Optimal Public Bank Bailout and Three Funding Alternatives: Can Moral Hazard be Prevented?

Michael Diemer
University of Leipzig
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Abstract
The moral-hazard-effect of bank bailouts is well known in the literature and played an important role in the bailout policy during the recent financial crisis. However, bank bailouts can have a value effect, too, which induces them to behave prudently. We account for both effects and ask how the optimal bailout policy should be designed and how it should be financed. To address these issues, we use a dynamic setup, similar to Repullo (2004), where banks compete for secured deposits and invest their funds in either a prudent or a gambling asset. By contrast to Cordella & Yeyati (2003), we differentiate between two state of natures, instable and stable financial markets, and take the banks’ planning horizon into consideration. Our findings suggest that banks with short-term planning horizon should always be liquidated in order to avoid excessive risk-taking. By contrast, banks with long-term planning horizon should be bailed out with probability one or the regulator should construct ambiguity and rescue them with probability between zero and one. The optimal bailout probability depends on the stability of the financial markets. Furthermore, we suggest that a bank levy and a profit tax surcharge are adequate funding alternatives in order to prevent moral hazard. However, depositors pay at least a part of the bill which is not the case if the regulator (partially) nationalizes insolvent banks.

Keywords: Bank levy, bank bailout, constructive ambiguity, moral hazard, monopolistic competition, profit tax
JEL classification: D82, G21, G28, E58
1. Introduction

During the recent financial crisis moral hazard played an important role in the debate about the welfare effects of bank bailouts. However, bank bailouts can also have a prudential effect as they might increase the banks’ value (Cordella & Yeyati (2003)). This effect is often neglected when bailout strategies are designed. In this paper, we account for both effects and ask how bank bailouts should be designed in order to prevent banks from excessive risk-taking.\footnote{We do not ask whether bank bailouts are necessary from a welfare point of view. For a short summary of arguments favouring a safety net, see, for instance, Freixas & Rochet (2008) and Beck, Coyle, Dewatripont, Freixas & Seabright (2010).} According to the International Monetary Fund (IMF), moral hazard should also be taken into consideration when the regulators decide about adequate instruments to fund these bank bailouts. Therefore, our second aim is to analyse the impact of the following funding alternatives on banks’ risk behaviour: a bank levy on liabilities, a profit tax surcharge and nationalizations of insolvent banks. While the first two instruments were proposed by the IMF, the third one must not be neglected as nationalizations of banks can contribute on a large extent to the funding of public bank bailouts (International Monetary Fund (2010); Sandal (2004)).\footnote{Sweden and Germany, for instance, introduced a bank levy depending on total assets less equity and in some countries, as in Germany, less customer claims. The IMF provides detailed information about several instruments introduced in European countries and the USA (International Monetary Fund (2010)). For detailed information about the German bank levy, see "Restrukturierungsgesetz" (RStruktG).}

Whether these instruments generate a sufficient revenue for an adequate bailout fund is not our main concern. To address these issues, we use similar framework as Repullo (2004) and model competition à la Salop (1979), where higher competition is equivalent to a higher number of banks.\footnote{This enables us to account for the possibility that banks pass the charges to their customers.} Our model is dynamic throughout the paper, but we vary the number of periods. We first consider banks with a short-term planning horizon (two periods). Thereafter, we enlarge the banks’ planning horizon and the number of periods to infinity. The optimal bailout policy is determined in both scenarios and we always account for the fact that banks’ expected profit is influenced by a state of nature which we consider as the degree of financial market’s stability. We differentiate between two state of natures: rather stable and rather instable financial markets.

The regulator announces ex ante his bailout policy, which is supposed to be credible, and implements either a bank levy on deposits, a profit tax surcharge or threats with nationalizations if banks become insolvent.\footnote{As in the bailout case we assume that the regulator’s threat is credible. We argue that the banks and the regulator can, for instance, sign a contract which forces the regulator to fulfill his policy announced ex ante.} Moreover, he requires a minimum amount of equity. Banks raise funds from depositors and shareholders and invest them in either a prudent or a gambling asset, though both assets are risky. If a bank is insolvent, the regulator follows his bailout policy announced ex ante. A bailed out bank receives the permission to continue and the regulator repays the liability claims to depositors. A liquidated bank loses its permission to operate and is replaced by another bank. We restrict our attention on two types of stationary equilibria: a prudent and a gambling equilibrium.

