Policy Risk and the Business Cycle*

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Abstract

The argument that policy risk, i.e. uncertainty about monetary and fiscal policy, has been holding back the economic recovery in the U.S. during the Great Recession has a large popular appeal. We analyze the role of policy risk in explaining business cycle fluctuations by using an estimated New Keynesian model featuring policy risk as well as uncertainty about technology. We directly measure uncertainty from aggregate time series using Sequential Monte Carlo Methods. While we find considerable evidence of policy risk in the data, we show that the "pure uncertainty"-effect of policy risk is unlikely to play a major role in business cycle fluctuations. In the estimated model, output effects are relatively small due to i) dampening general equilibrium effects that imply a low amplification and ii) counteracting partial effects of uncertainty. Finally, we show that policy risk has effects that are an order of magnitude larger than the ones of uncertainty about aggregate TFP.

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1 Introduction

The supposedly negative influence of “policy risk”, i.e. uncertainty about fiscal and monetary policy, has become a recurring theme in the political discourse. The popular argument espoused in speeches and newspaper articles by politicians and economists alike is that the uncertainty surrounding future policy stuns economic activity by inducing a “wait-and-see approach”.[1] In the following, we think of uncertainty as the dispersion of the economic shock distribution. Rational consumers and firms will react to the fact that future shocks will be drawn from a wider distribution. This reaction is distinct from the ex-post effect of higher uncertainty resulting from on average more extreme shock realizations.[2] The goal of the present study is to isolate the first effect and answer the question: Are uncertainty shocks to policy variables quantitatively important?

Clearly, during the so-called Great Recession U.S. citizens were facing a period of extraordinary uncertainty regarding economic policy. On the one hand, both the output decline due to the financial crisis and the fiscal stimuli designed to counteract this decline had led to a considerable deterioration of the U.S. fiscal situation. Given this unsustainable fiscal path, many commentators and politicians were arguing for a quick consolidation of government finances, possibly by raising taxes. On the other hand, the U.S. unemployment rate stood at 9.6% at the end of 2010, its highest value since 1983. Hence, there were considerable calls for more fiscal stimulus, preferably in the form of reduced taxes due to supposedly higher multipliers (see e.g. Romer and Romer, 2010). At the same time, Republicans and Democrats were fighting over the continuation of the Bush tax cuts. On the monetary side, the amount of policy risk was equally high. Hawks and doves at the Federal Reserve System fought over the extent of quantitative easing and the correct monetary stance given conflicting signals from core and headline inflation measures.

Scientific evidence on the aggregate effects of uncertainty is still inconclusive and mostly confined to TFP uncertainty. Empirical studies using different proxies and identification schemes to uncover the effects of uncertainty have produced a variety of results. One group of studies reports an important impact of uncertainty about productivity on real aggregate variables like GDP and employment (Alexopoulos and Cohen, 2009; Bloom, 2009; Bloom

[1]See e.g. The Wall Street Journal, October 29th, 2009: “*For these small businesses, and many others [...], there's an additional dark cloud: uncertainty created by Washington's bid to reorganize a wide swath of the U.S. economy.*” (Fields, 2009). For other proponents of this view, see Boehner (2010); Cantor (2010); Imrohoroglu (2010); Lowrie (2010); McKinnon (2010); see Klein (2010); Reeve (2010); Wingfield (2010) for dissenting opinions.

[2]Uncertainty shocks are mean preserving spreads to the shock distribution. They are not associated with the expectation of shocks going into a specific direction, like expecting an expansionary stimulus package. Hence, they are also distinct from news shocks (Beaudry and Portier, 2006; Schmitt-Grohé and Uribe, 2008), which are future level shocks of which both the sign and the magnitude are already perfectly known today.

et al., 2010). A one-standard deviation shock to uncertainty in these studies typically leads to a 1%-2% drop in GDP, followed by a recovery with a considerable overshooting. In contrast, a second group of studies reports little to no impact at all (Bachmann and Bayer, 2011; Bachmann et al., 2010; Bekaert et al., 2010; Chugh, 2011; Popescu and Smets, 2010). In the theoretical literature, while most studies have emphasized the contractionary effects of uncertainty on economic activity, it is generally acknowledged that there are different effects working in opposite directions, thereby making the overall effect ambiguous. For example, while an increase in uncertainty may depress investment due to a "wait-and-see approach", economic agents may want to self-insure by working more to build up a buffer capital stock, which ceteris paribus leads to an increase in investment.

We answer the question of whether policy risk shocks are quantitatively important in an estimated DSGE-model. We focus on aggregate uncertainty as it has been shown to have potentially important output effects (Fernández-Villaverde et al., forthcoming). We add to the previous literature in the following ways. First, we are to our knowledge the first to study the effect of policy risk on business cycles.[3] Second, we directly measure aggregate uncertainty from the respective time series without the need to resort to proxies. Third, we jointly consider level shocks and uncertainty shocks. Regarding uncertainty shocks, we focus on policy risk, i.e. uncertainty about future tax liabilities, government spending, and monetary policy, to test the hypothesis that policy risk may be an important factor in explaining the prolonged Great Recession. We also include uncertainty with respect to total factor productivity (TFP) and investment-specific technology in order to have a benchmark against which we can judge our findings. Fourth, we integrate these processes into a medium-scale New Keynesian DSGE-model of the type typically used for policy analysis (see e.g. Christiano et al., 2005; Smets and Wouters, 2007) and solve this model using third-order perturbation methods. We then estimate the model using the Simulated Method of Moments. This approach allows us to control for the effects of level shocks to TFP, investment-specific technology, government spending, monetary policy, and taxes when estimating the importance of policy risk.

We find that the role of policy risk in explaining the prolonged slump is largely overstated. Although the output effects of policy risk are an order of magnitude larger than the effects of TFP uncertainty, even a large (two-standard deviation) shock to policy risk decreases output by

[3]We have recently become aware of independently conducted work by Fernández-Villaverde et al. (2011), studying a similar issue in a calibrated model. The studies differ in the set of shocks considered and in the details of the model specification. However, the results are quite similar, with even large uncertainty shocks generating only a contained output decline. In their baseline calibration, a two-standard deviation policy risk shock decreases output by 0.06% compared to 0.025% in our estimated baseline specification. The advantage of our approach is that we estimate the parameters of our model. Moreover, we allow for time-varying volatility in technology, allowing us to relate our findings to the literature on TFP uncertainty and to "good luck" explanations of the Great Moderation.

a mere 0.025%. The reason for this result is the existence of strong general equilibrium effects that dampen the effects of aggregate uncertainty and imply a low shock amplification. Most notably, monetary policy reacts fast and decisively to current economic conditions, implying an interest rate response that dampens aggregate fluctuations arising from uncertainty shocks. If we allow for a stronger amplification, uncertainty shocks generate considerably larger output effects, but at the same time imply counterfactually volatile business cycles.

From a methodological viewpoint, the paper most closely related to our work is Fernández-Villaverde et al. (forthcoming). Their study also employs Sequential Monte Carlo Methods combined with third-order perturbation to estimate the effect of interest risk on the Argentinean economy. In terms of results, our paper is most closely related to Bachmann and Bayer (2011), who show for the case of idiosyncratic uncertainty about technology that general equilibrium effects may considerably reduce the effect of uncertainty shocks typically found in partial equilibrium models (e.g. Bloom, 2009). Our paper is also related to the work of Primiceri (2005) and Justiniano and Primiceri (2008). Using a time-varying Bayesian VAR and an estimated DSGE-model, respectively, the authors document the importance of time-varying volatility for explaining the time series behavior of output and inflation and the Great Moderation in particular. We differ from their work in two major points: first, we allow for a non-linear transmission of volatility shocks into the economy. Second, by using a third-order approximation instead of a first-order approximation, we are able to distinguish uncertainty-effects from the ex-post effect of uncertainty in the form of more extreme level shocks. We show that their result is mainly due to the differing size of the realized level shocks when the dispersion of the distribution from which they are drawn changes. In contrast, the pure uncertainty-effect is only of secondary importance.

The outline of the paper is as follows. Section 2 presents a short literature review on the transmission channels of uncertainty. In Section 3, we build a quantitative business cycle model featuring several channels identified in the theoretical literature through which aggregate uncertainty may impact economic activity. We measure policy risk and technological uncertainty directly from aggregate time series using Sequential Monte Carlo methods in Section 4. In Section 5, we feed the uncertainty processes estimated in Section 4 as driving processes into the model and fit it to U.S. data using a Simulated Method of Moments approach. With the estimated model at hand, we then study the effects of policy risk in Section 6. Section 7 concludes.

2 Uncertainty: Potential Transmission Channels

Three different mechanisms through which aggregate uncertainty may affect economic activity have been identified in the microeconomic literature: Hartman-Abel effects, real option effects, and precautionary savings. While these categories are helpful in shaping our thinking about the effects of uncertainty, they are partial equilibrium effects. In general equilibrium, each of these effects necessarily induces equilibrating price and quantity changes that may significantly dampen the aggregate effects. While in a partial equilibrium model uncertainty may have ceteris paribus largely contractionary effects on investment and output (e.g. Bloom, 2009), in general equilibrium wages and interest rates may adjust, thereby significantly reducing the resulting net effect (Bachmann and Bayer, 2011).

The first category are the so called Hartman-Abel-effects (Abel, 1983; Hartman, 1972). Under certain conditions,[4] it follows from the firms's FOC that the expected marginal revenue product of capital is convex in output prices and TFP.[5] Hence, due to Jensen's Inequality larger uncertainty about these variables increases the demand for capital and thus investment. In our model, while capital is predetermined, both the utilization of capital and labor input can be adjusted, opening up the possibility of expansionary Hartman-Abel effects.

Second, there may be real option effects at work (Bernanke, 1983), e.g. through investment being (partially) irreversible and/or partially expandable. For example, if the resale (purchase) price of capital in the future differs from the current acquisition price, a firm installing capital that it may sell later, effectively acquires a put option. Moreover, investment today destroys a call option, namely the opportunity to buy capital later at a possibly lower price. Hence, in the investment decision these option values have to be taken into account (Abel et al., 1996). Higher uncertainty decreases investment as the call option to purchase the capital later, which is "killed" by investing today, becomes more valuable. However, in the presence of partial reversibility, the value of the put option that is obtained by investing today increases with higher uncertainty. Hence, the total effect of uncertainty on investment in such a framework is generally ambiguous.

In our model, several features give rise to option effects. First, capital is predetermined for one period. Second, the relative price of investment and consumption is stochastic, thereby giving rise to potentially costly irreversibility and expandability. Third, through the presence

[4]Constant-returns-to-scale production function with i) a predetermined capital stock, ii) perfect competition, iii) risk neutrality, and iv) symmetric convex adjustment costs.

[5]The reason is that labor can flexibly react to shocks and hence the marginal revenue product reacts stronger than one for one to the movement in the respective variable. To see this, assume a fixed capital stock of capital and that the output price rises. There is a direct positive effect of this price increase on profits via quantity times price change. Additionally, there is a positive indirect effect through the increase in optimal output that is achieved by increasing labor.

Table 1: Overview: potential transmission mechanism

	Hartman-Abel effects	Real option effects			Precautionary savings
		Call	Put	Interest rate	
Investment	+	−	+	+/−	+
Consumption	?	?	?	?	−

Notes: + indicates a positive effect of uncertainty, − a negative effect, and +/− an ambiguous effect on the respective variable. ? denotes that the respective effect makes no prediction for this variable due to its partial equilibrium nature.

of depreciation allowances investment generates a tax shield at historical costs of investment so that investment effectively "kills" the option to purchase this tax shield later. Fourth, the interest rate in our model is stochastic, giving rise to additional countervailing option effects as discussed in Ingersoll and Ross (1992).

The third effect is the precautionary saving motive (Leland, 1968), defined as the "additional saving that results from the knowledge that the future is uncertain" (Carroll and Kimball, 2008). Faced with higher uncertainty, agents may both consume less and work more in order to self-insure against future shocks, i.e. they build a buffer stock.[6] As the preferences of the agents in our model feature prudence (Garcia et al., 2007; Kimball, 1990) uncertainty should increase precautionary savings in our model.

In the end, due to these three effects acting on different variables and potentially working in opposite directions as well as the presence of general equilibrium effects, only a rigorous quantitative evaluation can answer the question what the net effect of uncertainty on aggregate activity is. We pursue this question by estimating a structural model featuring time-varying volatility, which we present in the next section.

3 A DSGE-model with Policy Risk

We use a standard quantitative New Keynesian business cycle model (Smets and Wouters, 2007). The model economy is populated by a large representative family, a continuum of unions $j \in [0, 1]$ selling differentiated labor services to intermediate firms, a continuum of intermediate firms producing differentiated intermediate goods using bundled labor services and capital, and a final good firm bundling intermediate goods to a final good. In addition, the model features a government sector that finances government spending with distortionary taxation

[6] Real option effects and the precautionary saving motive are not disjunct effects. Consumption is completely irreversible as the consumed good is not available for consumption in later periods when the marginal utility of consumption may be high.

and transfers, and a monetary authority which sets the nominal interest rate according to an interest rate rule.

3.1 Household Sector

The economy is populated by a large representative family with a continuum of members, each consuming the same amount and working the same number of hours. Preferences are defined over per capita consumption C_t and per capita labor effort L_t. Following the framework in Schmitt-Grohé and Uribe (2006), labor is supplied to a continuum of unions $j \in [0,1]$, which are monopolistically competitive and supply differentiated labor services $l_t\ (j)$. Household members supply their labor uniformly to all unions. Hence, total labor supply of the representative family is given by the integral over all labor markets j, i.e. $L_t = \int_0^1 l_t\,(j)\,dj$. The labor market structure will be discussed in more detail below. We assume the preference specification of Jaimovich and Rebelo (2009), but allow for habits in consumption:

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{\left(C_t - \phi_c C_{t-1} - \gamma \frac{L_t^{1+\sigma_l}}{1+\sigma_l} S_t \right)^{1-\sigma_c} - 1}{1-\sigma_c} \right\}, \tag{1}$$

where $\phi_c \in [0,1]$ measures the degree of internal habit persistence, $\sigma_c \geq 0$ governs the intertemporal elasticity of substitution, $\sigma_l \geq 0$ is related to the Frisch elasticity of labor supply, and $\gamma \geq 0$ measures the relative disutility of labor effort. The term

$$S_t = \left(C_t - \phi_c C_{t-1} \right)^{\sigma_G} S_{t-1}^{1-\sigma_G} \tag{2}$$

makes the preferences non-separable in both consumption and work effort, where $\sigma_G \in [0,1]$ parameterizes the strength of the wealth effect on the labor supply. When $\sigma_G = 1$, the preference specification is equal to the one discussed in King et al. (1988), while with $\sigma_G = 0$ the preference specification of Greenwood et al. (1988) with no wealth effect on the labor supply is obtained.

The household faces the budget constraint

$$\begin{aligned} C_t + z_t^I I_t + \frac{B_{t+1}}{P_t} =& \left(1-\tau_t^n\right) \int_0^1 W_t\,(j)\, l_t\,(j)\, dj + \left(1-\tau_t^k\right) r_t^k u_t K_t \\ &+ \left(1-\tau_t^k\right) \left(R_{t-1}-1\right) \frac{B_t}{P_t} + \frac{B_t}{P_t} + \Phi_t + T_t + \left(1-\tau_t^k\right) \Xi_t\ , \end{aligned} \tag{3}$$

where the household earns income from supplying differentiated labor services $l_t(j)$ at the real wage $W_t\,(j)$ to union j, and from renting out capital services $u_t K_t$ at the rental rate r_t^k. In addition, it receives lump sum transfers T_t from the government and profits Ξ_t from owning the firms in the economy. All forms of income are taxed at their respective tax rates τ_t^n and τ_t^k. The term $\left(1-\tau_t^k\right)(R_{t-1}-1)\frac{B_t}{P_t}+\frac{B_t}{P_t}$ is the after-tax return on savings in bonds, where the net returns are taxed at the capital tax rate. Bonds are in zero net supply. The household spends its income on consumption C_t and investment $z_t^I I_t$, where I_t is gross investment and z_t^I denotes a shock to the relative price of investment in terms of the consumption good. This price is equal to the technical rate of transformation between investment and consumption goods. Due to the presence of a temporary shock, it is exogenous and stochastic. Changes in z_t^I do not affect the productivity of already installed capital, but do affect newly installed capital and become embodied in it. We assume the shock to follow an AR(2)-process[7]

$$\log z_t^I \;=\; \rho_1^{zI}\log z_{t-1}^I+\rho_2^{zI}\log z_{t-2}^I+e^{\sigma_t^{zI}}\nu_t^{zI}\ , \tag{4}$$

where σ_t^{zI} allows for time-varying volatility and is discussed in detail in Section 4. Apart from the fact that this form of investment-specific technology may be an important source of economic fluctuations (Greenwood et al., 1997, 2000), a stochastic relative price of investment introduces costly reversibility and expandability of investment into the model as the future purchase/resale price is stochastic.

The term Φ_t captures depreciation allowances, which are an important feature of the U.S. tax code. We assume depreciation allowances of the form $\Phi_t=\tau_t^k\sum\limits_{s=1}^{\infty}\delta_\tau(1-\delta_\tau)^{s-1}z_{t-s}^I I_{t-s}$, where δ_τ is the depreciation rate for tax purposes.[8] By providing new investment with a tax shield, depreciation allowances may be important in capturing the dynamics of investment following shocks (Christiano et al., 2007; Yang, 2005). Through this tax shield at historical investment prices, combined with a stochastic relative price of investment z^I, depreciation allowances contribute to costly reversibility and expandability of investment.

The household owns the capital stock K_t, whose law of motion is given by

$$K_{t+1}=\left[1-\left(\delta_0+\delta_1\left(u_t-1\right)+\frac{\delta_2}{2}(u_t-1)^2\right)\right]K_t+I_t-\frac{\kappa}{2}\left(\frac{I_t}{K_t}-\delta_0\right)^2K_t\ , \tag{5}$$

where I_t is gross investment. Household members do not simply rent out capital, but capital services u_tK_t, where u_t denotes the capital utilization, i.e. the intensity with which the existing capital stock is used. Without loss of generality, capital utilization in steady state is normalized to 1. Using capital with an intensity higher than normal incurs costs to the

[7]The lag lengths for the individual exogenous driving processes is chosen to provide a good empirical fit. Details are provided in Section 4.

[8]Following Auerbach (1989), we allow the depreciation rate for tax purposes to differ from the physical rate.

household in the form of a higher depreciation $\delta(u_t) = \delta_0 + \delta_1(u_t - 1) + \delta_2/2\,(u_t - 1)^2$, which, assuming $\delta_0, \delta_1, \delta_2 > 0$, is an increasing and convex function of the capital utilization. The last term in equation (5) captures capital adjustment costs at the household level of the form introduced by Hayashi (1982), where $\kappa \geq 0$ is a parameter governing the curvature of the cost function. This functional form implies that the capital adjustment costs are minimized and equal to 0 in steady state. We choose this type of adjustment costs for three reasons. First, while this functional form clearly is unable to explain some micro-level phenomena like lumpy investment, it has nevertheless been shown to provide a good fit of firm level investment data and performs better than the Christiano et al. (2005)-formulation with quadratic adjustment costs in investment changes (Eberly et al., 2008). Second, with the flow specification of Christiano et al. (2005), Tobin's marginal q would be independent of the capital stock, which would essentially shut off intertemporal linkages and thereby the option effects (Wu, 2009).

