6) and the Euler identity one arrives at the following condition on the survival threshold S^d:

$$S^d = \theta\left(S^d\right) - \frac{(1+r)q}{Lf\left(q+\hat{b}\left(S^d\right),m\right)}. \tag{7}$$

In (7), $\hat{b}(S^d)$ is the optimal choice of b implied by (6). The above condition is an equation for S^d whose solution depends on z, r and q as parameters. The problem is that, for typical distributions, production functions and a subset of otherwise realistic parameter values, this equation may have either two solutions or none at all. In the latter case, equilibrium

critical productivity value refers to systemic uncertainty realizations *conditional on the given firm-specific uncertainty*, whereas the named papers work with the firm-specific component.

equity+debt financing of such a firm cannot exist either, regardless of the presence of other firm types in the economy. In the former case, there emerges a possibility of two equilibria corresponding to high/low debt-financed levels of capital and high/low default probability in this firm type. Thus, our model is able to imitate real economic instability as a result of tiny financial shocks (see more in Section 4).

Now assume that the firm manager maximises the unconditional expectation of after-interest earnings (i.e. including the expectation over those *S*-realizations that would make net earnings negative in the absence of limited liability). Such a manager will borrow the following "unlimited liability" quantity of funds:

$$\bar{S}Lf_m(q+b,m) = (1+r)z\ , \qquad \text{(6aUL)}$$

$$\bar{S}Lf_k(q+b,m) = r \qquad \text{(6bUL)}$$

(Recall that $\bar{S}$ is the unconditional mean of systemic productivity component *S*.) That is, although the default consequences for the lender are the same as in the limited liability case, i.e. the bank seizes the output whose value is insufficient to repay the debt in full, the manager behaves "as if" he bore the full brunt of insolvency. To make managers behave like that, one would e.g. need a compensation scheme that is a function of after-interest earnings, e.g. a fixed fee, plus a percentage of actual – positive or negative – earnings. Similar remuneration schemes of "proportional liability" form, also in a much more general setting than the present one, have been considered by, for instance, Hui (2003).

In any event, firm choices based on (6UL) instead of (6) lead to the following analogue of (7):

$$S^d = \bar{S} - \frac{(1+r)q}{Lf(q+b,m)}. \qquad \text{(7UL)}$$

Now, the default threshold is uniquely determined by the endogenous variables *b*, *q* and *r* and the parameters of the model, i.e. the problem of equilibrium indeterminacy disappears. Unfortunately, managerial compensation schemes able to induce the said "unlimited liability behavior" are mostly a hypothetical possibility which one rarely encounters in corporate remuneration practice. Therefore, counting on financial intermediation disruptions following from the limited liability case (6), (7) is an empirical necessity ♦

2.3 Banks

The lending bank interaction with the borrower takes the form of a leader-follower game in which the bank is the leader and the firm is the follower. If a firm approaches a bank with a credit request, the latter makes an interest rate take-it-or-leave-it offer and the former decides on the loan volume based on this offer. That is, the firm formulates an optimal reaction to every value of the proposed lending rate (reaction function) and the bank sets the lending rate based on the information it has about this reaction function.[12]

[12] Since we make no further restrictions on the bank profit (e.g. no zero-profit assumption meant to imitate perfect competition), this set-up endows the bank with market power. The fact that, generically, bank-client relationship is not fully competitive on either part was recognized by the literature a long time ago. Santomero (1984) is an example of this early consensus. A more specific (and recent) example of imperfect competition modeling has to do with the concept of client "catch-up" in a specific bank, see e.g. Bonaccorsi di Patti and Dell'Ariccia (2004) or Dell'Ariccia and Marquez (2004).

The action of a bank depends on whether it is the original wholesale bank who negotiates the loan or the negotiations are delegated to a relationship banker. In the first case, the bank has a belief distribution over the borrower's productivity value *A* as a whole (convolution of beliefs about *S* and *L*). In the second case, we assume that the delegated loan manager knows type *L* exactly (just like the firm manager) and only faces systemic uncertainty regarding *S*. As a result, the wholesale banker sets a common interest rate for all borrowers whereas relationship bankers with delegation set separate rates for individual types.