These assumptions reveal the following findings. As in Repullo (2004), a sufficiently large number of banks leads to an equilibrium where all banks gamble whereas a prudent equilibrium exists if the number of banks is sufficiently low. Under symmetric information the regulator can differentiate between prudent and gambling banks and the optimal bailout policy suggests
to liquidate gambling and to rescue prudent banks with probability one independent of their planning horizon. This policy increases the range where banks choose the prudent asset.

While the regulator can discriminate between sound and unsound banks in case of symmetric information, asymmetric information forecloses this possibility. Under asymmetric information, the regulator should bail out insolvent banks with probability one in times of financial instability. However, even in rather stable periods he should not liquidate insolvent banks, but rescue them with probability one or construct ambiguity, thus rescue with probability between zero and one. The reason is that a higher bailout probability has two effects: it increases the bank value which induces banks to behave prudently (value effect), but it also decreases the impact of risk-decisions on the probability of surviving, which favours the gambling asset (moral-hazard-effect). In times of rather instable financial markets, the value effect outweighs the moral-hazard-effect and the regulator should rescue with probability one. In times of rather stable financial markets, the moral-hazard-effect outweighs the value effect if the regulator rescues with probability one, but the value effect dominates the moral-hazard-effect if the regulator liquidates with probability one. Consequently, a bailout probability between zero and one is the best policy. Cordella & Yeyati (2003) find a similar solution, although they assume that the regulator can respond to a continuum of state of natures. We rather suppose that he can only differentiate between crisis and non-crisis periods and take the banks’ planning horizon into consideration.

In order to finance bank bailouts, both a levy on deposits and a profit tax surcharge might be adequate instruments to prevent excessive risk behaviour. From a moral hazard point of view, a bank levy, which depends on the level of liabilities, is superior to a profit tax because the costs of the levy are completely passed to depositors which reduces their return and thus lowers their funding costs. The profit tax surcharge, by contrast, is partly, paid by shareholders. As a third instrument, we analyse the impact of (partial) bank nationalizations. We find that nationalizations of insolvent banks can prevent moral hazard without charging depositors. However, while an increase of the levy rate or the tax rate increases the fund level and the range where banks behave prudently, the regulator faces a trade-off between increasing funds and reducing moral hazard if he nationalizes insolvent banks.

The paper proceeds as follows. Section 2 provides an overview of the related literature. Section 3 introduces the model and section 4 derives the optimal bailout policy and analyses its effectiveness to decrease moral hazard. Section 5 determines the impact of a bank levy and a profit tax surcharge on banks’ risk behaviour and compares these instruments with nationalizations of insolvent banks. Section 6 concludes.

2. Related literature

We find three strands in the literature that are related to our model. One strand determines the optimal design of bailout policies. Freixas (1999) finds that bank bailouts are determined by rescue and liquidation costs. If rescue costs are sufficiently higher than liquidation costs, the regulator should implement "ambiguity", hence he should rescue a bank with probability greater than zero and smaller than one because otherwise the bank increases its size and bank liquidation becomes too costly. If rescue costs are sufficiently lower than liquidation costs, a

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5 The risk effect of a profit tax and a profit tax surcharge is the same. We sometimes write profit tax and sometimes profit tax surcharge.
bank should be bailed out with probability one. Our findings support the view that ambiguity can be constructive, though we do not take the costs of a bank bailout into account.