Thus, the household maximizes its utility (1) by choosing $C_t, B_{t+1}, u_t, K_{t+1}, I_t, S_t, L_t$, subject to the constraints (2) - (5) and the resource constraint for aggregate labor.

3.2 Labor Market

The household supplies labor $l_t(j)$ equally to a continuum of unions $j,\ j \in [0,1]$. This labor market structure allows to introduce differentiated labor services and staggered wage setting without letting idiosyncratic wage risk affect the household members, which would make aggregation intractable. Monopolistically competitive unions supply differentiated labor $l_t(j)$ to intermediate firms at wage $W_t(j)$. Every period, each union may re-optimize its wage with probability $(1-\theta_w), 0 < \theta_w < 1$. If a union j cannot re-optimize, its nominal wage is indexed to the price level according to $W_t(j)P_t = \Pi_{t-1}^{\chi_w} W_{t-1}(j)P_{t-1}$, where $\chi_w \in [0,1]$ measures the degree of indexing. Hence, when the union has not been able to re-optimize for τ periods, its real wage τ periods ahead is given by:

$$W_{t+\tau}(j) = \begin{cases} W_{t+\tau}^{opt}(j), & \text{if able to re-optimize in } t+\tau, \\ \prod\limits_{s=1}^{\tau} \frac{\Pi_{t+s-1}^{\chi_w}}{\Pi_{t+s}} W_t(j), & \text{otherwise.} \end{cases} \tag{6}$$

Household members supply the amount of labor services that is demanded at the current wage. The objective of each union able to reset its wage is to choose the real wage that maximizes the expected utility of its members, given the demand for its labor services $l_t(j) = (W_{jt}/W_t)^{-\eta_w} L_t^{comp}$, where L_t^{comp} is the aggregate demand for composite labor services and η_w is the substitution elasticity, the respective resource constraint $L_t = L_t^{comp} \int_0^1 (W_{jt}/W_t)^{-\eta_w} dj$, and the aggregate wage level $W_t = \left(\int_0^1 W_t(j)^{1-\eta_w} dj\right)^{\frac{1}{1-\eta_w}}$.

3.3 Firm Side

There is a continuum of monopolistically competitive intermediate goods firms i, $i \in [0,1]$, which produce differentiated intermediate goods Y_{it} using capital services $K_{it}^{serv} = u_{it} K_{it-1}$ and a composite labor bundle L_{it}^{comp} according to a Cobb-Douglas production function with capital share α

$$Y_{it} = z_t \left(K_{it}^{serv}\right)^{\alpha} \left(L_{it}^{comp}\right)^{1-\alpha} - \phi \ , \quad \text{if} \ \ z_t \left(K_{it}^{serv}\right)^{\alpha} \left(L_{it}^{comp}\right)^{1-\alpha} - \phi > 0 \tag{7}$$

and $Y_{it} = 0$ otherwise. The fixed cost of production ϕ is set to reduce economic profits to 0 in steady state, thereby ruling out entry or exit (Christiano et al., 2005). The stationary TFP shock z_t follows an $AR(2)$-process

$$\log z_t = \rho_1^z \log z_{t-1} + \rho_2^z \log z_{t-2} + e^{\sigma_t^z} \nu_t^z \ . \tag{8}$$

The composite labor bundle is built from differentiated labor inputs $L_{it}(j)$ according to a Dixit-Stiglitz aggregator $L_{it}^{comp} = \left(\int_0^1 L_{it}(j)^{\frac{\eta_w - 1}{\eta_w}} dj \right)^{\frac{\eta_w}{\eta_w - 1}}$.

We assume staggered price setting a la Calvo (1983) and Yun (1996). Each period, intermediate firms can re-optimize their prices with probability $(1-\theta_p), 0 < \theta_p < 1$. In between two periods of re-optimization, the prices are indexed to the aggregate price index P_t according to $P_{it+1} = \left(\frac{P_t}{P_{t-1}}\right)^{\chi_p} P_{it} = (\Pi_t)^{\chi_p} P_{it}$, where $\chi_p \in [0,1]$ governs the degree of indexation. Intermediate goods producers maximize their discounted stream of profits subject to the demand from composite goods producers, equation (10).

There is a competitive final goods firm which bundles a final good Y_t from a continuum of intermediate goods using a Dixit-Stiglitz aggregation technology with substitution elasticity η_p

$$Y_t = \left(\int_0^1 Y_{it}^{\frac{\eta_p - 1}{\eta_p}} di \right)^{\frac{\eta_p}{\eta_p - 1}} \ . \tag{9}$$

Expenditure minimization yields the optimal demand for intermediate good i as

$$Y_{it} = \left(\frac{P_{it}}{P_t} \right)^{-\eta_p} Y_t \quad \forall \, i \ . \tag{10}$$

3.4 Government Sector

Government spending, which may be thought of as entering the utility function additively separable, follows the process

$$\log\left(\frac{G_t}{\bar{G}}\right) = \rho_1^g \log\left(\frac{G_{t-1}}{\bar{G}}\right) + \rho_2^g \log\left(\frac{G_{t-2}}{\bar{G}}\right) + e^{\sigma_t^g}\nu_t^g \ , \tag{11}$$

where $\bar{G}$ is government spending in steady state. The government finances its expenditures by distortionary taxation of labor at the rate τ_t^n and capital and interest income at rate τ_t^k. We assume $AR(2)$-processes for the tax rates as this has been found to be a good empirical description for the U.S. (McGrattan, 1994; Mertens and Ravn, 2010)

$$\tau_t^k = (1-\rho_1^{\tau k}-\rho_2^{\tau k})\bar{\tau}^k + \rho_1^{\tau k}\tau_{t-1}^k + \rho_2^{\tau k}\tau_{t-2}^k + e^{\sigma_t^{\tau k}}\nu_t^{\tau k} \tag{12}$$

$$\tau_t^n = (1-\rho_1^{\tau n}-\rho_2^{\tau n})\bar{\tau}^n + \rho_1^{\tau n}\tau_{t-1}^n + \rho_2^{\tau n}\tau_{t-2}^n + e^{\sigma_t^{\tau n}}\nu_t^{\tau n} \ , \tag{13}$$

where $\bar{\tau}^n$ and $\bar{\tau}^k$ are the unconditional means of the labor and capital tax rates, respectively. The government also sets lump-sum transfers T_t to balance the budget. This assumed structure yields the government budget constraint

$$T_t + G_t + \Phi_t = \tau_t^n W_t L_t^{comp} + \tau_t^k\left(r_t^k u_t K_t + \Xi_t\right) \ . \tag{14}$$

Transfers plus government spending plus depreciation allowances equal tax revenues from taxing labor, capital income, and profits.

We close the model by assuming that the central bank follows a Taylor rule that reacts to inflation and output growth.

$$\frac{R_t}{\bar{R}} = \left(\frac{R_{t-1}}{\bar{R}}\right)^{\rho_R}\left(\left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\phi_\pi}\left(\frac{Y_t}{Y_{t-1}}\right)^{\phi_y}\right)^{1-\rho_R}\exp\left(m_t\right). \tag{15}$$

Here, ρ_R is a smoothing parameter introduced to capture the empirical evidence of gradual movements in interest rates (Clarida et al., 2000; Rudebusch, 1995), $\bar{\Pi}$ is the target interest rate set by the central bank, and the parameters ϕ_y and ϕ_Π capture the responsiveness of the nominal interest rate to deviations of inflation and output growth from their steady state values. We assume that the central bank responds to changes in output rather than its level as this specification conforms better with empirical evidence and avoids the need to define a measure of trend growth that the central bank can observe (see Lubik and Schorfheide, 2007).

Finally, m_t is a shock to the nominal interest rate that follows an $AR(1)$-process

$$\log m_t = \rho^m \log m_{t-1} + e^{\sigma_t^m} \nu_t^m \ . \tag{16}$$

The definition of equilibrium and the market aggregation are standard and omitted for brevity.

4 Policy Risk: Time Series Evidence

In this section, we present empirical evidence on the importance of time-varying volatility in modeling macroeconomic time series. We demonstrate that the data tend to reject the homoscedasticity of macroeconomic driving processes and show that a stochastic volatility (SV) model is able to capture the salient features of the data. Using a particle smoother, we are able to recover the historical series of uncertainty shocks and show that both "good luck" and "good policy" contributed to the Great Moderation.

4.1 Estimation Methodology

We perform a two-step estimation procedure. Due to the non-linear solution of the model required to capture uncertainty effects and the high-dimensional state space, it is computationally infeasible to jointly estimate all model parameters. Hence, we first estimate the exogenous stochastic driving processes of the model using Sequential Monte Carlo (SMC) methods. In the next section we feed these processes into the model presented in Section 3 and estimate the parameters of the remaining model equations with a Simulated Method of Moments (SMM) approach.

The model includes 6 exogenous stochastic driving processes with time-varying volatility, i.e. capital and labor tax rates, government spending, a monetary policy shock, total factor productivity, and investment-specific technology. We estimate these processes on quarterly U.S. time series, starting in 1960Q1 and using the longest available sample for each series. Details about the data sources can be found in Appendix A. Because we use a stationary model, we need to extract the deviations of the non-stationary time series from their respective trend. Hence, we apply a one-sided HP-filter to the logarithms of government spending and the two technology processes. Using a one-sided, i.e. "causal" filter (Stock and Watson, 1999) assures that the time ordering of the data remains undisturbed and the autoregressive structure is preserved. We allow for AR(2)-processes in all variables, except for the monetary policy shocks,[9] as the partial autocorrelations generally indicate the presence of a second

[9] Although theory suggests that monetary policy shocks in the Taylor rule should be unpredictable and thus i.i.d., we find a moderate degree of first-order autocorrelation.

root different from zero. Figure 1 shows the time series of the exogenous driving processes on which we estimate our laws of motion. In particular for monetary policy, the presence of time-varying volatility is immediately evident. In Appendix C.1, we provide further evidence for the presence of time-varying volatility.

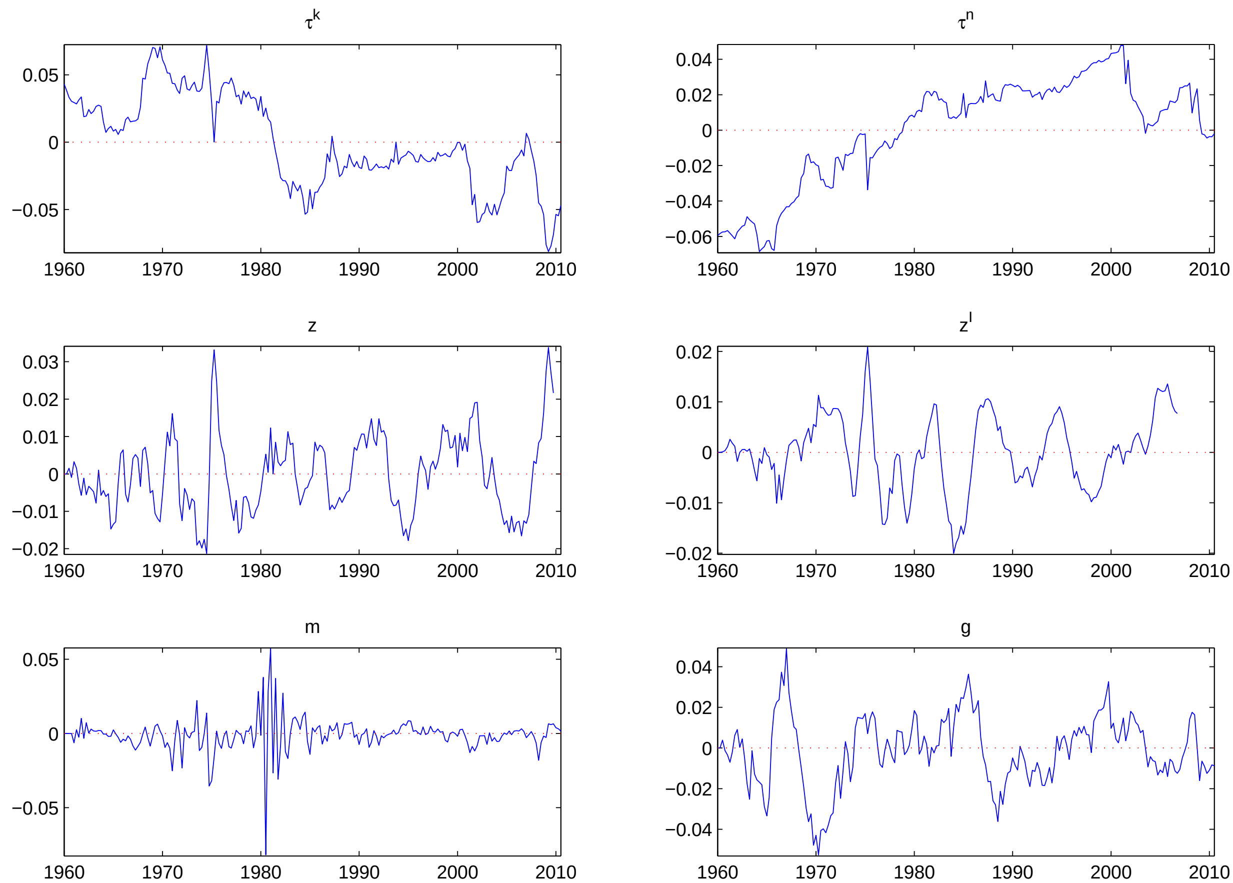

Figure 1: Time series of exogenous driving processes.
Notes: From left to right and top to bottom: capital taxes, labor taxes, TFP, investment-specific technology, monetary policy shocks, and government spending. Tax rates are demeaned; government spending and technology processes are detrended using one-sided HP-filter.

There are two major competing approaches to model time-varying standard deviations: GARCH models and stochastic volatility (SV) models (Fernández-Villaverde and Rubio-Ramírez, 2010). In the standard GARCH model, σ_t^2 is a function of the squared scaled lagged innovation in the level equation ν_{t-1}^2 and its own lagged value: $\sigma_t^2 = \omega + \alpha(\sigma_{t-1}\nu_{t-1})^2 + \beta\sigma_{t-1}^2$. The GARCH model has one important drawback: there are no distinct volatility shocks. The only innovations to the volatility equation are past level shocks, meaning that they cannot be separated from volatility shocks. As we are especially interested in the effects of shocks to the volatility, we cannot use a GARCH model but instead employ a stochastic volatility model. Specifically, we model the standard deviations σ_t^i as an AR(1) stochastic volatility process

(see e.g. Fernández-Villaverde et al., forthcoming; Shephard, 2008)

$$\sigma_t^i = (1-\rho^{\sigma_i})\,\bar{\sigma}^i + \rho^{\sigma_i}\sigma_{t-1}^i + \eta_i\varepsilon_t^i \,, \quad \varepsilon_t^i \sim \mathcal{N}(0,1), \tag{17}$$

where $\bar{\sigma}^i$ is is the unconditional mean of σ_t^i, $i \in \{\tau k, \tau n, g, m, z, zI\}$. The shock to the volatility ε_t^i is assumed to be independent from the level shock ν_t^i.

Due to the nonlinearity embedded in the stochastic volatility setup of the shocks, we cannot simply employ the Kalman filter as in the case of linearity and normally distributed shocks. For this case, Fernández-Villaverde and Rubio-Ramírez (2007) propose to use the *Sequential Importance Resampling* (*SIR*) particle filter, a special application of the more general class of *SMC* methods, to evaluate the likelihood.[10]

After obtaining the likelihood of the observables given the parameters, we use a *Tailored Randomized Block Metropolis-Hastings (TaRB-MH)* algorithm (Chib and Ramamurthy, 2010) to maximize the posterior likelihood. The prior distributions of the parameters, which are relatively weak, are given in Table 2.[11]

We are also interested in backing out the historical values of the latent state σ_t, given the whole set of observations. After filtering, it is straightforward to employ the *backward-smoothing routine* (Godsill et al., 2004) to obtain a historical distribution of the volatilities. The smoothed values were computed at the mean of the posterior distribution using 10,000 particles.

4.2 Estimation Results

The estimation results are presented in Table 2. Detailed convergence diagnostics are shown in Appendix C.2. In general, all parameters are quite precisely estimated as evidenced by the percentiles. All shocks, except for the monetary policy shock, exhibit a high degree of persistence in their levels, with less persistence in their volatilities. Moreover, the estimated processes show considerable evidence of uncertainty, with η^i ranging between 0.3 and 0.6. As a one-standard deviation uncertainty shock increases the volatility of the respective process by $(exp(\eta^i) - 1) \times 100$ percent, such a shock increases the variance of capital taxes, labor taxes, TFP, investment specific technology, monetary policy, and government spending by

[10] Technical details of the algorithms used in this subsection can be found in Appendices B.1-B.3.

[11] For the autoregressive parameters of the level equation ρ_1^i and ρ_2^i, we impose a uniform prior for each of the corresponding autoregressive roots over the stability region $(-1, +1)$. Let ξ_1 and ξ_2 be the roots of such an $AR(2)$-process. The autoregressive parameters corresponding to these roots can be recovered from: $\rho_1 = \xi_1 + \xi_2$ and $\rho_2 = -\xi_1\xi_2$. The posterior distribution was computed from a 20,500 draw Monte Carlo Markov Chain using 3,000 particles, where the first 2,500 draws were discarded as burn-in draws. Acceptance rates were generally between 20% and 45%. We also checked identifiability of the SV-process by simulating data from the process and trying to recover the true parameters from this artificial data.