Banks are assumed risk-neutral. The bank faces a cost of funds which, for simplicity, we denote by *i* (same as the deposit rate for retail investors) and assume a linear funding price regardless of volume. I Section 4, we will look at the consequences of relaxing the last of these three assumptions. Deviations from either of the first two assumptions can be easily accommodated in the model as well but are of subordinate importance for the subject of the paper.

We formulate the rate-setting problem of the delegated loan manager first. In the notation of the previous subsection, a firm of type *L* borrows $B=zm+b$, where the optimal quantities of both components are determined by the optimality conditions (6). Thus, from (6a), with $\hat{m}$ and $\hat{k}=q+\hat{b}$ being the optimal levels of, respectively, human and physical capital,

$$B=\frac{\theta\left(S^{d}\right)Lf_{m}(q+\hat{b},\hat{m})\hat{m}}{1+r}+\hat{b}\,. \tag{8}$$

Since we have agreed that *m* is in fixed supply for each firm (price *z* equalizes this supply with optimal demand), one can drop the hat in the notation: $\hat{m}=m$. Further, $\hat{b}$ can be expressed through *L, m, q, r* and $\theta=\theta(S^d)$ by using (6b). Often, the expression can be made explicit. For instance, for the Cobb-Douglas production function $f(k,m)=k^{\alpha}m^{1-\alpha}$ the named first order conditions imply that for optimally chosen physical and human capital, $\hat{k}$ and $\hat{m}$, $f\left(\hat{k},\hat{m}\right)=\left(\alpha\theta Lr^{-1}\right)^{\frac{\alpha}{1-\alpha}}\hat{m}$. Then, the preferred loan volume under lending rate *r* is equal to

$$B\left(\theta,L,m,q,r\right)=\frac{\alpha+r}{r(1+r)}\theta Lf(\hat{k},m)-q=\frac{\alpha+r}{r(1+r)}\theta^{\frac{1}{1-\alpha}}L^{\frac{1}{1-\alpha}}\alpha^{\frac{\alpha}{1-\alpha}}mr^{-\frac{\alpha}{1-\alpha}}-q\,.$$

In all cases, we will write $B=B(r)$ for the firm's choice of loan volume, by omitting the remaining arguments whenever it does not cause confusion.

Remark One can imagine situations in which the optimal level of physical capital is below the already available equity capital *q*, i.e. the firm does not need to finance physical capital by debt. It only has to borrow *zm* to finance "current first period expenditures", i.e. to pay for the human capital input. However, one can show that limiting lending to *zm* is infeasible as an equilibrium outcome for many important special cases. For instance, under Cobb-Douglas production banks, would be unwilling to lend at a finite rate to such firms. Therefore, we will not consider such cases in this paper. In the numeric examples to be discussed later, equilibrium debt levels turn out to by far exceed the current expenditure needs anyway ♦

The revenue from the loan is $(1+r)B(r)$ if the realization of S is above S^d (the firms survives) and $SLf(\hat{k},m)+\hat{k}$ if $S<S^d$. The cost is $(1+i)B(r)$ in both cases. The expected profit is taken over realizations of S (L is known) and can be written as

$$J^{RB}(L,r)=\Psi^-\left(S^d\right)L\hat{f}+X\left(S^d\right)\hat{k}+X^+\left(S^d\right)(1+r)B(r)-(1+i)B(r). \tag{9}$$

In (9), superscript RB refers to relationship banker and $\hat{f}$ is a shorthand for the production function value under the optimal choice of the firm. The loan manager chooses r to maximize the right hand side of (9) with the knowledge of loan demand function given by (8). When this maximization problem has a (finite) solution, and, under this solution, the firm equity is priced according to (3) or (3C), we obtain an equilibrium lending rate for the delegated loan management case, for the firms belonging to type (industry) L. This rate is type-dependent.