Cordella & Yeyati (2003) analyse the optimal bailout strategy for an insolvent institution in a dynamic framework where the regulator only cares about the impact of his policy on a bank’s credit risk. Following the optimal bailout strategy, the regulator rescues the bank in times of financial market distortions (recessions) and liquidates it otherwise. A bank bailout does not necessarily increase risk incentives because it implicates both a value effect and a moral-hazard-effect. On the one hand, a higher bailout probability raises the bank value and thus lowers gambling incentives (value effect). On the other hand, a higher bailout probability reduces the impact of the investment decision on the surviving probability which increases gambling incentives (moral-hazard-effect). If the value effect outweighs the moral-hazard-effect - which is the case in times of recessions - a bank bailout with probability one is the regulator’s optimal choice. As in Cordella & Yeyati (2003), we use a complete contract setting and a dynamic framework where the optimal bailout policy can depend on a state of nature. In addition, we use a monopolistic competition framework which allows us to measure the effectiveness of this bailout policy and we differentiate between a long-term and a short-term planning horizon of the banks.

Perotti & Suarez (2002) argue that a regulator should allow for acquisitions of failed banks by solvent institutions because it raises the charter value of acquiring banks and induces them ex ante to behave more prudent. Acharya & Yorulmazer (2007) find that the optimal bailout policy depends on the number of insolvent banks. If this number is sufficiently small, the private solution is feasible. However, a sufficiently high number of insolvent banks forecloses this solution since there are not enough solvent banks being able to afford an acquisition. As a consequence, failed banks have to be sold to investors outside the banking sector which leads to welfare losses since these investors do not have the same skills. In this case, a combination of private and public solution improves welfare and the regulator should assist solvent banks so that acquisitions become affordable (Acharya & Yorulmazer (2008)).

A second strand studies the relation between bank competition and banks’ risk behaviour. Boyd & De Nicoló (2005) analyse the effect of both loan and deposit market competition on banks’ risk-taking and find that higher competition can prevent moral hazard. On the one hand, higher competition on the deposit market increases deposit rates and decreases ceteris paribus expected profits which raises banks’ incentives to invest in riskier projects. On the other hand, an increasing number of banks decreases corporates’ loan rates which increases their incentives to stay solvent (risk-shift-effect). A lower loan rate, however, decreases the banks’ margin and therefore increases the incentives to gamble (margin effect). According to Martinez-Miera & Repullo (2010), in monopolistic markets it is the risk-shifting-effect where as in competitive markets it is the margin effect that dominates. De Nicoló & Lucchetta (2011) provide evidence that imperfect competition might be optimal from a welfare point of view if information technology features constant returns to scale while in case of increasing returns to scale perfect competition is desirable.

The ambivalent relation between competition and risk behaviour holds in empirical studies, too. Beck, Demirgüç-Kunt & Levine (2006) find that more concentrated banking systems suffer
less systemic crisis.\footnote{Concentration is measured by the share of assets of the three largest banks on total assets in banking system.} Jiménez, Lopez & Saurina (2007) show that higher market power leads to lower bank risk using data for Spanish banks. Berger, Klapper & Turk-Ariss (2009) identify a negative relationship between market power and risk exposure, though higher market power leads to higher loan risks.\footnote{Suppose, for instance, that a bank has several branches which are located next to each other on a circle line. Then only two branches compete with other banks. This relation, however, changes if banks occupy a sequence of adjacent locations.} The reason is that higher loan risk is compensated by higher capital asset ratio which decreases the overall level of risk.

As in this model, some papers also use the Salop model and analyse the relation between competition and financial stability. Allen & Gale (2004) show that the trade-off between competition and stability depends on the locations a bank is allowed to occupy. If two banks occupy alternate locations, competition does not have a negative effect on stability.\footnote{See also Bolt & Tieman (2004).} According to Besanko & Thakor (1992), a relaxation of bank markets entry barriers improves the welfare of borrowers and depositors and decreases the welfare of shareholders. Matutes & Vives (1996) assert that the fragility of financial institutions is due to depositors' differentiated expectations. If depositors perceive a bank to be safe, the bank can receive a higher margin and tends to decrease risk level. Consequently, the probability of failure results from the self-fulfilling expectation of depositors. Our paper is close to the paper of Hellmann, Murdock & Stiglitz (2000) and Repullo (2004) who show that competition on the deposit market leads to higher risk-taking. They suggest higher capital reserves in order to induce prudent behaviour.\footnote{See also Bolt & Tieman (2004).} However, Hellmann et al. (2000) assert that capital controls are not sufficient to achieve a pareto efficient outcome and deposit rate controls are necessary. Gale (2010) argues that capital controls can increase investment risk if banks try to maximize rates of return. The relation between competition and financial stability is not our main concern, though we also use the Salop model and account for the possibility that banks pass a part of their funding charges to their customers. In addition to Hellmann et al. (2000) and Repullo (2004), we show that bank bailouts can have a stabilizing effect, too.