Table 2: Prior and Posterior Distributions of the Shock Processes

Parameter	Prior distribution			Posterior distribution		
	Distribution	Mean	Std. Dev.	Mean	5 Percent	95 Percent
			Capital Tax Rates			
ρ_1	Uniform*	0.00	0.577	0.856	0.819	0.893
ρ_2	Uniform*	0.00	0.577	0.103	0.070	0.137
ρ_σ	Beta*	0.90	0.100	0.795	0.745	0.860
η_σ	Gamma	0.50	0.100	0.379	0.333	0.426
$\bar{\sigma}$	Uniform	-7.00	5.333	-5.071	-5.361	-4.786
			Labor Tax Rates			
ρ_1	Uniform*	0.00	0.577	1.051	1.018	1.084
ρ_2	Uniform*	0.00	0.577	-0.052	-0.085	-0.019
ρ_σ	Beta*	0.90	0.100	0.581	0.514	0.670
η_σ	Gamma	0.50	0.100	0.651	0.587	0.718
$\bar{\sigma}$	Uniform	-7.00	5.333	-5.901	-6.253	-5.531
			Total Factor Productivity			
ρ_1	Uniform*	0.00	0.577	1.021	0.965	1.080
ρ_2	Uniform*	0.00	0.577	-0.175	-0.230	-0.125
ρ_σ	Beta*	0.90	0.100	0.679	0.611	0.781
η_σ	Gamma	0.50	0.100	0.320	0.272	0.369
$\bar{\sigma}$	Uniform	-7.00	5.333	-5.349	-5.555	-5.138
			Investment-Specific Technology			
ρ_1	Uniform*	0.00	0.577	1.420	1.369	1.468
ρ_2	Uniform*	0.00	0.577	-0.501	-0.536	-0.461
ρ_σ	Beta*	0.90	0.100	0.807	0.765	0.861
η_σ	Gamma	0.50	0.100	0.332	0.295	0.368
$\bar{\sigma}$	Uniform	-7.00	5.333	-6.206	-6.427	-5.983
			Government Spending			
ρ_1	Uniform*	0.00	0.577	0.919	0.866	0.972
ρ_2	Uniform*	0.00	0.577	-0.028	-0.079	0.018
ρ_σ	Beta*	0.90	0.100	0.719	0.623	0.865
η_σ	Gamma	0.50	0.100	0.295	0.227	0.368
$\bar{\sigma}$	Uniform	-7.00	5.333	-4.887	-5.193	-4.585
			Monetary Policy Shock			
ρ_1	Uniform*	0.00	0.577	0.427	0.385	0.469
ρ_σ	Uniform*	0.90	0.100	0.921	0.895	0.947
η_σ	Beta*	0.50	0.100	0.364	0.330	0.400
$\bar{\sigma}$	Gamma	-7.00	5.333	-5.188	-5.512	-4.849

Notes: Beta* indicates that the parameter divided by 0.999 follows a beta distribution. Uniform* indicates that the roots of the autoregressive process are estimated instead of the autoregressive coefficients and follow the specified prior distribution.

46%, 92%, 38%, 39%, 34%, and 45%, respectively.[12] Appendix C.3 shows the results of model misspecification tests applied to the SV model. In general, the model fits the data well and cannot be rejected.

The relevance of stochastic volatility in modeling the behavior of the exogenous driving processes can be seen in the smoothed estimates of the historical variances of the shocks in Figure 2. The end of the 1960s and particularly the 1970s were plagued by high shock volatilities, both in the technology and the policy shocks. Particularly during the 1970s, the volatilities increased and reached their sample maxima for both tax rates and technology shocks. In contrast, the decade from 1985 to 2000 was characterized by shock volatilities to the technology variables well below their unconditional mean, indicating the role of "good luck" in explaining the Great Moderation. However, from about 1990 on "good policy" also contributed to this phenomenon as is evidenced by the low volatilities of the tax and government spending shocks, although the change in volatility is not as pronounced for the latter. For monetary policy shocks, there is clear evidence of a lower shock volatility following the Volcker disinflation from 1979-1983, a trend that also continued under Greenspan. In contrast, the early tenure of Volcker experienced a volatility of monetary shocks considerably larger than during the first oil price shock. With the height of the dot-com bubble the volatility of TFP shocks somewhat increased again, while the investment-specific technology growth remained tranquil over the whole 2000s. The largest changes in volatility in the 2000s came under George W. Bush who considerably changed the tax law, resulting in a pronounced increase in the volatility of tax rates. At the end of our sample, the Great Recession again results in an increase in policy risk with a rise in the volatility of government spending, tax rates, and monetary policy to comparable levels as after 9/11. For government spending and taxes, this mostly reflects the provisions in the *American Recovery and Reinvestment Act* that contained $288 billion in tax relief to companies and individuals, e.g. in the form of $116 billion in payroll tax relief.

Note that the SV-framework used in the present study does not imply a mechanical link between the level shocks and the volatility shocks as a GARCH-model would do. Of course, as a comparison of Figures 1 and 2 shows, a large level shock tends to coincide with an increase in the conditional variance. However, the reason for this increase in the estimated conditional variance is not a mechanical effect of this level shock subsequently entering the volatility equation. Rather, the Bayesian estimation of the SV-model weighs the likelihood of observing such a large shock being drawn from a narrow distribution, i.e. without observing a simultaneous/previous volatility shock, against the likelihood of observing a shock of this size

[12] Thus, e.g. a one-standard deviation monetary policy risk shock increases the volatility of the monetary policy shocks from $exp(-5.19) = 0.56\%$ to $exp(-5.19 + 0.364) = 0.8\%$.

that is drawn from a wider distribution due to the occurrence of a variance shock.

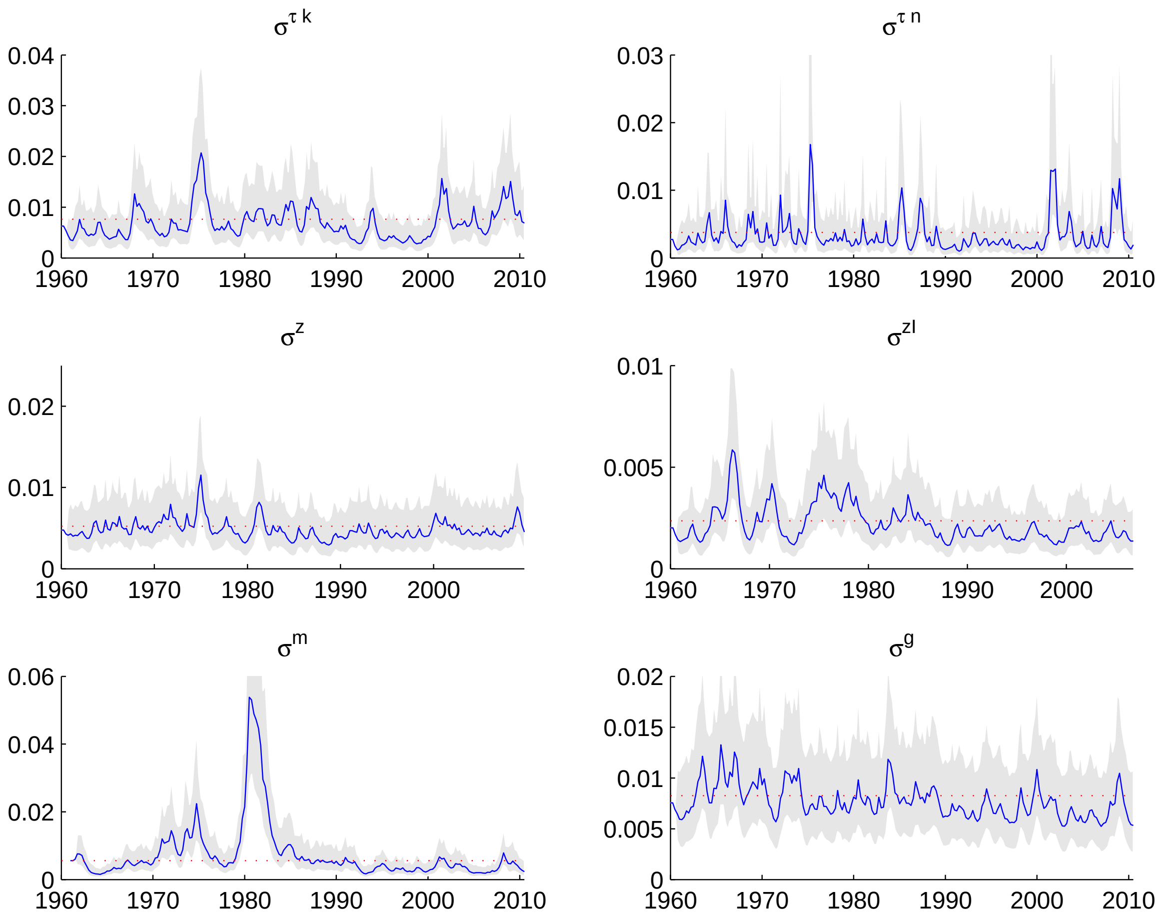

Figure 2: Smoothed standard deviations. From left to right and top to bottom: capital taxes, labor taxes, TFP, investment-specific technology, monetary policy shocks, and government spending.

Notes: Red dotted line: unconditional mean; shaded area: two standard deviation bands.

5 Fitting the Model to the Data

Using the parameter estimates of the stochastic driving processes obtained in the previous section, we are now in a position to estimate the deep parameters of the model presented in Section 3.

5.1 Simulated Method of Moments Estimation

We use the Simulated Method of Moments (SMM) approach as proposed in Ruge-Murcia (2010). Intuitively, this method minimizes the weighted distance between the empirical

Table 3: Parameters fixed prior to estimation

Parameter	Value	Target/Motivation	Parameter	Value	Target/Motivation
$\bar{\Pi}$	1	Zero infl. steady state	σ_G	0.001	Jaimovich-Rebelo (2009)
β	0.99	Standard value	η_p	10	11% Markup
δ_0	0.025	10% annual deprec.	η_w	10	11% Markup
δ_1	0.0351	$\bar{u} = 1$	α	0.295	Sample mean
δ_τ	0.05	Auerbach (1989)	τ^n	0.1984	Sample mean
ϕ	0.038	0 profits in SS	τ^k	0.388	Sample mean
γ	19.1	SS labor of 0.2	G/Y	0.2031	Sample mean
σ_c	2	Standard value			

moments and the moments resulting from artificial data simulated from the model (details can be found in Appendix B.5).

In order to simulate data, we first need to solve the model non-linearly. Due to the high-dimensional state space of our model, we employ perturbation methods to obtain an approximation of the policy function around the deterministic steady state (see e.g. Judd, 1998). Specifically, we need to obtain a third-order approximation, because we are interested in the pure effects of volatility shocks, i.e. when holding the level shocks constant. Loosely speaking, a first-order approximation yields no effects of uncertainty; a second-order approximation yields both a constant effect and an effect mediated through the corresponding level shock. Only in the third-order approximation does time-varying uncertainty play a separate role (for a more detailed explanation, see Appendix B.4).

Table 3 presents the values of parameters we fix prior to the estimation. We set gross steady state inflation $\bar{\Pi}$ to 1 and the discount factor β to 0.99. Regarding the depreciation parameters, $\delta_0 = 0.05$ is chosen to imply a 10% annual depreciation rate, $\delta_1 = 0.0351$ sets the steady state capital utilization to 1, and the depreciation rate for tax purposes δ_τ is set to twice the rate of physical depreciation (Auerbach, 1989). The fixed-cost parameter $\phi = 0.038$ implies that firms make zero profit in steady state and the labor disutility parameter $\gamma = 19.1$ sets the steady state share of hours worked to total time to 20%. Regarding the preference parameters, we set the parameter governing the intertemporal elasticity of substitution σ_c to 2 and set $\sigma_G = 0.001$, the value chosen in Jaimovich and Rebelo (2009).[13] Hence, preferences are close to the GHH-specification and imply a small wealth effect on the labor supply, which is consistent with evidence from studies focusing on the effects of news (Schmitt-Grohé and Uribe, 2008) and government spending (Monacelli and Perotti, 2008). The elasticity of substitution parameters for differentiated labor services and intermediate goods are set to

[13] When attempting to estimate this parameter, it hit the lower bound of 0 as in Schmitt-Grohé and Uribe (2008). Hence, we fix the parameter to a small value that still assures a balanced growth path.

10, resulting in a steady state markup of 11%. The capital share α, the steady state tax rates τ^k and τ^n, and the steady state share of government spending to output are set to their respective sample means.

The empirical moments to be matched are the standard deviations and first- and second-order autocovariances of output, consumption, investment, inflation, the real wage, and the nominal interest rate. Moreover, we target the covariance of output with the other variables. All variables are logged and detrended using a one-sided HP-filter with smoothing parameter $\lambda = 1600$. The second and fourth columns of Table 5 display the respective sample moments.[14]

5.2 Parameter Estimates

Table 4: Parameters estimated by SMM

Parameter	Description	Mean	-1 std.-dev.	+1 std.-dev.
ϕ_c	Consumption habits	0.9665	0.9660	0.9671
δ_2/δ_1	Capital utilization costs	0.0414	0.0314	0.0546
κ	Capital adjustment costs	10.0857	0.8007	127.0438
θ_p	Calvo parameter prices	0.9644	0.9641	0.9646
θ_w	Calvo parameter wages	0.7785	0.7615	0.7947
χ_p	Price indexation	0.4170	0.3809	0.4539
χ_w	Wage indexation	0.9751	0.9725	0.9774
σ_l	Frisch elasticity parameter	0.0683	0.0652	0.0716
ρ_R	Interest smoothing	0.4889	0.4541	0.5238
ϕ_π	Taylor rule inflation	1.9691	1.9058	2.0422
ϕ_y	Taylor rule output growth	1.2195	0.8416	1.7671

The parameter estimates are shown in Table 4. All parameters except for the capital adjustment cost parameter κ are precisely estimated as seen in columns 4 and 5.[15] Consumers have strong habits in consumption with $\phi_c = 0.97$, which is at the upper end of values generally considered plausible. Capital utilization costs show little convexity with $\delta_2/\delta_1 = 0.04$, while capital adjustment is costly as indicated by $\kappa = 10.09$, ensuring that investment is not excessively volatile. Prices are estimated to be quite sticky with $\theta_p = 0.96$, while the degree of wage stickiness is moderate with an average duration of 4.3 quarters. The high degree of price stickiness compared to e.g. Smets and Wouters (2007) reflects the absence of real

[14]Some of the target moments are transformed to correlations for better interpretation. The relative standard deviations with respect to the standard deviation of output are only implicitly targeted through the standard deviations of the respective series.

[15]The confidence bands rely on the asymptotic normality of the estimator as shown in equation (B.18). However, this is only a rough approximation as most parameters, e.g. the Calvo parameters, have bounded support. Unfortunately, SMM is computationally too intensive to rely on bootstrapping the standard errors.

rigidities like a non-constant elasticity of substitution in our setup. The degree of indexation to past inflation is considerably higher for wages than for prices, with the former being almost perfectly indexed to past inflation. An estimated value of $\sigma_l = 0.07$ indicates almost linear disutility of labor. In the Taylor rule, there is a moderate degree of interest smoothing. The reaction coefficients of monetary policy are in line with values found in the literature.

Table 5: Simulated and Empirical Moments

	Model	Data	Model	Data	Model	Data	Model	Data	Model	Data
	$\sigma(x_t)$		$\rho(x_t, y_t)$		$\sigma_{x_t}/\sigma_{y_t}$		$\rho(x_t, x_{t-1})$		$\rho(x_t, x_{t-2})$	
Y	1.44%	1.57%	1.00	1.00	1.00	1.00	0.93	0.90	0.84	0.75
C	0.93%	0.95%	0.71	0.85	0.65	0.60	0.99	0.90	0.95	0.74
I	5.74%	5.30%	0.91	0.85	3.98	3.37	0.88	0.93	0.74	0.80
Π	0.22%	0.27%	0.23	0.17	0.16	0.17	0.91	0.50	0.75	0.32
W	0.82%	0.90%	0.23	0.10	0.57	0.57	0.97	0.84	0.91	0.69
R	0.40%	0.39%	0.28	0.34	0.28	0.25	0.73	0.86	0.49	0.67

Notes: Time Series X_t are output (Y_t), consumption (C_t), investment (I_t), inflation (Π_t), the real wage (W_t), and the nominal interest rate (R_t). Small letters denote variables that are logged and detrended using a one-sided HP-filter with smoothing parameter $\lambda = 1600$.

The first and third column of Table 5 show the fit of the model. Output is 92% as volatile in the simulated as model as in the data, while investment is 108% as volatile. The volatility of consumption is well-matched, while its correlation with output is too low. The volatilities of the real wage, inflation, and the nominal interest rate are on target. Their correlation with output is also well matched. Only the real wage is somewhat too procyclical. The autocorrelations are also in general well-matched. Only consumption exhibits a slightly too high autocorrelation.

5.3 The Effects of Time-Varying Volatility

With the estimated model at hand, we can perform a simple counterfactual experiment to demonstrate the importance of time-varying volatility for explaining U.S. macroeconomic time series. However, the effects of time-varying volatility reflect both the ex-ante uncertainty effect of knowing that the shocks are drawn from a wider distribution and the ex-post effect of more extreme shock realizations. In the next section, we will therefore separate these two by using the model to keep the level shocks constant.

In Figure 2, we found clear evidence of a decrease in the variance of both the technological shocks and the policy shocks since the mid 1980s, which contributed to the lower volatility of output and inflation during the Great Moderation. Using our estimated DSGE-model, we can ask what a counterfactual economy without time-varying volatility would have looked like. For this purpose, we completely shut off time-varying volatility by setting uncertainty shocks to zero. We then simulate the model again using the new set of driving forces where both the uncertainty effect and the effects of the corresponding more extreme level shocks are absent due to $\sigma_t^i = \bar{\sigma}^i$ for all $i \in \{\tau k, \tau n, g, m, z, zI\}$. This unconditional sample mean of the log-volatility of the level shocks $\bar{\sigma}^i$ lies between the high volatility pre-Great Moderation period's value and the value in the subsequent low volatility Great Moderation phase. The corresponding simulated moments are presented in Table 6. The co-movement of the model variables still fits the data quite well. However, compared to the actual data, such an economy fails to generate sufficient volatility: output, consumption, and investment are only about 65%, 73%, and 75% as volatile as the data, respectively.[16] In contrast, as seen in Table 5, the model with time-varying volatility captures the data moments well. These results clearly indicate the importance of time-varying volatility in explaining U.S. macroeconomic time series (see e.g. Justiniano and Primiceri, 2008; Primiceri, 2005).

Table 6: Simulated and empirical moments for the model without time-varying volatility

	Model	Data	Model	Data	Model	Data	Model	Data	Model	Data
	$\sigma(x_t)$		$\rho(x_t, y_t)$		$\sigma_{x_t}/\sigma_{y_t}$		$\rho(x_t, x_{t-1})$		$\rho(x_t, x_{t-2})$	
Y	0.99%	1.57%	1.00	1.00	1.00	1.00	0.94	0.90	0.85	0.75
C	0.71%	0.95%	0.67	0.85	0.72	0.60	0.99	0.90	0.95	0.74
I	3.91%	5.30%	0.89	0.85	3.97	3.37	0.92	0.93	0.79	0.80
Π	0.18%	0.27%	-0.19	0.17	0.18	0.17	0.91	0.50	0.76	0.32
W	0.53%	0.90%	0.56	0.10	0.54	0.57	0.97	0.84	0.91	0.69
R	0.30%	0.39%	-0.11	0.34	0.30	0.25	0.78	0.86	0.61	0.67

Notes: Time Series X_t are output (Y_t), consumption (C_t), investment (I_t), inflation (Π_t), the real wage (W_t), and the nominal interest rate (R_t). Small letters denote variables that are logged and detrended using a one-sided HP-filter with smoothing parameter $\lambda = 1600$.

6 The Aggregate Effects of Policy Risk

We now turn to analyzing the effects of aggregate uncertainty on business cycle fluctuations. First, having estimated the deep parameters of the model, we conduct policy experiments to trace out the effects of uncertainty shocks. We then study their transmission into the economy and analyze the underlying amplification mechanisms. We find that the model is in

[16] If we had used a linearized version of the model, this effect would not have been observed, as periods of high volatility would offset periods of low volatility. However, due to the non-linearity of our model, this is not the case here.

principle able to generate large effects of uncertainty, but that the estimated parameterization implies that the aggregate effects of uncertainty are quantitatively small. The reason for the small aggregate response to uncertainty shocks is the presence of general equilibrium effects that imply only a weak amplification.