When the wholesale bank sets the rate for all firms itself without delegation, it has the objective function obtained by taking expectation over L of the right hand side of (9). That is (superscript AL refers to the arm's length handling of credit provision),

$$J^{AL}(r)=\int J^{RB}(L,r)\psi(L)dL, \tag{10}$$

where ψ is the probability density function of the public (hence also wholesale banks') beliefs about L. Both in the retail and the wholesale bank cases, the stock price p (equivalently, the amount of physical capital financed by equity q) of the loan applicant is taken as given.

Functions J^{RB} and J^{AL} both have at most one internal maximum $r^*(q)$ in r for every value of q. It is given by the obvious first order condition

$$J_r(r^*)=N_r(r^*)-B_r(r^*)i=0. \tag{11}$$

In (11), the superscript is dropped for notational economy and N denotes the sum of the four first terms on the right hand side of (9) in the relationship banking, and their L-expectation, as given by the right hand side of (10) in the wholesale banking, cases. Subscripts denote partial derivatives.

For the equilibrium to exist, the curves $r^*(q)$ and $q^*=q_0+p^*(r)$ must intersect in the (q,r)-plane. (Here, p^* is the stock price of the borrower determined in Subsection 2.2 as a function of lending rate r; this is a function if the price satisfies (3) and a correspondence if it satisfies (3C)). If the curves don't intersect, the equilibrium does not exist. If they intersect in more than one point, there are multiple equilibria.

3 Equilibria with and without biased sentiment

We go over to discussing quantitative properties of the model equilibria, which we have obtained by numerically solving the equation system (3), (11) with respect to variables q (equity capital) and r (lending rate). Equilibria are naturally split into two categories. The first is arm's length (henceforth denoted AL) loan management, when there is one lending rate for all borrowers. The second is relationship banking (RB), when there is one lending rate for each borrower type L. Recall that the stock price, equal to share capital less foundation stake ($p=q-q_0$) is in both cases common to all firm types, since retail investors in every stock have the same imperfect information about type as wholesale banks. In the following, we show the results for the simplified situation of just two productivity types, deviating down- or upwards from the average (so that $L \in \{L_d, L_u\}$, $L_d < L_u$), in which loan management is either AL or RB for all firms at once. If there were more than two elements in set **L**, one could also consider different wholesale banks choosing different subsets of **L** in which to try out delegation, but this ramification is left outside of the present analysis.

Information held by retail investors and wholesale banks alike is parameterized by the value λ giving the perceived proportion of high-productivity firms in the economy. We think of this information as being a result of a Bayesian update of some prior belief distribution common to all agents.[13] Every firm, although unable to communicate its productivity type credibly to anyone but its relationship banker, is nonetheless able to send an unbiased, even if noisy, public signal about its type. Then, the Bayesian belief update procedure results in a reduction (depending on the relative variances of the signal noise and the prior belief distributions), albeit never a complete elimination, of the prior error in the public perception.

Note that, when the solution of the equation system (3), (11) is being sought, the relevant value of λ is the one characterizing the beliefs and not the actual proportion of high-productivity firms (by the law of large numbers, it should be the same thing when the bias is zero). This is because the perceived λ enters both the retail investor and the wholesale banker decision problem (delegated loan managers already know the exact borrower type, so that for them, the value of λ is irrelevant). The true λ is important for determining economy-wide aggregates (for e.g. investment, bank credit and average output) after individual decision problems have been solved and equilibrium established.

We begin with showing the results of equilibrium calculation in the unbiased sentiment case and then discuss the changes caused by either optimistic or pessimistic prejudice.[14]

The results for the unbiased sentiment case are shown in Table 1. As was to expect, more high-productivity firms (i.e. higher value λ, both perceived and actual as long as there is no prior bias) in the economy mean more equity investment, but also higher lending rates (for everyone in the AL case and on average for the RB case). A less obvious outcome is a fall in bank credit, investment and output for each individual type at the same time as aggregate values of these fundamentals grow with λ. This is a sort of "income effect": under more numerous high-productivity firms, less effort is needed to attain a given level of expected output.

Further, looking specifically at equilibria in the relationship banking environment, one sees that lending rates for low-productivity firms fall (moderately) with growing λ, whereas they grow with λ for high-productivity firms. At the same time, higher λ corresponds to higher levels of bank credit, investment and output in the low- productivity segment, but lower levels of the same fundamentals, in the high-productivity segment.