6.1 Impulse Response Analysis

We first analyze the pure uncertainty effect resulting from time-varying volatility by separating it from the ex-post effect of more extreme shock realizations. We do so by computing impulse response functions to uncertainty shocks while keeping constant the realizations of the level shocks.

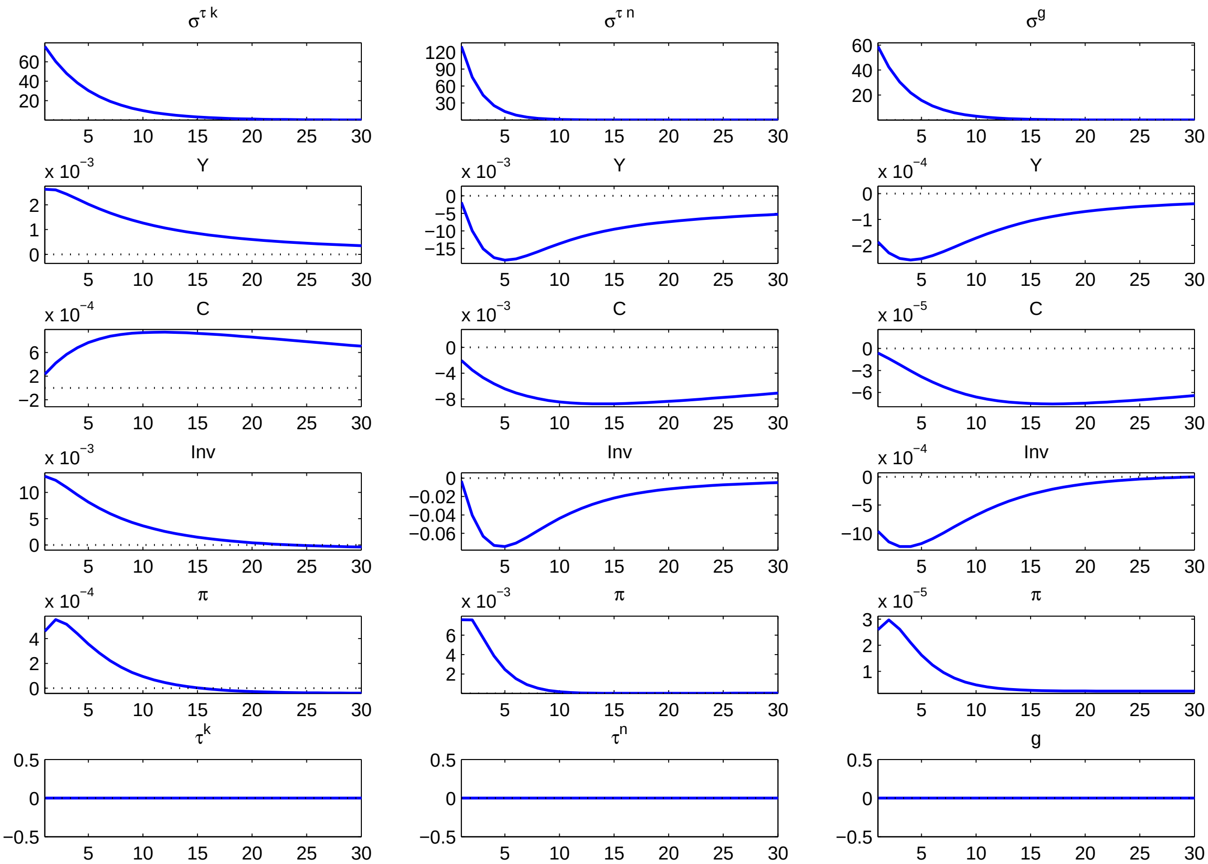

Figure 3: Impulse responses to a two-standard deviation uncertainty shock to capital taxes, labor taxes, and government spending (from left to right column).
Notes: Level shocks are held constant. All responses are in percent, except for π which is in percentage points.

Figures 3 and 4 show the impulse response functions to two-standard deviation policy risk and technology risk shocks with each column representing the impulse responses to a different shock. The ex-post level effect has been shut off, which is reflected in the flat impulse

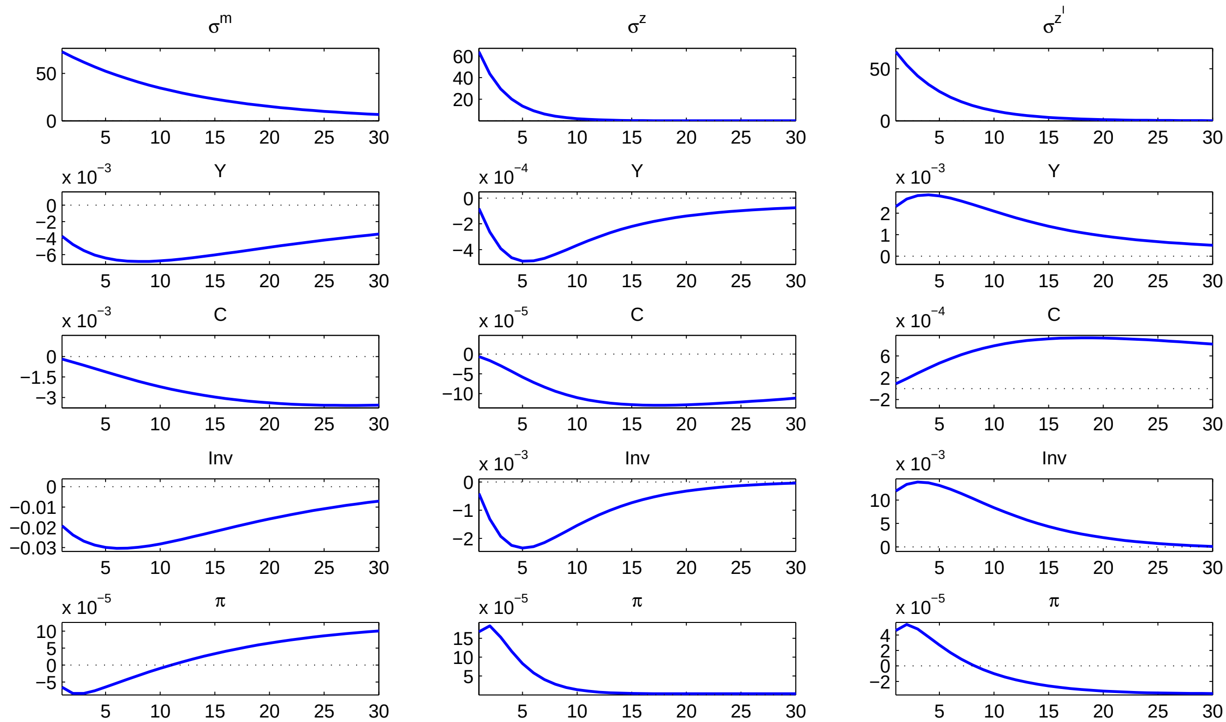

Figure 4: Impulse responses to a two-standard deviation uncertainty shock to monetary policy, TFP, and investment-specific technology (from left to right column).
Notes: Level shocks are held constant. All responses are in percent, except for π which is in percentage points.

response for τ^k, τ^n, g, m, z, and z^I depicted in the bottom row.[17] The left column of Figure 3 shows that a capital tax risk shock acts like a positive demand shock. Output and inflation both increase on impact and slowly return to zero. Initially the output response is mostly driven by the positive response of investment, which has a peak response on impact of 0.014%. Consumption increases less strongly and follows a hump-shape, peaking after 12 quarters. Due to the estimated strong degree of habit persistence in consumption, the consumption response decays only slowly and drives the output response after about four years, when investment is already almost back to its initial level. The middle and right columns show the impulse responses to labor tax risk and government spending risk, respectively. Both emulate the characteristics of a negative supply shock, with output, consumption, and investment exhibiting a hump-shaped decline, while inflation rises.

Labor tax risk induces the strongest output response of all uncertainty shocks considered, with output showing a peak decline of 0.02% and investment dropping by four times as much. The reason for this relatively strong response, compared to e.g. the government spending risk shock, is that a two-standard deviation labor tax risk shock increases uncertainty about

[17]In the subsequent graphs, we generally omit the flat level impulse responses.

labor taxes by about 120%, compared to around 60% for the other uncertainty shocks. Due to the relatively low persistence of the underlying shock process for labor tax risk, the effect on inflation subsides after 10 quarters, while the effect on consumption is again considerable more drawn out.

The left column of Figure 4 displays the response to a two-standard deviation monetary policy risk shock. This shock has a contractionary effect on output, mostly driven by a decline in investment that peaks at -0.03% after 7 quarters. In contrast, consumption reacts sluggishly, peaking only after 30 quarters. Inflation initially drops, overshoots after 10 quarters and then slowly returns, driven by a large persistence in the underlying risk shock process.

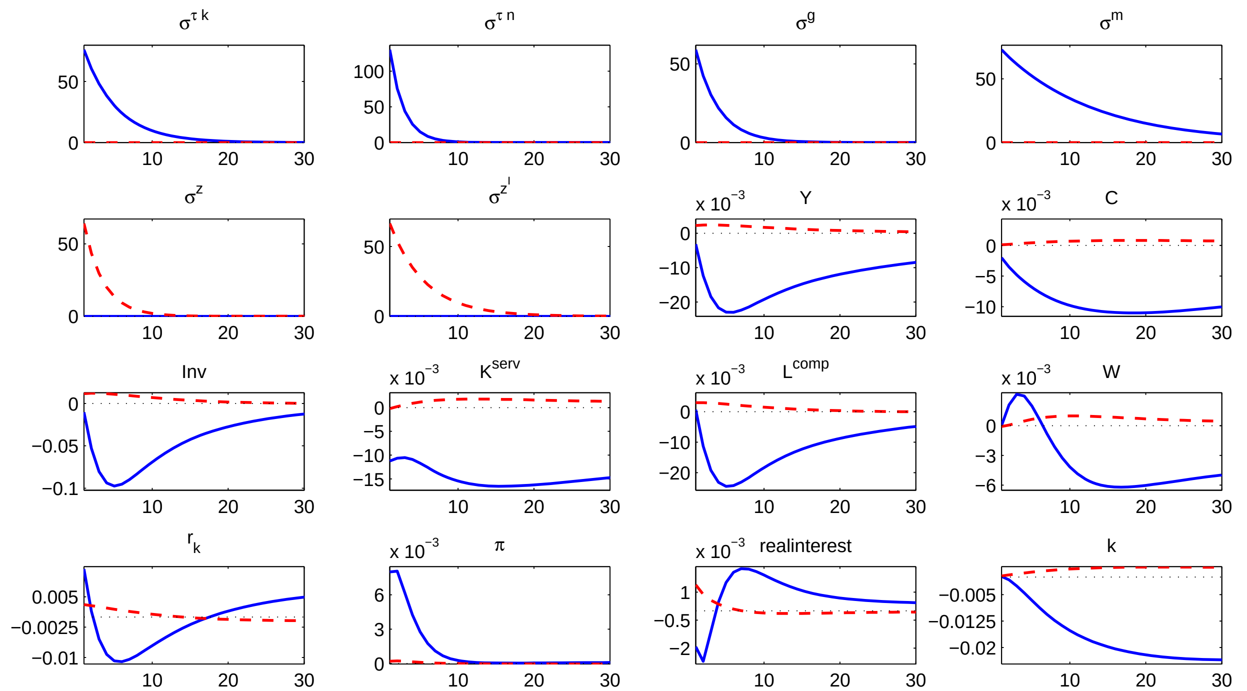

Figure 5: Impulse responses to a joint two-standard deviation policy risk shock (solid blue line) and to a joint technology risk shock (dashed red line).
Notes: Level shocks are held constant. All responses are in percent, except for π and *realinterest* which are in percentage points.

The historical volatility estimates shown in Figure 2 indicated that uncertainty about the future path of economic policy increased for all policy instruments during the Great Recession. We simulate such a situation in the form of a simultaneous two-standard deviation increase in policy risk.[18] Results are shown in Figure 5. A simultaneous two-standard deviation policy risk shock (solid lines) acts like a negative supply shock. It leads to an immediate decrease in output of 0.025%, before output slowly returns to its initial level as the shock subsides. This

[18]Due to the nonlinearity inherent in our model and the solution method that preserves this nonlinearity up to third order, the resulting impulse responses are not necessarily identical to the sum of the impulse responses to the individual uncertainty shocks.

decrease in output is driven by both consumption and investment, with investment dropping initially by 0.1%. While the capital stock reacts sluggishly due to the presence of relatively high capital adjustment costs, capital services decline immediately due to an accompanying decline in capital utilization. At the same time inflation rises. As a consequence, the real wage rises for a few periods, reflecting the indexation to the rising inflation, and then starts to decrease, reaching its minimum after 15 quarters. Due to monopolistic competition in the labor market and the non-separability of the utility function, the initial increase in the real wage does not induce an increase in labor supplied by the household. Rather, household members decrease their labor supply and consume more leisure. The real interest rate, computed as the difference between the policy rate and inflation, declines initially and then follows a hump-shaped pattern, reaching its peak after 7 quarters. The initial decline in the real interest rate reflects both the interest smoothing present in the estimated Taylor rule as well as the response of the central bank to the initial decline in output. Only when output starts to recover does the real interest rate rise to bring down inflation. The similarity in both the size and the shape of the impulse response functions of a policy risk shock and the labor tax risk shock indicates that the latter dominates the effects of the other policy risk shocks.[19]

It is instructive to compare the policy risk results to the benchmark of uncertainty about technology. The middle and right columns of Figure 4 show the impulse responses to a two-standard deviation risk shock to total factor productivity and investment-specific technology, respectively. The response to TFP risk is qualitatively similar to what could have been expected from the previous literature: it triggers an investment driven decline in output while inflation increases. In contrast, investment-specific technology risk triggers exactly the opposite effect: output increases initially and peaks after 4 quarters, with the response again being mainly driven by the investment response. It is noteworthy that the response to TFP uncertainty is an order of magnitude smaller than the effects of uncertainty about the investment-specific technology shocks. This result suggests that the role of investment-specific technology risk might be underappreciated in the uncertainty literature.[20] Figure 5 also shows the impulse responses to a joint technology risk shock of the type occurring in the middle of the 1970s. The comparison of technology risk (dashed lines) with policy risk (solid lines)

[19] While strictly speaking the impulse responses to single shocks are not additive, the opposite signs of the output response for some sources of uncertainty have important consequences for periods of generally heightened uncertainty. The simultaneous increase in uncertainty from different sources does not necessarily translate into a large output response. In times like the Great Recession, where policy risk jointly increased, different sources of uncertainty may partially offset each other, resulting in a low overall effect. For example, Figure 3 documents that capital taxation risk acts expansionary and could more than offset the negative effect of government spending risk on output and investment.

[20] While the effects of level shocks to investment-specific technology have received considerable attention in recent years (Fisher, 2006; Justiniano et al., 2010; Schmitt-Grohé and Uribe, 2011), we are to our knowledge the first to study the effects of uncertainty about investment-specific technology.

shows that policy risk generates responses that are one order of magnitude larger.

Summarizing, our results show that the finding of relatively minor effects of uncertainty on aggregate activity for the case of TFP (Bachmann and Bayer, 2011; Bachmann et al., 2010; Bekaert et al., 2010; Chugh, 2011; Popescu and Smets, 2010) also holds true for policy risk and investment-specific technology risk.

6.2 What Drives the Response to Policy Risk?

Of the transmission channels discussed in Section 2, the precautionary savings motive does not play a dominant role. In all sets of impulse responses, consumption and investment move in the same direction, while in the case of a dominant precautionary savings motive we would expect agents to decrease their consumption in order to self-insure against aggregate uncertainty by investing in a buffer-stock. Of course, it is conceivable that the precautionary savings motive counteracts the observed effects, which then would have been larger in its absence.

While it is virtually impossible to disentangle the different real option, Hartman-Abel, and general equilibrium effects, we can gain some insight into the transmission of uncertainty by shutting off various features of the model. First, as can be seen by fixing the relative price of investment to consumption at 1, the real option effect embedded in the depreciation allowances via the stochastic resale price of capital hardly plays a role. However, while their role in providing current investment with a tax shield at historical investment prices does not seem to create strong real option effects in our model, this does not mean that depreciation allowances do not play an important role. With their effect on Tobin's marginal q and the capital utilization decision, they have an important amplifying effect on the investment response and hence on output. When shutting them off completely, i.e. setting $\delta_\tau = 0$, capital drops less and the negative consumption response is cut in half (figures omitted for brevity).

Second, the low wealth effect on the labor supply implied by the preferences being close to the GHH-form ($\sigma_G \approx 0$) has a considerable effect on the responses to uncertainty, amplifying the response to some shocks and dampening the one to others. As shown in Figure 6, when setting the preferences to the standard King-Plosser-Rebelo specification ($\sigma_G = 1$), the negative response to labor tax risk declines by two orders of magnitude. At the same time, the effect of uncertainty shocks that mainly affect the capital margin, i.e. capital tax and TFP risk, substantially increases, with the former now being the dominant policy risk factor. The output response to government spending, monetary policy and investment-specific technology risk stays largely unaltered (figures omitted for brevity).[21]

[21] This finding of an important role of the preference specification for the transmission of uncertainty shocks suggests that adopting a certain form of utility function may already predetermine the sign of the output

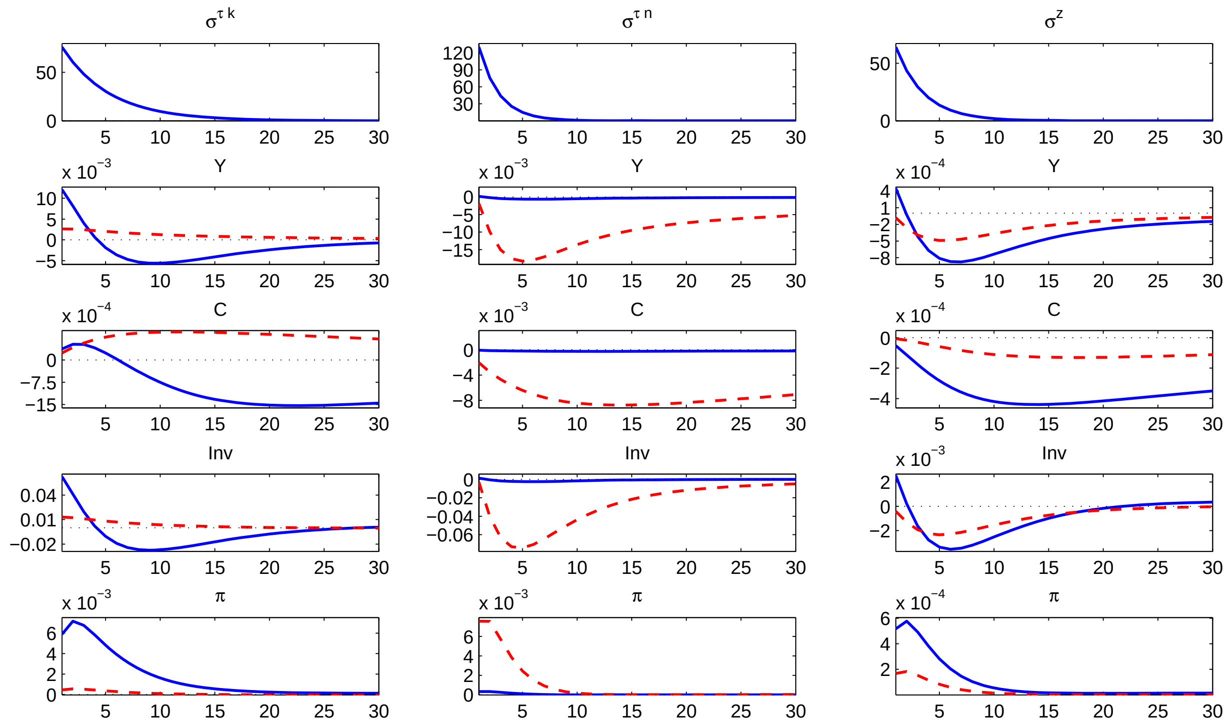

Figure 6: Impulse responses to a two-standard deviation uncertainty shock to capital taxes, labor taxes, and TFP (from left to right column).
Notes: solid blue line: KPR-preferences ($\sigma_G = 1$); red dashed line: preferences close to GHH ($\sigma_G \approx 0$). Level shocks are held constant. All responses are in percent, except for π which is in percentage points.

As noted in Section 2, the theoretical literature predicts an ambiguous effect of uncertainty as real option, Hartman-Abel, and general equilibrium effects drive the dynamics and may work in opposite directions. That this is actually the case for the specific types of uncertainty considered can be seen from, e.g., the impulse response of consumption to a capital tax shock depicted in the middle left panel of Figure 6. The consumption response is mostly negative for the case of $\sigma_G \approx 0$ but unambiguously positive for $\sigma_G = 1$. This suggests that different partial effects are dominating the respective responses for the different parameterizations. While a contractionary effect dominates in the GHH-case, an expansive effect prevails in the KPR-case. The strong dependence of uncertainty effects on the specific parameterization underscores the need for model estimation as opposed to calibration in order to trace out the aggregate effects of uncertainty.

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response to an uncertainty shock. Hence, future studies dealing with the effects of uncertainty should devote more attention to tracing out which preference specification may be the most suitable one. Our estimation results hint at a utility function featuring a low wealth effect on the labor supply. This is in line with an increasing number of studies from the fiscal policy (Monacelli and Perotti, 2008), open economy (Chang and Fernández, 2010; Garcia-Cicco et al., 2010), and news literature (Jaimovich and Rebelo, 2009; Schmitt-Grohé and Uribe, 2008), which also suggest the presence of a low wealth effect on the labor supply.

Table 7: Counterfactual calibration implying large uncertainty effects

Parameter	Description	Estimated mean	Counterfactual
ϕ_c	Consumption habits	0.96	0.9
κ	Capital adjustment costs	10.1	5
θ_p	Calvo parameter prices	0.96	0.9
σ_l	Frisch elasticity parameter	0.07	4
ρ_R	Interest smoothing	0.49	0.9
ϕ_y	Taylor rule output growth	1.22	0

6.3 Why are the Effects of Uncertainty small?

We identify strong general equilibrium effects – constraining the amplification of uncertainty shocks – as the main reason for the small effect of uncertainty on economic activity. While the model is in principle capable of generating large real effects of uncertainty, strong stabilizing effects are required to match the data moments. Therefore, SMM estimates the model parameters to imply strong equilibrating effects.

Consider the simple counterfactual experiment displayed in Table 7. Here, we decrease habit persistence, capital adjustment costs, price rigidities, and the Frisch elasticity of labor supply. To dampen the general equilibrium response of the nominal interest rate, we shut off the reaction to output growth and considerably increase the interest smoothing. In this case, as shown Figure 7, policy risk leads to a drop in output of 1.5%, which is mostly driven by a large decline in investment. While this calibration allows for larger effects of uncertainty, it comes at a cost: the model with this calibration implies unrealistically large business cycles. As shown in Table 8, output would be almost three times as volatile as found in the data, investment five times, and wages almost four times as volatile.

Hence, given the estimated exogenous driving processes, SMM estimates the parameters to imply a shock amplification more in line with the actually observed data. First, consumption habits, capital adjustment costs, and price rigidities are estimated to be quite high, generating a high persistence and thereby limiting the reaction of consumption, investment, and inflation to shocks and thus the deviations from the ergodic mean that are realized over time. Second, the parameter governing the Frisch elasticity of labor supply is estimated to be low so household's labor supply reacts quite flexibly to shocks. Third and most importantly, monetary policy reacts fast and decisively to current economic conditions and in particular to output. The resulting transmission of both uncertainty and level shocks into the economy then implies less pronounced business cycles.

The decisive reaction to output growth is evident from the large coefficient estimate in the Taylor rule. The monetary authority's aggressive reaction to changes in output has a

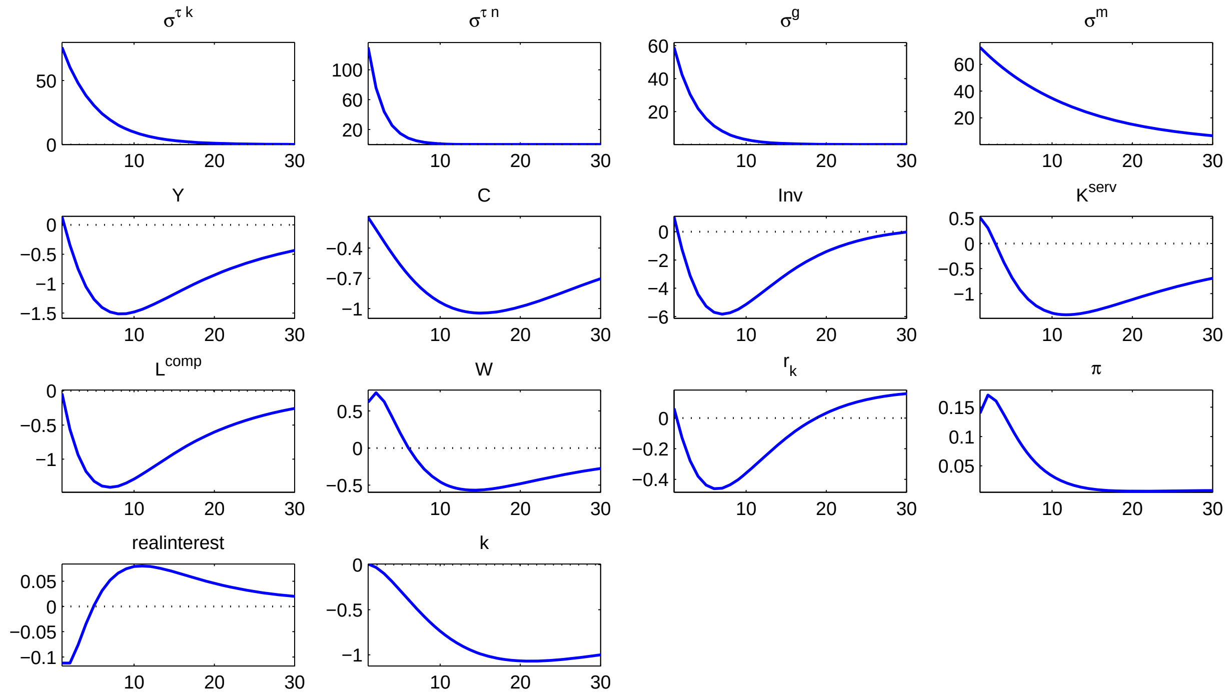

Figure 7: Impulse responses to a two-standard deviation policy risk shock under counterfactually volatile calibration.
Notes: Level shocks are held constant. All responses are in percent, except for π and *realinterest* which are in percentage points.

considerable dampening effect on the business cycle as it prevents output from deviating too far from steady state. When keeping all parameters at their baseline values but setting $\phi_y = 0$, thus shutting off the response of interest rates to output growth, triples the negative output response following a policy risk shock (figures omitted for brevity). The main reason for this behavior is the response of the real interest rate. The uncertainty shock acts like a negative supply shock, agents reduce their labor and capital input, and inflation rises. The monetary authority responds to this increase in inflation by raising the nominal interest rate without considering the negative impact on output. As a result, the real interest rate now has a positive impact response, amplifying the original shock's contractionary effect on output. In contrast, if the monetary authority also reacts to changes in output, the interest rate hike is more muted and the negative output response lower. The real interest initially declines to counteract the contractionary effect on output and only rises after several quarters.

The fast reaction of nominal interest rates to exogenous shocks can be seen from the relatively low degree of interest smoothing, meaning that current economic conditions affect nominal interests more than past interest rates. This low amount of interest smoothing exerts a considerable influence on the economy's response to uncertainty shocks, allowing a stronger counteracting reaction of the nominal interest rate, which dampens the uncertainty effects in

Table 8: Simulated and Empirical Moments: Counterfactual with stronger amplification

	Model	Data	Model	Data	Model	Data	Model	Data	Model	Data
	$\sigma(x_t)$		$\rho(x_t, y_t)$		$\sigma_{x_t}/\sigma_{y_t}$		$\rho(x_t, x_{t-1})$		$\rho(x_t, x_{t-2})$	
Y	4.47%	1.57%	1.00	1.00	1.00	1.00	0.66	0.90	0.38	0.75
C	0.65%	0.95%	0.45	0.85	0.15	0.60	0.95	0.90	0.85	0.74
I	24.90%	5.30%	0.99	0.85	5.58	3.37	0.63	0.93	0.34	0.80
Π	0.29%	0.27%	0.76	0.17	0.07	0.17	0.80	0.50	0.53	0.32
W	3.55%	0.90%	0.85	0.10	0.79	0.57	0.83	0.84	0.57	0.69
R	0.31%	0.39%	-0.90	0.34	0.07	0.25	0.82	0.86	0.57	0.67

Notes: Time Series X_t are output (Y_t), consumption (C_t), investment (I_t), inflation (Π_t), the real wage (W_t), and the nominal interest rate (R_t). Small letters denote variables that are logged and detrended using a one-sided HP-filter with smoothing parameter $\lambda = 1600$.

a similar way as the output feedback of monetary policy. When giving more weight to past interest rates compared to the currently desired nominal interest, the nominal interest rate responds more sluggishly to shocks to the system, thereby temporarily allowing for larger deviations from steady state.

Hence, our result lend support to the findings of Bachmann and Bayer (2011). Their study showed for the case of idiosyncratic uncertainty about technology that general equilibrium effects, most importantly the endogenous feedback to wages and interest rates may considerably dampen the output effects of uncertainty shocks. Our results indicate that this also holds true for the case of aggregate uncertainty in an estimated DSGE-model.

These results suggest a potential issue for studies using a "proof-of-concept"-approach. Such studies typically show that uncertainty may matter by putting one source of uncertainty along one level shock into a model and then designing a transmission mechanism that enables this source to explain the whole business cycle. Our findings indicate that more attention needs to be devoted to what happens if other shocks, both uncertainty and level are present. As soon as other competing sources of aggregate fluctuations documented in the literature are added to these models, the effects of uncertainty are bound to decrease. Moreover, the approach of considering only one source of uncertainty and designing a particular amplification mechanism to generate an output drop in response may neglect that specially designed amplification mechanisms may interact with other types of shocks in undesired ways.[22]

[22] For example, expansionary output effects of uncertainty, which in our model e.g. arise with capital tax risk, might be amplified in the same way.

7 Conclusion

The current paper analyzes the effects of policy risk, i.e. aggregate uncertainty about labor and capital tax rates, monetary policy, and government spending on aggregate activity. We find that aggregate policy risk has only minor effects on the business cycle. Although its effects are an order of magnitude larger than the ones of technological uncertainty, a two standard-deviation policy risk shock still only generates a 0.025% drop in output. The reason for this small effect is that our parameter estimates imply strong general equilibrium effects that dampen the aggregate effects of uncertainty on economic activity. Most notably, the monetary authority's estimated strong and rapid response to current conditions implies a nominal interest rate reaction that considerably reduces aggregate fluctuations. While our model is capable of generating strong uncertainty effects, such a calibration would imply unrealistically large business cycle fluctuations. Thus, SMM estimates the amplification of uncertainty shocks to be rather low.

The small effect of uncertainty on output does not imply that time-varying volatility is unimportant. In accordance with the previous literature (e.g. Justiniano and Primiceri, 2008; Primiceri, 2005), our findings suggest that the Great Moderation can be explained through a combination of "good luck" and "good policy". The historical variance estimates indicate that the standard deviation of both technology and policy shocks significantly decreased since the mid-1980s. However, most of the effect of this time-varying volatility comes in the form of a different size of the realized level shocks instead of through the uncertainty-effect.

As our analysis focuses on aggregate uncertainty, it does not necessarily contradict studies finding large effects of idiosyncratic uncertainty. However, these studies clearly require different transmission mechanisms that do not give rise to large general equilibrium effects (see also Bachmann and Bayer, 2011).

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A Data construction

Unless otherwise noted, all data are from the Bureau of Economic Analysis (BEA)'s NIPA Tables and available in quarterly frequency from 1960Q1 until 2010Q3.

A.1 Data for the exogenous processes

Capital and labor tax rates. Our approach to calculate average tax rates closely follows Mendoza et al. (1994), Jones (2002), and Leeper et al. (2010). We first compute the average personal income tax rate

$$\tau^c = \frac{IT}{W + PRI/2 + CI} \ ,$$

where IT is personal current tax revenues (Table 3.1 line 3), W is wage and salary accruals (Table 1.12 line 3), PRI is proprietor's income (Table 1.12 line 9), and $CI \equiv PRI/2 + RI + CP + NI$ is capital income. Here, RI is rental income (Table 1.12 line 12), CP is corporate profits (Table 1.12 line 13), and NI denotes the net interest income (Table 1.12 line 18).

The average labor and capital income tax rates can then be computed as

$$\tau^p = \frac{\tau^p(W + PRI/2) + CSI}{EC + PRI/2} \ ,$$

where CSI denotes contributions for government social insurance (Table 3.1 line 7), and EC is compensation of employees (Table 1.12 line 2), and

$$\tau^k = \frac{\tau^p CI + CT + PT}{CI + PT} \ ,$$

where CT is taxes on corporate income (Table 3.1 line 5), and PT is property taxes (Table

3.3 line 8).

Government spending. Government spending is the sum of government consumption (Table 3.1 line 16) and government investment (Table 3.1 line 35) divided by the GDP deflator (Table 1.1.4 line 1) and the civilian noninstitutional population (BLS, Series LNU00000000Q).

Monetary policy shock. Computed as the residual from a Taylor rule as in Clarida et al. (2000) (see Appendix B.7). The sample only starts in 1961Q1 as we lose the first year of data due to the use of four time lags as instruments in the GMM estimation.

Total factor productivity (TFP). The construction of TFP closely follows Beaudry and Lucke (2010), i.e.

$$TFP_t = \frac{Y_t}{K^{\alpha} H^{1-\alpha}} \ .$$

To construct K, we use data on capital services for the private non-farm business sector (Bureau of Labor Statistics (BLS), Historical Multifactor Productivity Tables),[23] multiply it by the total capacity utilization rate (Federal Reserve System, Statistical Release G.17 - Industrial Production and Capacity Utilization), and divide it by the civilian noninstitutional population above 16 years of age (BLS, Series LNU00000000Q). Real GDP per capita Y is nominal GDP (Table 1.1.5 line 1) divided by the GDP deflator (line 1 in Table 1.1.4) and the population, and per capita hours H are non-farm business hours worked (BLS, Series PRS85006033) divided by the population. The capital share α is set at 0.295, the mean over the sample compiled by the BLS (Bureau of Labor Statistics (BLS), Historical Multifactor Productivity Tables). The TFP-series ends in 2009Q4.

Relative price of investment. The relative price of investment is taken from Schmitt-Grohé and Uribe (2011) and only available until 2006Q4. They base their calculations on Fisher (2006).

The different sample lengths are not an issue as we estimate each exogenous process separately. Using the longest available sample assures that we make optimal use of the available information for each series.

A.2 Data for SMM

Output. Nominal GDP (Table 1.1.5 line 1) divided by the GDP deflator (Table 1.1.4 line 1) and the civilian noninstitutional population (BLS, Series LNU00000000Q).

Investment. Sum of Residential fixed investment (Table 1.1.5 line 12) and nonresidential fixed investment (Table 1.1.5 line 9) divided by the GDP deflator (Table 1.1.4 line 1) and the civilian noninstitutional population (BLS, Series LNU00000000Q).

[23]Quarterly data is interpolated from the annual series using cubic spline interpolation.

Consumption. Sum of personal consumption expenditures for nondurable goods (Table 1.1.5 line 5) and services (Table 1.1.5 line 6) divided by the GDP deflator (Table 1.1.4 line 1) and the civilian noninstitutional population (BLS, Series LNU00000000Q).
Real wage. Hourly compensation in the nonfarm business sector (BLS, Series PRS85006103) divided by the GDP deflator (Table 1.1.4 line 1).
Inflation. Computed as the log-difference of the GDP deflator (Table 1.1.4 line 1).
Nominal interest rate. Geometric mean of the effective Federal Funds Rate (St.Louis FED - FRED Database, Series FEDFUNDS).

A.3 Additional data for GMM

Interest term spread. We use the difference of the quarterly geometric mean of the 10-Year Treasury Constant Maturity Rate (FRED Database, Series GS10) and the quarterly geometric mean of the 3-Month Treasury Bill: Secondary Market Rate (FRED Database, Series TB3MS).
Money growth rate. Growth rate of the M2 Money Stock (FRED Database, Series M2SL).
Commodity inflation. Commodity inflation is computed as the growth rate of the X12-seasonally adjusted Producer Price Index: All Commodities (FRED Database, Series PPI-ACO).
Output gap. The output gap is constructed as the percentage difference between real GDP (FRED Database, Series GDPC96) and Real Potential Gross Domestic Product (FRED Database, Series GDPPOT).

B Econometric Methods

B.1 The Particle Filter

For ease of exposition, let x_t be a generic observable AR(1) process

$$x_t = \rho x_{t-1} + e^{\sigma_t} \nu_t \ , \quad \nu_t \sim \mathcal{N}(0,1) \tag{B.1}$$

where the unobserved/latent state σ_t follows a stochastic volatility process

$$\sigma_t = (1 - \rho^\sigma)\,\bar{\sigma} + \rho^\sigma \sigma_{t-1} + \eta \varepsilon_t \ , \quad \varepsilon_t \sim \mathcal{N}(0,1), \tag{B.2}$$

where $\bar{\sigma}$ is is the unconditional mean of σ_t. The shock to the volatility ε_t is assumed to be independent from the level shock ν_t.