Finally, aggregate investment, bank credit and output (we will refer to them collectively as "economic activity") are lower in RB-economies compared to AL-ones (for each fixed λ).

[13] The assumption of common prior beliefs was made to simplify the analysis of *public sentiment* implications. It can be easily relaxed if there are reasons to consider belief differentials across important subcategories of economic agents.

[14] All calculations were conducted using Mathematica®.

This is true when public sentiment is unbiased but can be violated under some realizations of prior prejudice. That is, it turns out that in the world we have created, it is often welfare improving to know less, i.e. not to delegate lending to loan managers who know the borrower type, rather than more (the effect is due to cheaper credit that, on average, the imperfectly informed wholesale banks charge everybody as opposed to relationship banks; the latter charge low-productivity borrowers a disproportionally high risk premium). So, in our model, not unlike the developed economies shortly before the outburst of the latest crisis, banks are tempted to resign on the costly use of intermediary agents with superior information and instead grant loans based on general formal rules (this is the essence of the AL-approach). Potentially, AL could also mean transferring loans to third parties whose ability to gain "soft" information on borrowers is minimal.

Note that all the discussed effects result from a complex interplay between equity and debt markets. i.e. they cannot be obtained by simply combining two partial-equilibrium models of each market separately. In the latter, investment and output would always fall with the lending rate (like in the IS-equation of the old Keynesian models), and the same is true for the equity price. Looking at Table 1, one immediately sees that our approach renders substantially different reduced-form behavior patterns of the basic fundamentals.

Next, let us turn to the role of bias in the public perception. The results illustrating the corresponding economic sentiment effect are collected in Table 2. Within each borrower type, determination of equilibrium equity price and lending rate depends on the perception (not the actual λ), i.e. does not depend on prior bias as such. The difference between subjective beliefs and reality matters for the observed economic aggregates. As expected, aggregate bank credit, as well as investment and output grow along with the actual proportion of high-productivity firms. On the other hand, for every fixed value of actual λ, economic activity *falls with growing perceived* λ. In other words, there exists an aggregate cost of incorrect economic sentiment. In this respect, RB-economies are slightly less sensitive to the prior bias than AL-economies, and it may also occasionally happen that RB-output under a particular sentiment value exceeds the AL-output (as when perceived λ is 0.4 and the actual one is 0.6 in our example). In all cases, inspection of Table 2 suggests that, for a fixed absolute size of sentiment error, it is socially preferable when people are pessimistic. This follows from comparing economic activity for, say, combination of actual λ=0.4, perceived λ=0.5 with the combination actual λ=0.5, perceived λ=0.4, etc. It remains to be seen to what extent this particular result is influenced by the utilized orthodox efficient market paradigm of equity pricing. The use of the latter paradigm may be also responsible for a relatively high sensitivity of economic activity values to sentiment changes: whereas the interest rate changes by 0.1 per cent, output values shift by 3 per cent and more under a 0.1-size change of sentiment (i.e. the perceived λ-value).

4 Macro-prudential capital charges and economic activity

In this section, we will test the ability of the constructed model to address a highly topical policy issue positioned, same as the model itself, on the borderline between macroeconomics and finance: the real effects of macroprudential regulation of financial intermediaries.

There exists enough reliable evidence that debt volume grows and its quality deteriorates, much faster in the run-up to a financial crisis than in normal times. Therefore, policymakers have for some time been looking for an adequate means to dampen unusual debt expansions and prevent credit bubbles without tethering "genuine" growth. One of the instances of this search is the discussion of an "anti-cyclical" reform of Basel II capital requirements on banks. At the moment, reliable recipes of separation of bubbles from sustainable growth are unavailable. So, most probably, in the pursuit of their financial stability goals, most regulators would resort to simple penalties of suspicious credit expansions by mandating attribution of increased risk weights to all lending that visibly exceeds the accepted target. That is, macroprudential policy instruments one is most likely to see are capital requirements on – and hence additional costs of funding of – incompletely collateralized loans, the requirements that would grow convexly with loan volume. In the present model, we can accommodate such an instrument by replacing the linear cost of funds term in the bank objective function by a linear-quadratic term that contains a surcharge on loan volume in proportion to growing borrower leverage.