Hence, a filter is required to obtain the so-called filtering density $p\left(\sigma_t | \, x^t; \Theta\right)$. Due to the nonlinearity embedded in the stochastic volatility setup of the shocks, we cannot simply employ the Kalman filter as in the case of linearity and normally distributed shocks. Instead, we employ the *Sequential Importance Resampling* (SIR) particle filter, a special application of the more general class of *Sequential Monte Carlo* methods, to evaluate the likelihood (Fernández-Villaverde and Rubio-Ramírez, 2007; Fernández-Villaverde et al., forthcoming). Given the structure in (B.1) and (B.2) and some initial value x_0, the factorized likelihood of observing x^T can be written as

$$\begin{aligned} p\left(x^T;\Theta\right) &= \prod_{t=1}^{T} p\left(x_t|x^{t-1};\Theta\right) \\ &= \int p\left(x_1|x_0,\sigma_0;\Theta\right) d\sigma_0 \ \prod_{t=2}^{T} \int p\left(x_t|x_{t-1},\sigma_t;\Theta\right) p\left(\sigma_t|x^{t-1};\Theta\right) d\sigma_t \\ &= \int \frac{1}{e^{\sigma_0}\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{x_1-\rho x_0}{e^{\sigma_0}}\right)^2\right] d\sigma_0 \\ &\times \prod_{t=2}^{T} \int \frac{1}{e^{\sigma_t}\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{x_t-\rho x_{t-1}}{e^{\sigma_t}}\right)^2\right] p\left(\sigma_t|x^{t-1};\Theta\right) d\sigma_t \ , \end{aligned} \tag{B.3}$$

where x^t is a $(t \times 1)$ vector that stacks the observations on x up to time t, Θ stacks the parameters, and the last equality follows from the assumption of normally distributed shocks. Although we do not have an analytical expression for $p\left(\sigma_t|x^{t-1};\Theta\right), t = 1,\ldots,T$, and can therefore not compute it directly, we can employ the particle filter to estimate the likelihood by iteratively drawing from $p\left(\sigma_t|x^{t-1};\Theta\right)$.

The underlying idea of the particle filter is to use an approximation of the filtering density $p\left(\sigma_t|\, x^t;\Theta\right)$ with a simulated distribution generated from empirical data. This distribution can be formed from mass points, or particles,

$$p\left(\sigma_t|\, x^t;\Theta\right) \simeq \sum_{i=0}^{N} \omega_t^i \delta_{\sigma_t^i}\left(\sigma_t\right), \quad \sum_{i=0}^{N} \omega_t^i = 1, \quad \omega_t^i \geqslant 0 \tag{B.4}$$

where δ is the Dirac delta function and ω_t^i is the weight attached to the respective draw/particle σ_t^i (Godsill et al., 2004). We can then use a *Sequential Importance Resampling* (SIR)-approach to update particles from time t to $t+1$ and obtain the new filtering distribution at $t+1$ (see e.g. Fernández-Villaverde et al., forthcoming). A convenient by-product of this filtering approach is that we also approximate $p\left(\sigma_t|x^{t-1};\Theta\right)$, the distribution we need to build the likelihood.

The SIR is a two-step procedure that, by using a prediction and a resampling/filtering step for each time period, ultimately allows to iteratively draw from $p\left(\sigma_t|x^{t-1};\Theta\right)$. Starting with $p\left(\sigma_0|\, x^0;\Theta\right) = p\left(\sigma_0;\Theta\right)$, the prediction step uses the law of motion for the states $f(\sigma_{t+1}|\sigma_t)$, equation (B.2), to obtain the conditional density $p\left(\sigma_1 | x^0; \Theta\right) = p\left(\varepsilon_1\right) p\left(\sigma_0 | x^0; \Theta\right)$. That is, given N draws $\left\{\sigma_{t|t}^i\right\}_{i=1}^N$ from $p\left(\sigma_t | x^t; \Theta\right)$, (here $p\left(\sigma_0 | x^0; \Theta\right)$) and a draw of exogenous shocks $\varepsilon_t^i \sim \mathcal{N}(0,1)$, we can use equation (B.2) to compute $\left\{\sigma_{t+1|t}^i\right\}_{i=1}^N$.[24]

Next, the resampling/filtering step uses importance resampling to update the conditional probability from $p\left(\sigma_t | x^{t-1}; \Theta\right)$ to $p\left(\sigma_t | x^t; \Theta\right)$. The crucial idea is that if $\left\{\sigma_{t|t-1}^i\right\}_{i=1}^N$ is a draw from $p\left(\sigma_t | x^{t-1}; \Theta\right)$ and $\{\widetilde{\sigma}_t^i\}_{i=1}^N$ is a draw with replacement from $\left\{\sigma_{t|t-1}^i\right\}_{i=1}^N$ using the resampling probabilities

$$\omega_t^i = \frac{p\left(x_t | x^{t-1}, \sigma_{t|t-1}^i; \Theta\right)}{\sum_{i=1}^N p\left(x_t | x^{t-1}, \sigma_{t|t-1}^i; \Theta\right)} , \tag{B.5}$$

then $\left\{\sigma_{t|t}\right\}_{i=1}^N = \{\widetilde{\sigma}_t^i\}_{i=1}^N$ is a draw from $p\left(\sigma_t | x^t; \Theta\right)$. The resampling with probabilities given in (B.5) serves two purposes. First, the reweighting implements an importance sampling approach, i.e. draws are obtained from a proposal density that is easy to draw from and are then subsequently reweighted to reflect the density to be approximated (see Arulampalam et al., 2002, for a derivation).[25] Second, without the resampling step, there would be an increase in the unconditional variance of ω_t over time, yielding only one particle with non-zero weight (known as degeneracy or sample impoverishment, see Arulampalam et al. (2002)). By resampling, we keep only those particles with high ω_t^i (i.e. those that are closer to the true state vector). Having now obtained draws from $p\left(\sigma_t | x^t; \Theta\right)$, we can again start with the prediction step to obtain draws for time period $t+1$.

After T iterations, we get an estimate of our likelihood as[26]

$$\begin{aligned} p\left(x^T; \Theta\right) \simeq & \frac{1}{N} \sum_{i=1}^N \frac{1}{e^{\sigma_{0|0}} \sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{x_1 - \rho x_0}{e^{\sigma_{0|0}}}\right)^2\right] \\ & \times \prod_{t=2}^T \frac{1}{N} \sum_{i=1}^N \frac{1}{e^{\sigma_{t|t-1}} \sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{x_t - \rho x_{t-1}}{e^{\sigma_{t|t-1}}}\right)^2\right] . \end{aligned} \tag{B.6}$$

B.2 Particle Smoother

We employ the *backward-smoothing routine* suggested by Godsill et al. (2004) to draw from the smoothing density $p(\sigma^T | x^T; \Theta)$ to get a historical distribution of the volatilities. Specifically,

[24]The notation $t+1|t$ indicates a draw at time $t+1$ conditioned on the information available at time t.

[25]In our case, we use the prior density $p\left(\sigma_t | \sigma^{t-1}; \Theta\right)$ as the importance density.

[26]See Fernández-Villaverde and Rubio-Ramírez (2007) and Doucet and Johansen (2009) and the references contained therein for the conditions required for a central limit theorem to apply, yielding a consistent estimator of $p\left(x^T; \Theta\right)$.

we start with the factorization

$$p(\sigma^T|x^T;\Theta) = p(\sigma_T|x^T;\Theta)\prod_{t=1}^{T-1} p(\sigma_t|\sigma_{t+1:T},x^T;\Theta) \ . \tag{B.7}$$

The second factor can be further simplified

$$\begin{aligned} p(\sigma_t|\sigma_{t+1:T},x^T;\Theta) &= p(\sigma_t|\sigma_{t+1},x^t;\Theta) \\ &= \frac{p(\sigma_t|x^t;\Theta)f(\sigma_{t+1}|\sigma_t)}{p(\sigma_{t+1}|x^t)} \\ &\propto p(\sigma_t|x^t;\Theta)f(\sigma_{t+1}|\sigma_t) \ , \end{aligned} \tag{B.8}$$

where the first equality results from the Markovian properties of the model and f denotes the state transition density following from equation (B.2). Equation (B.4) describes how to approximate $p(\sigma_t|x^t;\Theta)$ by forward filtering. Therefore, we can approximate $p(\sigma_t|\sigma_{t+1:T},x^T;\Theta) \propto p(\sigma_t|x^t;\Theta)f(\sigma_{t+1}|\sigma_t)$ by

$$p(\sigma_t|\sigma_{t+1},x^T;\Theta) \simeq \sum_{i=1}^{N} \omega^i_{t|t+1}\delta_{\sigma^i_t}(\sigma_t) \ , \tag{B.9}$$

where the new weights $\omega^i_{t|t+1}$ are given by

$$\omega^i_{t|t+1} = \frac{\omega^i_t f(\sigma_{t+1}|\sigma^i_t)}{\sum_{j=1}^{N}\omega^j_t f(\sigma_{t+1}|\sigma^j_t)} \ . \tag{B.10}$$

and the ω^i_t are the weights obtained in the filtering step. Denote with $\tilde{\sigma}^i_t$ the i^{th} draw from the smoothing density at time t. At time T, we can obtain draws $\tilde{\sigma}^i_T$ by drawing from $p(\sigma_T|x^T)$ with the weights ω^i_T. Then, going backwards in time, we can use the above recursions to iteratively obtain draws $\tilde{\sigma}^i_t$ by resampling using the weights given in (B.10).

B.3 Tailored Randomized Block Metropolis Hastings Algorithm

Let Θ, $p\left(x^T|\Theta\right)$, and $\pi(\Theta)$ denote the vector of parameters to be estimated, the likelihood function, and the prior distribution of the parameters, respectively. The posterior distribution $\pi(\Theta|x^T)$ can be computed as

$$\pi\left(\Theta|x^T\right) \propto p\left(x^T|\Theta\right)\pi\left(\Theta\right) \ . \tag{B.11}$$

Given this usually analytically intractable posterior, most macroeconomic applications employ a Random Walk Metropolis-Hastings (RW-MH) algorithm to generate draws from the posterior distribution. However, the standard RW-MH algorithm often has poor mixing properties, leading to highly autocorrelated draws, and is therefore often very inefficient. Hence, to increase the efficiency, we use the Tailored Randomized Block Metropolis Hastings (TaRB-MH) algorithm proposed by Chib and Ramamurthy (2010).[27] Instead of in each iteration step simultaneously drawing an entire new parameter vector from a proposal density, the parameter vector is randomly split up into several blocks. Each block is then subsequently updated by a separate MH run, conditional on the previous step's values of the parameters in the other blocks. Ideally, the blocks should be formed according to the correlation between parameters, with highly correlated parameters belonging to the same block. However, we have no a priori knowledge about the correlation between parameters and resort to a blocking scheme where both the number of blocks and its composition are randomized in each step. This algorithm provides a good compromise between the standard RW-MH and tailored multiple block MH algorithms that use multiple blocks, which are particularly designed for the problem at hand. The second feature that improves on the standard RW-MH is that in each step the proposal density is "tailored" to the location and the curvature of the posterior density in that block by using a non-derivative based global optimizer. We deviate from Chib and Ramamurthy (2010) by using the CMAES algorithm (Hansen et al., 2003) instead of a simulated annealing as the former has been shown to be more efficient (Andreasen, 2010).[28] Moreover, it requires considerably less tuning than a simulated annealing. The TaRB-MH algorithm proceeds as follows.

1. At each iteration step n, $n = 1, \ldots, N$, the elements of the parameter vector θ are separated into random blocks $(\theta_{n,1}, \theta_{n,2}, \ldots, \theta_{n,p_n})$ by perturbing their initial ordering and assigning the first parameter in the perturbed vector to the first block and each following parameter with probability $p = 0.5$ to a new block, leaving us with 2.5 blocks on average as we estimate 5 parameters.

2. At each iteration step n, each block $\theta_{n,l}, l = 1, \ldots, p_n$ is sampled by a Metropolis-Hastings step using a proposal density adapted to the posterior in the following way. Denote with $\theta_{n,-l}$ the most current value of all blocks except for the lth one, i.e. their value at the end of step $n-1$. To generate a new draw for $\theta_{n,l}$, the CMAES-algorithm is used to find
$$\hat{\theta}_{n,l} = \arg\max_{\theta_{n,l}} \log\left[p\left(x^T|\theta_{n,l}, \theta_{n,-l}\right)\pi\left(\Theta\right)\right]. \tag{B.12}$$
That is, we use a global optimizer to maximize the posterior over the current block l, given the value of all other parameters at the end of step $n-1$. Having found the

[27]Using the TaRB-MH decreased the inefficiency factors from values around 10 to below 2.
[28]For an intuitive introduction to the working of the CMAES algorithm, see Binsbergen et al. (2010).

"conditional mode" $\hat{\theta}_{n,l}$, we compute the curvature of the target posterior distribution in the standard way as the negative inverse of the Hessian at the "conditional mode"

$$V_{n,l} = \left(- \frac{\partial \log \left[p \left(x^T | \theta_{n,l}, \theta_{n,-l} \right) \pi \left(\Theta \right) \right]}{\partial \theta_{n,l} \theta'_{n,l}} \right)^{-1} \Bigg|_{\theta_{n,l} = \hat{\theta}_{n,l}} . \tag{B.13}$$

Following Chib and Ramamurthy (2010), we use a multivariate t-distribution with ν degrees of freedom as proposal density for $\theta_{n,l}$, $q_l \left(\theta_{n,l} | \theta_{n,-l}, x^T \right)$. Mean and variance are set to the "conditional mode" and the negative inverse of the Hessian at this point:

$$q_l \left(\theta_{n,l} | \theta_{n,-l}, x^T \right) = t \left(\theta_{n,l} | \hat{\theta}_{n,l}, V_{n,l}, \nu \right) . \tag{B.14}$$

In the Metropolis-Hastings-step, a proposed value $\theta^*_{n,l}$ is accepted as the new value of the block with probability

$$\alpha_l \left(\theta_{n,l}, \theta^*_{n,l} \middle| \theta_{n,-l}, x^T \right) = \min \left[\frac{p \left(x^T | \theta^*_{n,l}, \theta_{n,-l} \right) \pi \left(\theta^*_{n,l} \right)}{p \left(x^T | \theta_{n,l}, \theta_{n,-l} \right) \pi \left(\theta_{n,l} \right)} \frac{t \left(\theta_{n,l} | \hat{\theta}_{n,l}, V_{n,l}, \nu \right)}{t \left(\theta^*_{n,l} \middle| \hat{\theta}_{n,l}, V_{n,l}, \nu \right)}, 1 \right] . \tag{B.15}$$

If the proposed value $\theta^*_{n,l}$ is rejected, we set $\theta_{n+1,l} = \theta_{n,l}$. This step is repeated for all p_n blocks before the algorithm starts over with step 1.

Setting $\nu = 5$ and iterating over steps 1 and 2, we can - after a suitable burn-in-period - obtain samples from the desired posterior distribution, which is the invariant distribution of the resulting Markov Chain. In our case, a burn-in of 2500 proved sufficient.

B.4 Model Solution

Let s_t denote the $n_s \times 1$ vector of state variables in deviations from steady state, including the exogenous shocks and the perturbation parameter Λ, and let s^i_t denote its ith entry. The policy function/law of motion for an arbitrary model variable $\widehat{X}_t$ then has the form

$$\widehat{X}_t = \sum_{i=1}^{n_s} \xi^X_i s^i_t + \frac{1}{2} \sum_{i=1}^{n_s} \sum_{j=1}^{n_s} \xi^X_{i,j} s^i_t s^j_t + \sum_{i=1}^{n_s} \sum_{j=1}^{n_s} \sum_{l=1}^{n_s} \xi^X_{i,j,l} s^i_t s^j_t s^l_t \ , \tag{B.16}$$

where the ξ's are scalars that depend on the deep parameters of the model and hats denote percentage deviations from steady state. Equation (B.16) shows why lower-order approximations would not be sufficient for our purpose.

As is well known, a first-order approximation exhibits certainty equivalence. This implies $\xi^X_v = 0$, where v denotes the position of a volatility shock in the state vector s. That is, up to first order, uncertainty shocks do not enter the policy function at all.

For a second-order approximation, it is well known from Schmitt-Grohé and Uribe (2004) for the homoscedastic case that uncertainty only enters the policy function through a constant term via the second derivative with respect to the perturbation parameter, i.e. through $\xi_{\Lambda,\Lambda} \neq 0$. However, things are more complicated in the heteroscedastic case where shocks to the variance occur, leading to an additional effect. Fernández-Villaverde et al. (2010) prove that in this case, the volatility shocks additionally only enter the policy function with non-zero coefficients in their interaction term with the respective level shock. Algebraically, only the cross-product of $\hat{\sigma}^i \times \hat{\nu}^i$ is different from 0. In contrast, all other cross-terms with the uncertainty shocks are zero, i.e $\xi^X_{v,j\neq u} = 0$, where v and u denote the positions of a volatility and its corresponding level shock in the state vector s, respectively. Hence, the effect of uncertainty is always mediated through level shocks. It is not possible to shock the variance of the level shocks independently from the level shock as its effect would be 0 by construction.

Only in the third-order approximation do the volatility shocks enter the policy function separately from the level shocks in a non-constant form. Most importantly, the term $\xi_{i,\Lambda,\Lambda}$ is in general different from 0 for all volatility shocks.

B.5 Simulated Method of Moments

The idea of the Simulated Method of Moments (SMM) is the following. Let x_t be a time t vector of observables from a stationary and ergodic distribution and let $\{x_t\}_{t=1}^{T}$ be the corresponding sequence. Furthermore, let $m(x_t)$ denote a $k \times 1$ vector of empirical moments computed from this data. Denote with $\{x_t^{sim}(\theta)\}_{t=1}^{aT}$ the corresponding time series of length aT generated from simulating the model using the $p \times 1$ parameter vector $\theta \in \Theta$, with $\Theta \subset R^p$. Let $m(x_t^{sim}(\theta))$ be the vector of simulated moments computed from the artificial data. The SMM estimator is the value of θ that satisfies

$$\widehat{\theta} = \underset{\theta\in\Theta}{\arg\min} \left[m(x_t) - m\left(x_t^{sim}(\theta)\right)\right]' W \left[m(x_t) - m\left(x_t^{sim}(\theta)\right)\right] \ , \tag{B.17}$$

where W is a $p \times p$ positive definite weighting matrix. Under the assumption that the model with $\theta = \theta_0$ is a correct representation of the true process that generated $m(x_t)$ and the regularity conditions spelled out in Duffie and Singleton (1993), $\hat{\theta}$ is a consistent estimator of θ_0 with asymptotic distribution

$$\sqrt{T}\left(\widehat{\theta} - \theta_0\right) \xrightarrow{d} \mathcal{N}\left(0, (1 + 1/\tau)\left(J'WJ\right)^{-1} J'WSWJ\left(J'WJ\right)^{-1}\right), \tag{B.18}$$

where

$$S = \lim_{T\to\infty} Var\left((1/\sqrt{T})\sum_{t=1}^{T} m(x_t)\right), \tag{B.19}$$

and $J = E(\partial m(x_t^{sim})/\partial\theta)$ (see Ruge-Murcia, 2010).