Formally, we introduce the following macro-prudential control mechanism into the model. If the target level of physical capital of the borrower is *k* and the loan size is *B*, the bank is subject to an additional charge (in the form of regulatory capital) that leads to extra funding costs equal to $\frac{a}{2k}B^2$, where *a* is a positive constant. That is, the funding cost term $(1+i)B$ in (9) (and (10)) is replaced by $(1+i)B+\frac{a}{2k}B^2$. This means that

(a) funding costs are growing as a convex function of the loan volume and not linearly as the original equation (9) stated
(b) the surcharge is proportional to the product of the loan volume and the borrower's debt-to-physical capital ratio; every additional unit of credit is penalized unless offset by physical capital collateral financed with equity;
(c) the unit of penalty for uncollateralized credit is *a*, usually a small number in the order of single digit percentage points.[15]

Calculation of equilibrium under prudential capital surcharges can proceed in the same way as before, with only term *N* in equation (11) to be modified in accordance with the new definition of funding costs. We show the results for the base case of equal borrower type weights and no prior bias, in Table 3. For the sake of accurate comparison with the original model without prudential policy instruments, we assume that extra funding costs carried by the banks are turned over back to the private sector in the form of transfers (e.g. tax relief) and, therefore, are included in the aggregate output measures.

What we see upon inspecting Table 3 is that macroprudential policies in the defined form are a significant extra component in the price of credit and a heavy burden on economic activity. The bulk of this burden is carried by high-productivity borrowers, so that their distance from low-productivity ones in terms of investment and output is now smaller. In the present model, high-productivity firms take on more risks and default more frequently than those in the low-productivity segment. So, if the objective of macroprudential policies is to put a check on the expansion of the riskiest segments of the bank loan market, it is being achieved through dampening economic activity in the high-productivity segment. On the other hand, if the stabilization objective of the macroprudential instrument involves the number of defaults (this can be the case if defaults carry a negative externality that enters the social planner's objective function), then its introduction in our environment is clearly counter-productive: the number of defaults is now higher. And, since the relationship banking regime in general is more favorable to high-productivity firms, the costs of new policies are higher in RB-economies as

[15] In calculations, we have taken a=0.01.

well. The presence of biased sentiment (not shown) does not change much in the nature of the said results.

To sum up, the following advantages and disadvantages of the considered capital charge on bank loans materialize in the model. The two main advantages follow from the fact that application of the considered macro-prudential instrument

- helps the investors coordinate on an equilibrium mix of equity and debt financing in situations in which equilibria in the absence of this instrument do not exist; in our model this happens particularly when firm productivity types are distributed very unevenly or when the public economic sentiment is highly biased
- reduces imprudent leverage of highly productive borrowers.

On the contrary, the main problems associated with the use of the instrument can be identified as

- a uniform and significant increase in lending rates for all borrowers
- an increase in default rates (which may be a problem if those are associated with welfare externalities not considered in the model)
- excessive sensitivity of investment and output to small changes of the capital charge rate (this has to do with the additional transmission channel through equity markets).

Naturally, the actual *raison d'être* for the macroprudential tool of the above type is in its ability to stabilize inflation and output in the medium run, i.e. it can only fully transpire in a dynamic environment. Nevertheless, the consequences of its application in terms of expensive credit and low investment are likely to carry over from our present two-period to a multiperiod model designed along the same lines. Therefore, it would be always welfare-improving if one were able to come up with such a capital charge mechanism that could minimize side effects for quality borrowers. To cover this ground, we have considered a variant of the present model with proportional liability rules for firm management remuneration, which effectively induces unlimited-liability decisions on capital structure and input purchases. Our conjecture is that the road towards a welfare-improving capital requirement policy goes in the direction of encouraging lending to personally liable borrowers and penalizing excessive exposure towards borrowers with conventional limited liability. Supporting evidence in the present setting is provided by the comparison of the outcomes of the benchmark model (limited-liability borrowers) with the ones of the model under the said imitation of unlimited liability behavior (cf. the remark at the end of Subsection 2.2).[16] The comparison (under equal productivity type weights and unbiased public sentiment) is summarized in Table 4. We see that the induced mimicry of unlimited liability behavior has four major consequences compared to the benchmark:

(a) lending rates of both productivity types get quite close in the RB case, and approach the common lending rate of the LA case
(b) there is a sharp increase of the equity value of the high-productivity type, and a minor decrease of equity value of the low-productivity type

[16] Recall that the considered behavior of the borrower firm does not mean that the lender gets a full repayment in all states of nature, of which in adverse states of nature (output less than the debt service, i.e. default) a part of the compensation comes from the borrower's private wealth. As before, in default the jointly available assets of the firm and its management are insufficient to service the debt. All that is assumed here is that the firm manager compensation is an affine function of the firm earnings less the debt service. In that case, the manager would select production inputs *as if* the firm operated under unlimited liability.

(c) default probability of the high-productivity type falls substantially, whereas for the low-productivity type, although formally increasing, this probability remains negligible
(d) economic activity experiences a minor reduction compared to the limited liability borrower behavior.

If the prime concern of the policymaker is finding a macroprudential policy without a major negative impact on economic activity, the unlimited liability mimicry by borrowers, if one could get near it, would have an advantage over the previously considered convex capital surcharge instrument, provided one were at the same time able to encourage *delegated loan management* in banks. Indeed, suppose that unlimited liability mimicry is impossible to implement in the AL regime but possible – in the RB regime (that is, the relationship banker is able to influence the manager incentive structure in the borrowing firm). In that case, the trade-off for the macroprudential policy is between convex capital surcharge in the wholesale banks against unlimited borrower liability mimicry in the RB regime without capital surcharges. Comparing the upper right panel of Table 3 with the lower right panel of Table 4, one sees that losses in economic activity caused by abandonment of limited liability are more than compensated by the possibility to give up additional capital requirements. A reduction in default rates comes as a bonus on top of that.

More generally, there seem to be limits, in terms of economic activity and ex ante welfare costs, to promoting financial stability through policies directed at *credit providers*. At the same time, policies with the unchanged ultimate objective of credit bubble prevention, but directed at *credit consumers*, have largely remained unexplored (let alone exploited). Our results indicate that the potential gain from such re-orientation from regulating credit supply to educating credit demand may be worthwhile, notwithstanding numerous implementation difficulties.

5 Conclusion

We have constructed a model of financial sector as an interface between conventional optimizing general equilibrium macro models and partial equilibrium models of financial intermediation driven by information asymmetries. The three main building blocks of this interface are

- own human capital input-dependent return on equity, hence a skill-conditioned constraint on diversification of stock holdings by retail investors
- exclusive access to financial intermediation technology by wholesale banks, including the ability of the latter to choose between delegated and arm's length loan management
- prior beliefs (public sentiment) about productivity risks of the firms feeding into investment, interest rates and output.

We formulate an equation system characterizing equilibrium for the cases with and without delegation of lending by wholesale banks. Then, we conduct comparative statics exercises with the help of numeric solutions to this system for a number of important cases. Those include: variations in prior economic sentiment of the public, introducing non-linear regulatory capital requirements linked to unsecured loans, and relaxation of the conventional limited liability constraint on borrower behavior.

The quantitative characteristics of equilibrium in our model are quite sensitive to the level of initial own capital of the borrower. Varying levels of this “foundation stake” can produce an equilibrium breakdown in the arm’s length lending case, generate multiple equilibria both in the latter and the delegated loan management cases and, in a synergetic effect with prior bias, can cause large swings of interest rates and output. The effect is particularly strong in the textbook Cobb-Douglas production environment, but might be somewhat mitigated in the presence of convex capital installation costs.

A similar sensitivity exists with respect to the liability status of the borrower. Limited liability is one of the main sources of equilibrium fragility: under many combinations of parameter values, an equilibrium does not exist for pure limited liability borrowers but exists when borrowers mimic unlimited liability in their decisions under the pressure of specially designed incentives.