This estimator is asymptotically efficient when using the weighting matrix

$$W = \left(V^{longrun}\right)^{-1} = \left[\lim_{T\to\infty} Var\left(\frac{1}{\sqrt{T}}\sum_{t=1}^{T} m\left(x_t\right)\right)\right]^{-1}. \tag{B.20}$$

The ideal weighting matrix places the most weight on the linear combination of moments that are the most precisely measured in the data. However, for two reasons, we use only the diagonal of the optimal weighting matrix:

$$W^{diag} = \text{diag}\left(V^{longrun}\right)^{-1}. \tag{B.21}$$

First, we would like to put more weight on moments that are actually observed in the data and that are economically meaningful, rather than on a linear combination of moments (see also Cochrane, 2005). Second, in practice, fully specified weighting matrices often lead to diverging parameter estimates. As shown in Ruge-Murcia (2010), using only the main diagonal of the optimal weighting matrix leads to a loss in efficiency but nevertheless delivers good results in most cases.

The simulation proceeds as follows. Starting at the deterministic steady state, we simulate the model for 3015 quarters using shocks drawn from the estimated shock distributions. Shocks larger than two standard deviations are trimmed. To assure non-explosive behavior of the simulations, we use the pruning algorithm of Kim et al. (2008). We discard the first 2000 quarters as a burn-in in order to reach the ergodic distribution. We then use the remaining 1015 quarters to compute the respective moments. The results are robust to using a longer burn-in period. The choice of using five times the length of the original data sample (i.e. $a = 5$) to compute the moments is motivated by the simulations in Ruge-Murcia (2010), who finds this choice to deliver a good balance between the precision of the estimates and computation time.

B.6 Impulse Responses

The nonlinearity of our model complicates the computation of impulse responses compared to linear models. We follow Fernández-Villaverde et al. (forthcoming) and generate impulse responses as the response to a two standard deviation shock to uncertainty at the ergodic mean. First, we simulate the model for 2,000 quarters by drawing shocks from the respective estimated distributions. Shocks larger than two standard deviations are trimmed to assure convergence, which technically depends on the shocks being bounded. To assure non-explosive behavior of the simulations, we use the pruning algorithm of Kim et al. (2008). We discard the first 2,000 quarters as a burn-in in order to reach the ergodic distribution and use the next 675 quarters to compute the ergodic mean. Starting at the ergodic mean, we compute the IRFs as the percentage difference of the respective variables between the system shocked with the respective shock and the baseline model response, i.e. the model response without shocks. To account for sampling uncertainty, we generate 50 different IRFs with different starting values of the random number generator and take the cross-sectional average as our impulse response.

B.7 GMM

We construct the monetary policy shocks by specifying the Federal Reserve's policy reaction function and estimating it by the generalized method of moments (GMM). Our approach is similar to the one used in Clarida et al. (2000), with the difference that Clarida et al. (2000) use a forward-looking policy reaction function, while we use a rule that reacts to contemporaneous variables to stay consistent with our DSGE-model. Specifically, the policy reaction function to be estimated is given by

$$r_t = \rho r_{t-1} + (1-\rho)\left[\bar{r} + \phi_\pi\left(\pi_t - \bar{\pi}\right) + \phi_y y_t^{gap}\right] + \varepsilon_t \ , \tag{B.22}$$

where π_t is inflation with target rate $\bar{\pi}$, y_t^{gap} is the output gap, r_{t-1} allows for interest smoothing, $\bar{r}$ ist the target nominal interest rate, and ε_t is an error term. Using the vector of instruments $\mathbf{z}_t$, the set of moment conditions for our GMM estimation procedure can be written as

$$E\left[\left\{r_t - \rho r_{t-1} - \alpha - \beta\pi_t - \gamma y_t^{gap}\right\} z_t\right] = 0 \tag{B.23}$$

where $\alpha = (1-\rho)\left(\bar{r} + \phi_\pi\bar{\pi}\right)$ collects all constant terms, $\beta = (1-\rho)\phi_\pi$, and $\gamma = (1-\rho)\phi_y$.

Hence, we regress the average effective Federal Funds Rate in the first month of the quarter on the lagged FFR, the inflation rate, and the output gap, where all rates are annualized. The set of instruments includes four lags of the FFR, the inflation rate, the output gap, commodity price inflation, money growth, and the interest term spread. Because we are only interested in the residuals of the policy reaction function $\widehat{\varepsilon}_t$, we do not need to separately identify the target nominal rate $\bar{r}$ and target inflation $\bar{\pi}$.

Table 9 presents the estimation results, which are all in the range typically reported in the literature. There is strong evidence of interest smoothing with $\rho = 0.898$. The point estimates

Table 9: GMM Estimation of Taylor Rule

Coefficient	Mean	Std. Error	t-Statistic	Prob.
ρ	0.898	0.018	48.926	0.000
α	0.001	0.001	0.874	0.383
β	0.1741	0.027	6.361	0.000
γ	0.102	0.017	5.950	0.000
R-squared	0.890	Mean dependent var		0.058
Adjusted R-squared	0.888	Sum squared resid		0.027
S.E. of regression	0.012	J-statistic		18.545
Durbin-Watson stat	2.314	pval(J-statistic)		0.552

Note: Kernel: Bartlett, Bandwidth: Fixed (4), No prewhitening; Simultaneous weighting matrix & coefficient iteration; Convergence achieved after: 28 weight matrices, 29 total coef iterations.

of the feedback parameters are $\phi_\pi = 1.718$ and $\phi_y = 1.003$. The test of overidentifying restrictions shows that the model cannot be rejected at conventional significance levels.

C Diagnostics

C.1 Testing for Heteroscedasticity

Table 10 presents evidence of the need to model time-varying volatility. Despite our relatively short sample size and the low power of tests for heteroscedasticity, the null hypothesis of homoscedastic shocks can be rejected at the 10% level for all series except labor taxes. This result is consistent with evidence that the standard deviation of structural shocks has changed over time (see e.g. Justiniano and Primiceri, 2008; Primiceri, 2005).

Table 10: Tests for Heteroscedasticity

	τ^k	τ^n	z	z_I	g	m
White	0.000*	0.932	0.001*	0.042*	0.360	0.068*
WW	0.169	0.523	0.265	0.005*	0.076*	0.068*
BPK	0.004*	0.890	0.126	0.770	0.511	0.298

Notes: Asterisks indicate significance at the 10% level. White refers to the standard White (1980)-test, WW refers to the Wooldridge (1990)-version of this test, and BPK refers to the Breusch and Pagan (1979)/Koenker (1981)-test.

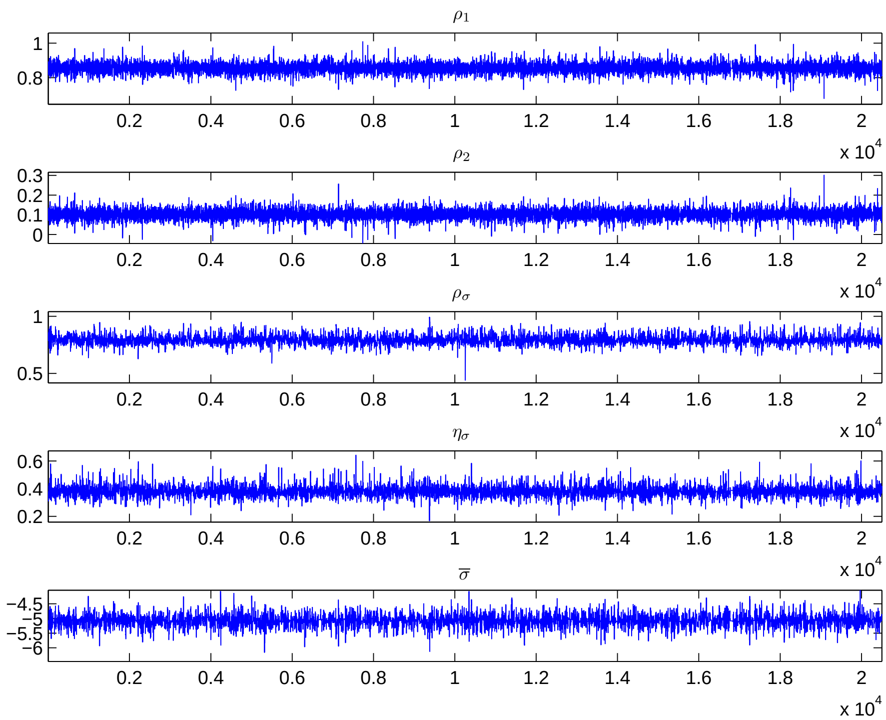
C.2 Convergence Diagnostics

Table (11) shows the results from the Geweke (1992)-convergence diagnostics that compares the means of the first 20% of draws with that of the last 50% of the draws. In general, all MCMC chains have converged to their stationary distribution as indicated by the p-values of the χ^2-test for equal means. Figures 8 to 13 show the corresponding mean plots.

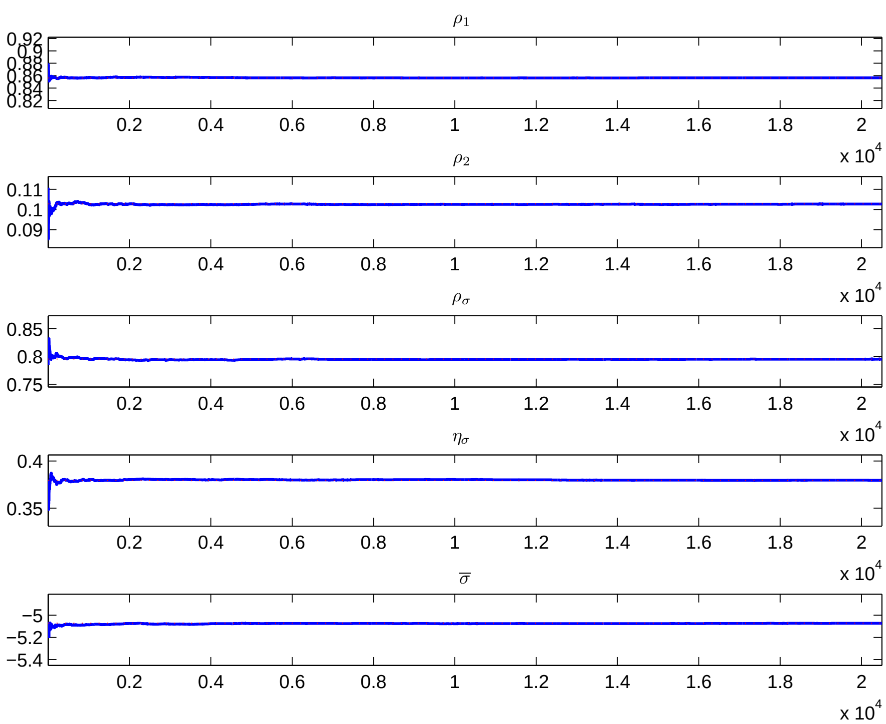
Table 11: Geweke (1992) Convergence Diagnostics

Parameter	4% taper	8% taper	15% taper	4% taper	8% taper	15% taper
	Capital Tax Rates			Labor Tax Rates		
ρ_1	0.160	0.165	0.145	0.909	0.890	0.887
ρ_2	0.947	0.941	0.937	0.926	0.913	0.904
ρ_σ	0.623	0.596	0.566	0.648	0.652	0.653
η_σ	0.929	0.927	0.919	0.327	0.319	0.271
$\bar{\sigma}$	0.760	0.744	0.738	0.922	0.921	0.917
	Total Factor Productivity			Investment Specific Technology		
ρ_1	0.891	0.887	0.879	0.199	0.174	0.124
ρ_2	0.679	0.681	0.665	0.353	0.340	0.297
ρ_σ	0.643	0.615	0.583	0.546	0.534	0.520
η_σ	0.456	0.453	0.391	0.638	0.649	0.638
$\bar{\sigma}$	0.772	0.765	0.706	0.304	0.260	0.187
	Government Spending			Monetary Policy Shock		
ρ_1	0.608	0.598	0.572	0.192	0.200	0.181
ρ_2	0.605	0.606	0.558			
ρ_σ	0.550	0.561	0.562	0.231	0.227	0.155
η_σ	0.293	0.267	0.232	0.885	0.870	0.860
$\bar{\sigma}$	0.412	0.402	0.369	0.066	0.078	0.071

Notes: Numbers are p-values of the χ^2-test for equal means of the first 20% of draws and the last 50% of the draws (after the first 2500 draws are discarded as burn-in).

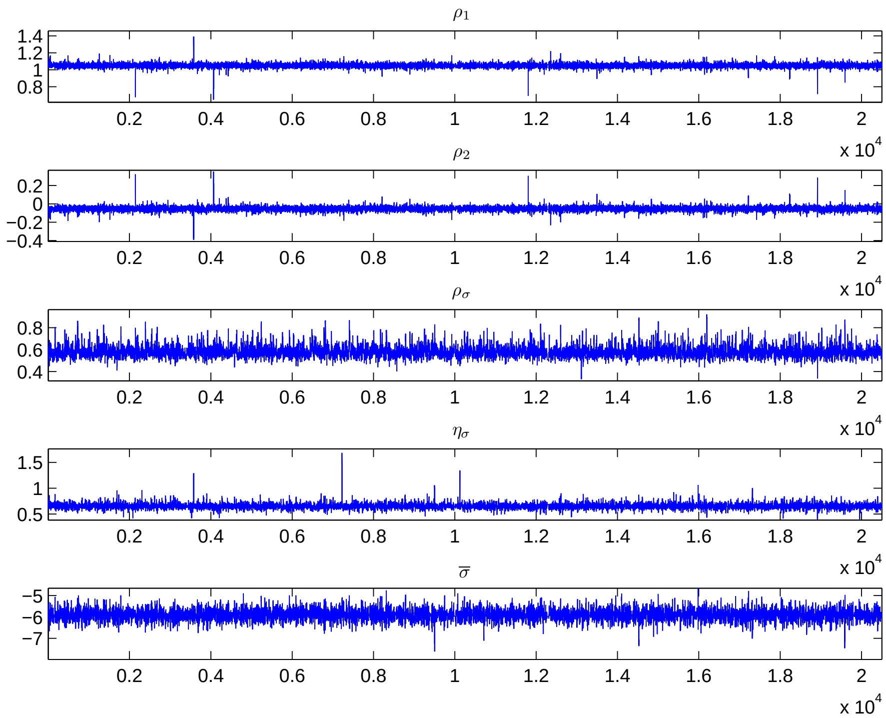


(a) MCMC draws.

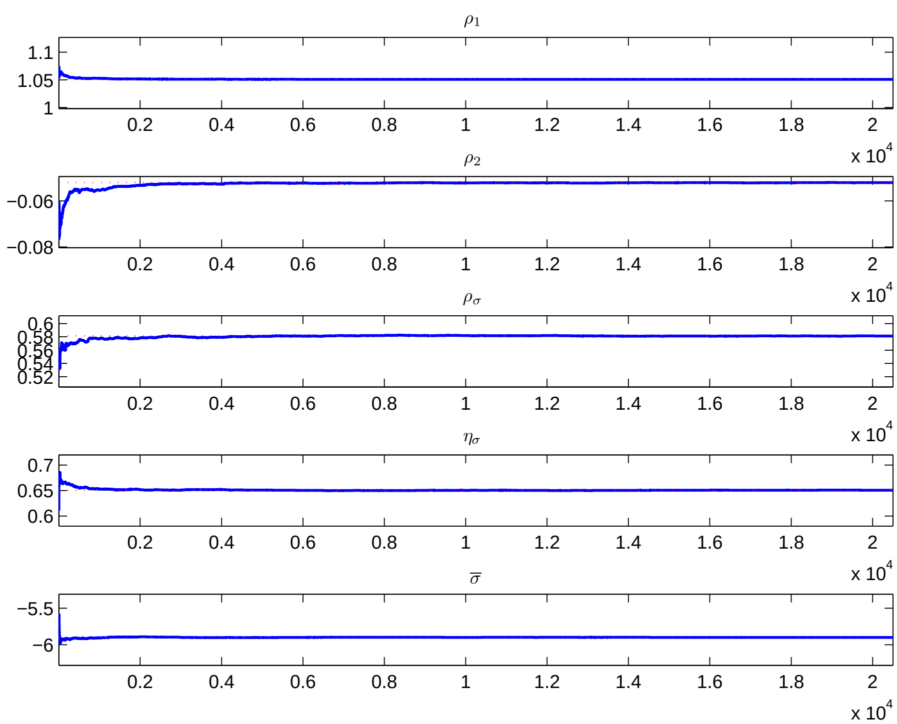


(b) Mean of the parameters over time.

Figure 8: Evolution of MCMC sampler over time for τ^k.

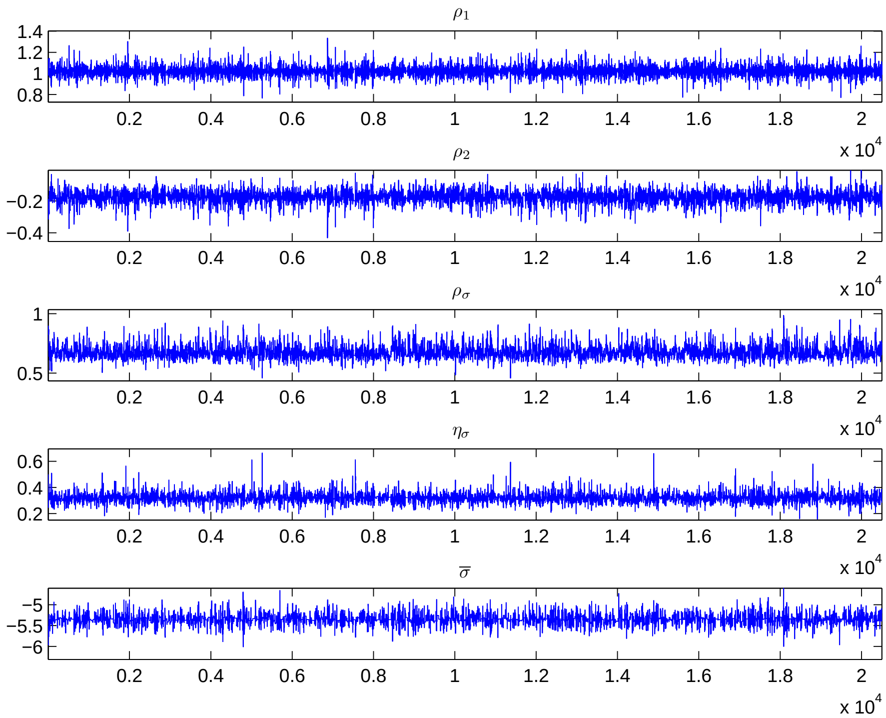


(a) MCMC draws.