Nevertheless, one shall keep in mind that in all cases in which equilibrium either becomes indeterminate or falls apart, this happens in the environment governed by the standard (“Walrasian”) asset pricing paradigm. The latter is used to price company equity and hence also determine – in a one-to-one relation – the level of equity-financed physical capital. It is possible that an adjustment of the model that relaxes this unrealistic frictionless link between stock price and investment will be also instrumental in restoring a well-defined unique equilibrium. This question is left to future research.

We believe that the modeling ideas outlined in the paper should help future macro-models address the empirically tangible spillovers from financial sector shocks to the real economy in a more convincing way than has been the case so far. In particular, our model accommodates an endogenously mixed equity-debt financing of production. Further, it contains a flexible agency-based rationale for the role of banks and a workable quantification of the economic sentiment notion (usually considered a “soft”, non-quantifiable concept) within integrated macro-financial modeling setups. Additionally, we are able to investigate real economic implications of macroprudential policies motivated by financial stability considerations.

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Table 1 Economic fundamentals in equilibrium with unbiased sentiment

Proportion of high-productivity borrowers →		λ=0.4		λ=0.5		λ=0.6	
Indicator ↓			*Aggregate*		*Aggregate*		*Aggregate*
AL	q	3.244		3.328		3.398	
	r	0.074		0.075		0.076	
	B_d	16.391	19.595	16.055	19.823	15.649	19.949
	B_u	24.402		23.590		22.815	
	k_d	17.256	20.072	17.017	20.324	16.698	20.467
	k_u	24.296		23.632		22.980	
	y_d	21.088	24.443	20.832	24.809	20.489	25.057
	y_u	29.475		28.786		28.102	
RB	q_d	3.009		3.138		3.267	
	q_u	3.389		3.413		3.440	
	r_d	0.081		0.080		0.079	
	r_u	0.070		0.072		0.075	
	B_d	14.304	19.061	14.433	19.516	14.575	19.805
	B_u	26.197		24.599		23.292	
	k_d	15.055	19.495	15.299	19.988	15.555	20.307
	k_u	26.155		24.677		23.475	
	y_d	18.718	23.820	18.981	24.446	19.257	24.884
	y_u	31.475		29.912		28.635	

Notes: The foundation stake q_0 in firm equity is at level 0.2. The cost of lendable funds (deposit rate) is 0.03. For firms of type #, $q_\#$ is total equity capital, $r_\#$ is the borrowing rate, $B_\#$ is the volume of credit taken, $k_\#$ is the total investment in physical capital, $y_\#$ is expected gross output (when the systemic productivity factor takes its expected value of 1), AL is arm's length loan management, RB is relationship banking (delegated loan management)

Table 2 Main fundamentals under changing sentiment

Perceived proportion of high-productivity borrowers →		λ=0.4			λ=0.5			λ=0.6		
True value of λ →		0.4	0.5	0.6	0.4	0.5	0.6	0.4	0.5	0.6
	Indicator ↓									
AL	q	3.244	3.244	3.244	3.328	3.328	3.328	3.398	3.398	3.398
	r	0.074	0.074	0.074	0.075	0.075	0.075	0.076	0.076	0.076
	B	19.595	20.396	21.197	19.069	19.823	20.576	18.516	19.232	19.949
	k	20.072	20.776	21.480	19.663	20.324	20.986	19.211	19.839	20.467
	y	24.443	25.282	26.120	24.014	24.809	25.605	23.534	24.296	25.057
RB	q_d	3.009	3.009	3.009	3.138	3.138	3.138	3.267	3.267	3.267
	q_u	3.389	3.389	3.389	3.413	3.413	3.413	3.440	3.440	3.440
	r_d	0.081	0.081	0.081	0.080	0.080	0.080	0.079	0.079	0.079
	r_u	0.070	0.070	0.070	0.072	0.072	0.072	0.075	0.075	0.075
	B	19.061	20.396	21.197	18.499	19.516	20.533	18.062	18.933	19.805
	k	19.495	20.605	21.715	19.050	19.988	20.926	18.723	19.515	20.307
	y	23.820	25.096	26.372	23.353	24.446	25.539	23.009	23.946	24.884

Notes: The foundation stake q_0 in firm equity is at level 0.2. The cost of lendable funds (deposit rate) is 0.03. For firms of type #, $q_\#$ is total equity capital, $r_\#$ is the borrowing rate; variables without subscripts denote economy-wide aggregates; B is the volume of credit taken, k is the total investment in physical capital, y is expected gross output (when the systemic productivity factor takes its expected value of 1), AL is arm's length loan management, RB is relationship banking (delegated loan management).

Table 3 Economic fundamentals in equilibrium with and without prudential capital surcharges

Prudential capital surcharge →			**No**		**Yes**	
Indicator ↓				*Aggregate*		*Aggregate*
AL	q		3.328		2.999	
	r		0.075		0.085	
	Default probability	L_d	+0		+0	
		L_u	0.025		0.039	
	B_d		16.055	19.823	13.304	16.637
	B_u		23.590		19.970	
	k_d		17.017	20.324	14.101	16.984
	k_u		23.632		19.867	
	y_d		20.832	24.809	17.684	21.201
	y_u		28.786		24.709	
RB	q_d		3.138		2.756	
	q_u		3.413		3.099	
	r_d		0.080		0.094	
	r_u		0.072		0.081	
	Default probability	L_d	+0		+0	
		L_u	0.022		0.034	
	B_d		14.433	19.516	11.437	16.230
	B_u		24.599		21.022	
	k_d		15.299	19.988	12.116	16.540
	k_u		24.677		20.964	
	y_d		18.981	24.446	15.528	20.717
	y_u		29.912		25.906	

Notes: Results are shown for the perceived share λ=0.5 of high-productivity borrowers and no prior bias. The base capital surcharge a is 1 per cent per 1st unit of credit uncollateralized by physical capital. The foundation stake q_0 in firm equity is at level 0.2. The cost of lendable funds (deposit rate) is 0.03. For firms of type #, $q_\#$ is total equity capital, $r_\#$ is the borrowing rate, $B_\#$ is the volume of credit taken, $k_\#$ is the total investment in physical capital, $y_\#$ is expected gross output (when the systemic productivity factor takes its expected value of 1), AL is arm's length loan management, RB is relationship banking (delegated loan management)

Table 4 Economic fundamentals in equilibrium when borrower incentives replicate either limited or unlimited liability conditions

Borrower incentives → / Indicator ↓			LL-consistent	LL-consistent *Aggregate*	UL-consistent	UL-consistent *Aggregate*
AL	q		3.328		3.916	
	r		0.075		0.075	
	Default probability	L_d	+0		+0	
	Default probability	L_u	0.025		0.00086	
	B_d		16.055	19.823	15.505	18.916
	B_u		23.590		22.326	
	k_d		17.017	20.324	17.053	20.047
	k_u		23.632		23.042	
	y_d		20.832	24.809	20.870	24.535
	y_u		28.786		28.200	
RB	q_d		3.138		3.072	
	q_u		3.413		3.881	
	r_d		0.080		0.076	
	r_u		0.072		0.077	
	Default probability	L_d	+0		0.000051	
	Default probability	L_u	0.022		0.00062	
	B_d		14.433	19.516	15.879	18.522
	B_u		24.599		21.164	
	k_d		15.299	19.988	16.606	19.256
	k_u		24.677		21.907	
	y_d		18.981	24.446	20.390	23.684
	y_u		29.912		26.979	

Notes: Results are shown for the perceived share λ=0.5 of high-productivity borrowers and no prior bias. The foundation stake q_0 in firm equity is at level 0.2. The cost of lendable funds (deposit rate) is 0.03. For firms of type #, $q_{\#}$ is total equity capital, $r_{\#}$ is the borrowing rate, $B_{\#}$ is the volume of credit taken, $k_{\#}$ is the total investment in physical capital, $y_{\#}$ is expected gross output (when the systemic productivity factor takes its expected value of 1), AL is arm's length loan management, RB is relationship banking (delegated loan management). Borrower management incentives are either consistent with limited liability (LL) or imitate unlimited liability (UL)