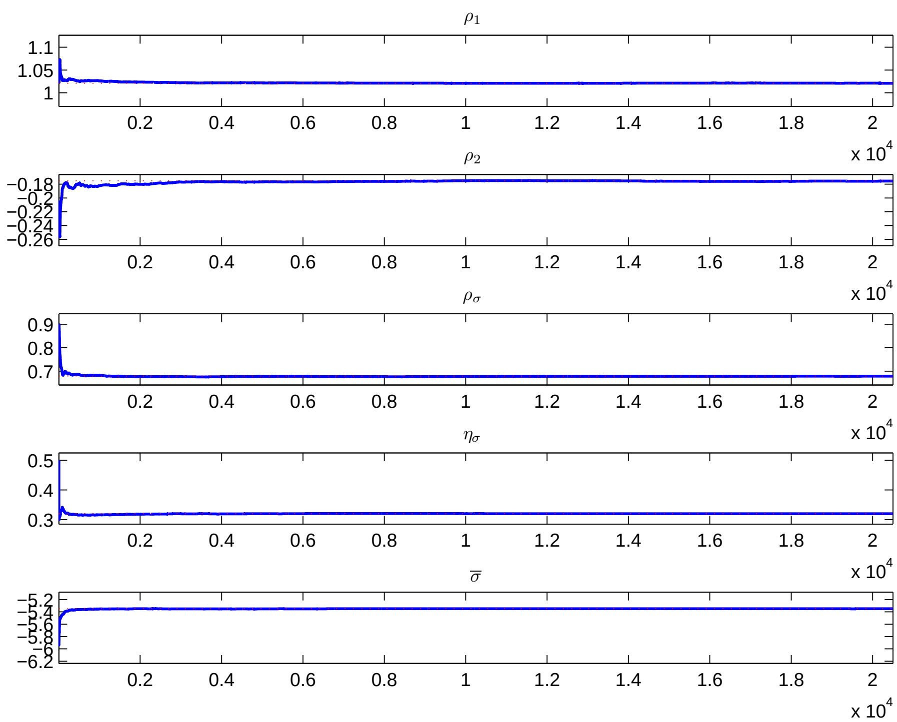


(b) Mean of the parameters over time.

Figure 9: Evolution of MCMC sampler over time for τ^n.

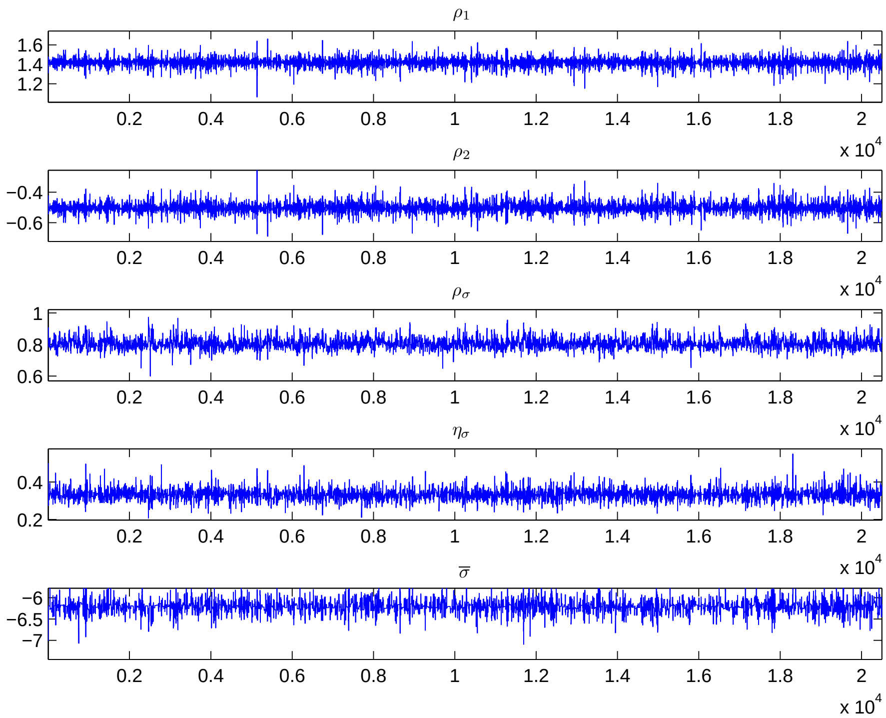


(a) MCMC draws.

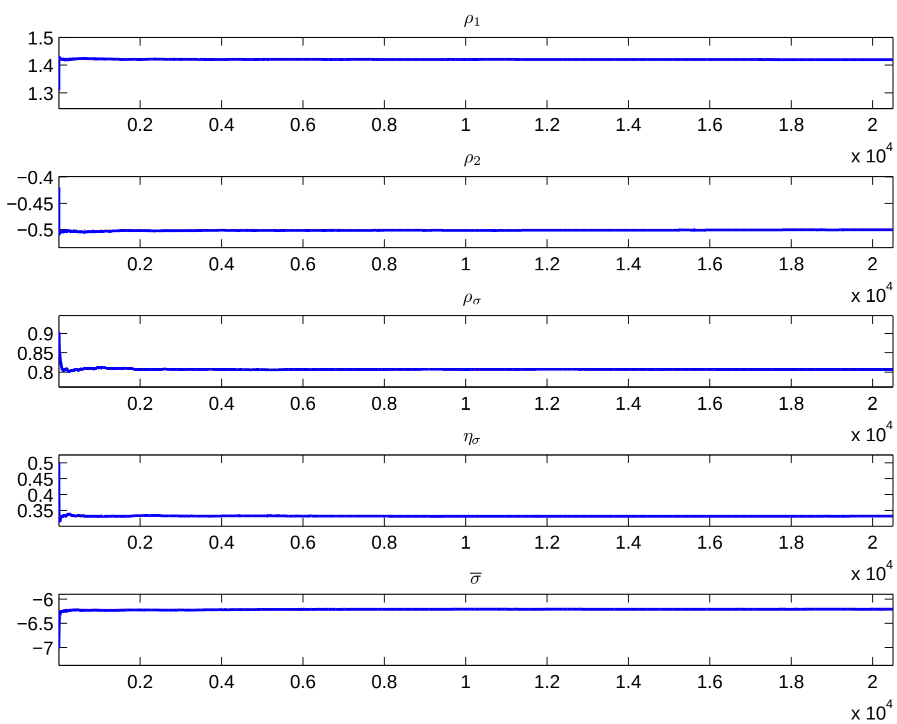


(b) Mean of the parameters over time.

Figure 10: Evolution of MCMC sampler over time for z.

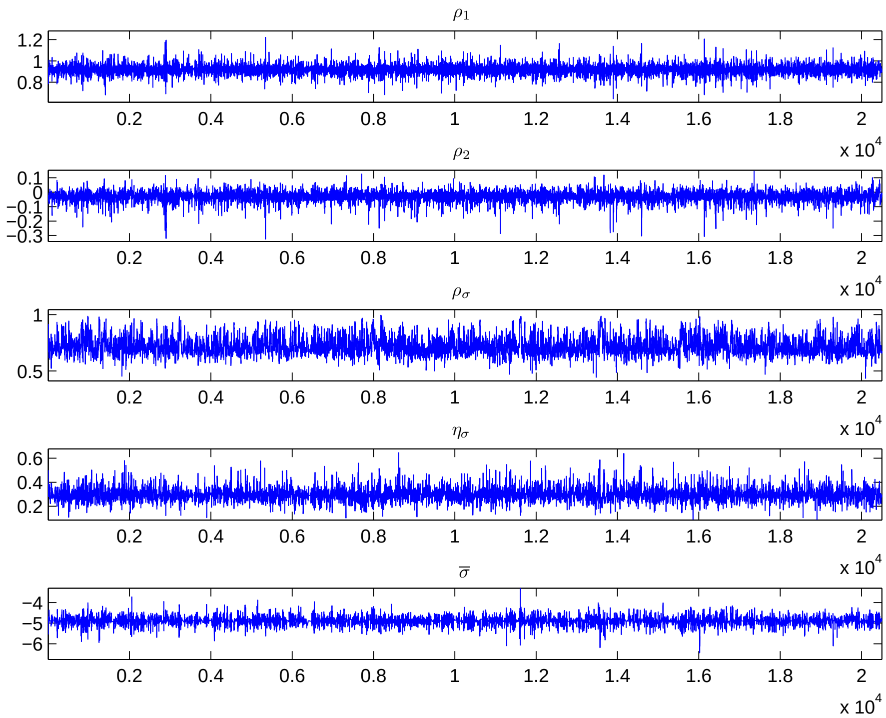


(a) MCMC draws.

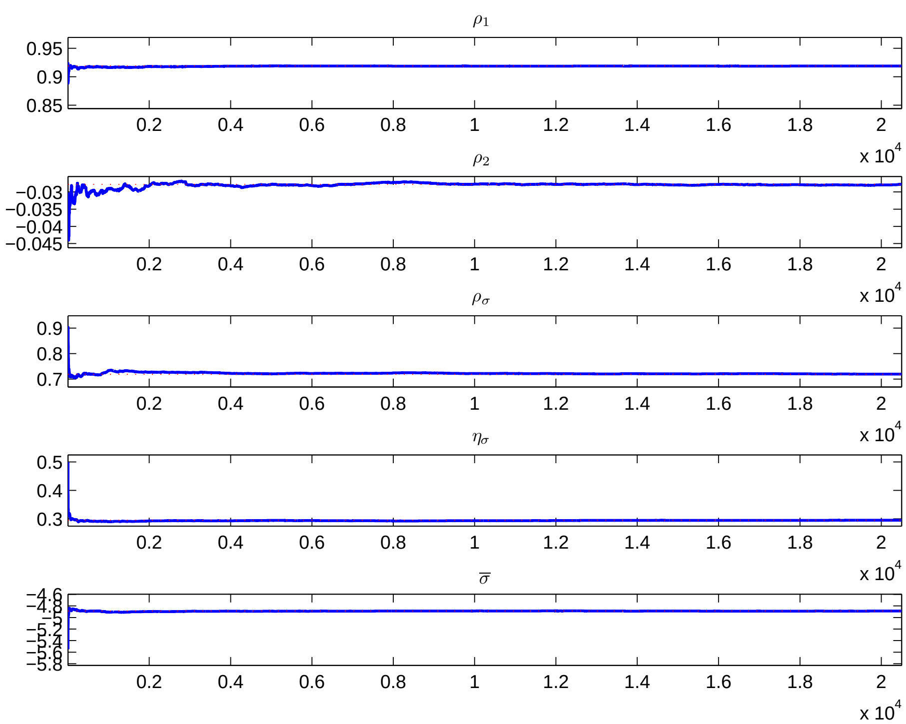


(b) Mean of the parameters over time.

Figure 11: Evolution of MCMC sampler over time for z^I.

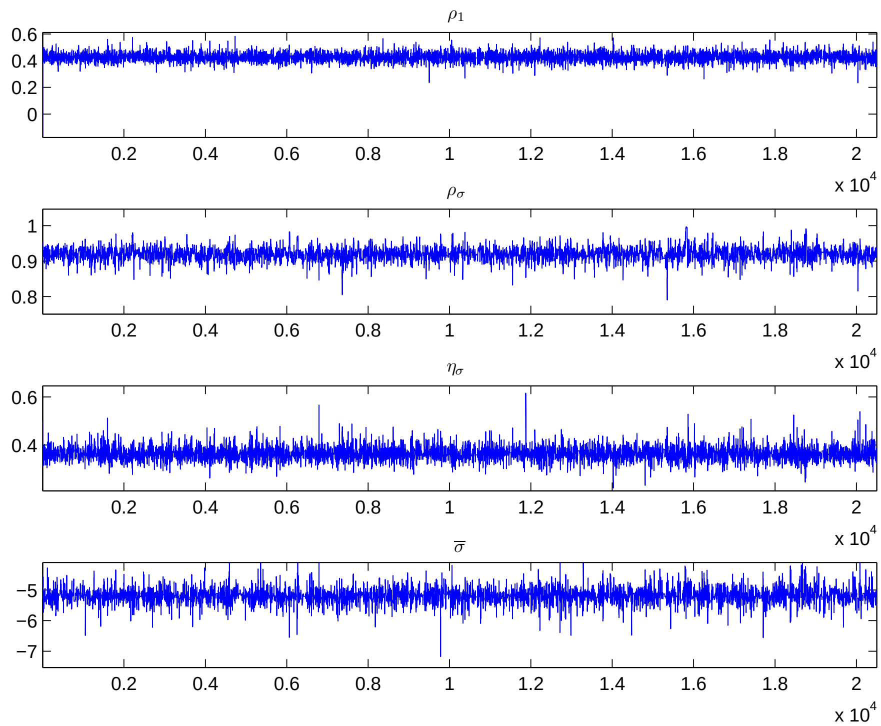


(a) MCMC draws.

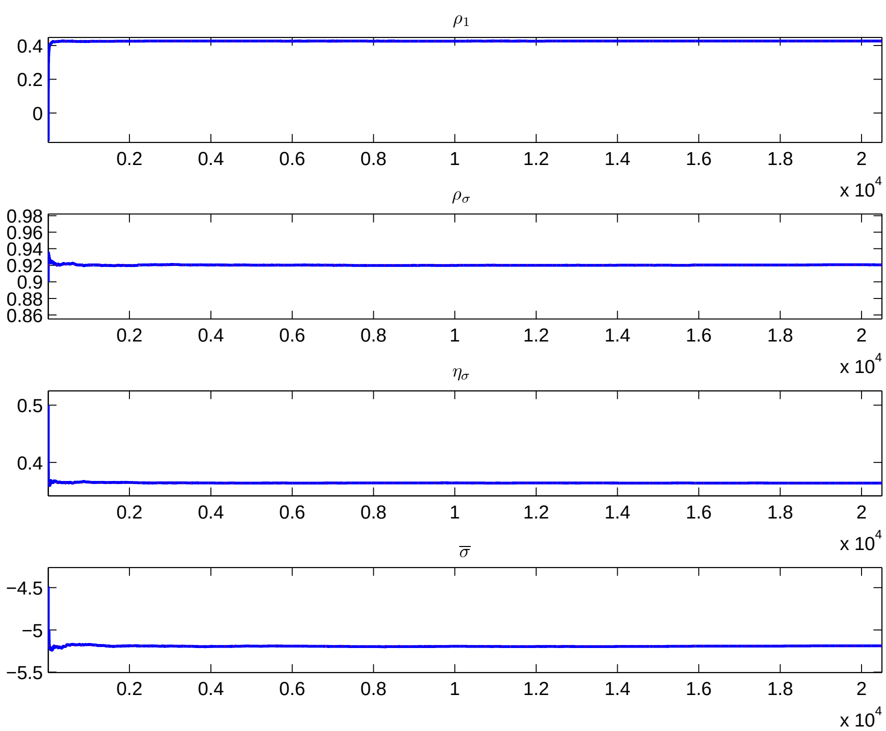


(b) Mean of the parameters over time.

Figure 12: Evolution of MCMC sampler over time for g.



(a) MCMC draws.



(b) Mean of the parameters over time.

Figure 13: Evolution of MCMC sampler over time for m.

C.3 Model Misspecification Diagnostics

Following Kim et al. (1998), we can test the specification of our SV-model. Using N draws from the prediction density $p\left(x_t|x^{t-1};\Theta\right)$, we can compute the probability that x^2_{t+1} will be less or equal than the actually observed value of $\left(x^{obs}_{t+1}\right)^2$:

$$\Pr\left(x^2_{t+1} \leqslant \left(x^{obs}_{t+1}\right)^2 \middle| x^t\ ;\Theta\right) \simeq u_{t+1} = \frac{1}{N}\Pr\left(x^2_{t+1} \leqslant \left(x^{obs}_{t+1}\right)^2 \middle| x^t, \sigma_{t+1|t}\ ;\Theta\right)\ , \tag{C.1}$$

$\forall t = 1, \ldots T-1$. If the SV-model is correctly specified, the sequence of u_t converges in distribution to $i.i.d.$ uniform variables as the number of particles N goes to infinity (Rosenblatt, 1952). Under the null hypothesis of a correctly specified model, the u_t can be transformed to $i.i.d.$ standard normal variables using the inverse normal CDF. Hence, we can perform a simple test for misspecification by testing the resulting series for their normality. Figure 14 shows the corresponding QQ-plots.

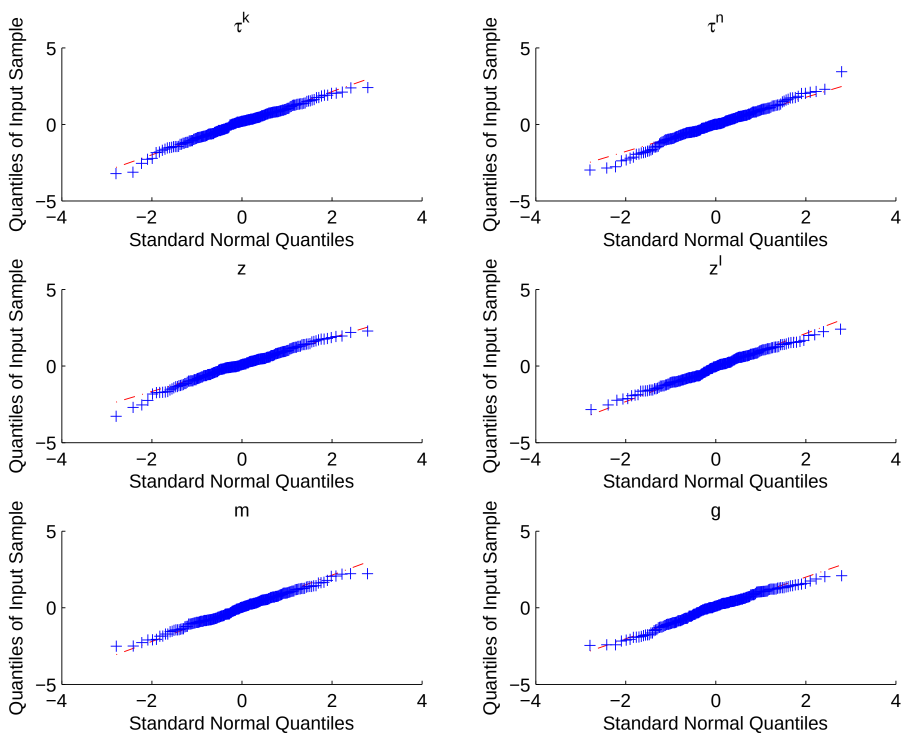


Figure 14: QQ-plots. From left to right and top to bottom: capital taxes, labor taxes, TFP, investment-specific technology, monetary policy shocks, and government spending.

Table 12 presents the results from three commonly used normality tests. In general, a correct specification of the model tends to not be rejected. Only for z, the Jarque-Bera and the Kolmogorov-Smirnov tests reject normality. However, this effect is driven by the outliers visible in the bottom left corner of Figure 14. In contrast, when shutting off the time-varying volatility and setting the volatility to its unconditional mean, the specification is generally rejected (results are not shown here).

Table 12: Tests for Model Misspecification

	JB	KS	SW
τ^k	0.066	0.039**	0.125
τ^n	0.141	0.960	0.135
z	0.037**	0.035**	0.085
z^I	0.377	0.076	0.586
g	0.500	0.747	0.528
m	0.052	0.377	0.012**

Note: Asterisks indicate significance at the 5% level. JB refers to the Jarque and Bera (1987)-test, KS refers to the Kolmogorov (1933)/Smirnov (1948)-test, and SW refers to the Shapiro and Wilk (1965)-test.