A third strand analyses how bank bailouts should be financed. Shin (2010) suggests a tax on non-core liabilities. In a boom period when loan demand is growing rapidly, retail deposits (core liabilities) may be insufficient to fund these loans which makes alternative fundings, such as non-core liabilities, necessary. However, the required returns on these types of liabilities are higher and induce banks to engage in riskier projects. Weder di Mauro (2010) recommends a systemic risk charge. Accordingly, only systemic relevant institutes should be charged by a levy on all liabilities excluding insured deposits. Due to the fact that only a part of the institutions are charged, taxed institutions cannot pass the burden to creditors. However, we show that the possibility to pass the charge to creditors is a driving force of moral hazard prevention. By contrast, if this way is prohibited, for example by charging only some institutions, the instruments might increase the probability of a failure. Perotti & Suarez (2009) suggest liquidity risk charges that should depend on the maturity of fundings because short-term uninsured liabilities induce fire sales in crisis and thus enforce financial distress.
3. The model

3.1. Agents and strategy sets

We consider three groups of risk-neutral players: secured depositors, shareholders who run \( n \geq 2 \) banks and a regulator who decides about the policy instruments (bailout probability and funding instruments).\(^{11}\) The banks are symmetrically located on a circle line of length one. Each bank \( j = 1, \ldots, n \) chooses the amount of secured deposits \( D_j \) and decides about the project type - gambling or prudent - at the beginning of each period \( t = 1, 2.12\) Each generation of depositors and shareholders lives for one period, does not have alternative investment possibilities and desires to consume at the end of its life, thus at the end of the investment period. This means that at the end of each investment period the payoffs are distributed to the depositors and the shareholders and a new investment begins which starts again with the mobilisation of a new generation of depositors and shareholders.

3.1.1. Depositors

The depositors are continuously distributed on the circle line and depositing funds in a bank induces transaction costs \( \pi > 0 \), where the transaction costs can, for instance, be interpreted as the heterogeneity of banks.\(^{13}\) The aggregate demand of deposits, \( D = \sum_{j=1}^{n} D_j \), is constant and normalized to one. The amount of deposits offered by bank \( j \) is determined by \( j \)'s and its neighbours’ (to the left and to the right) interest rates on deposits and by the transaction costs \( \pi \). We denote \( r_j \in (0, 1) \) as the deposit rate of bank \( j \) and \( r_{j+1} \) and \( r_{j-1} \) as the deposit rates of bank \( j \)'s neighbours \( j+1 \) and \( j-1 \). A depositor located between bank \( j \) and bank \( j+1 \) is indifferent between these banks if the net returns are the same, thus if

\[
\pi z = r_{j+1} - \pi \left[ \frac{1}{n} - z \right]
\]

holds, where \( z \) is the distance between the indifferent depositor and bank \( j \) and \( \frac{1}{n} - z \) the distance between the indifferent depositor and bank \( j+1.14\) Solving for \( z \) reveals the "amount" of depositors on the line between bank \( j \) and \( j+1 \) who decide to deposit their funds in bank \( j \):

\[
z (r_j, r_{j+1}) = \frac{1}{2n} + \frac{r_j - r_{j+1}}{2\pi}.
\]

Due to the symmetrical order of the banks, the distance between bank \( j \) and \( j+1 \) is equal to the distance between bank \( j \) and \( j-1 \) and therefore the total demand of depositors, that lend their funds to bank \( j \), amounts to:

\(^{11}\)As we do not model principal agent problems between shareholders and bank managers we sometimes write shareholders and in some cases we just write banks.

\(^{12}\)In sections 4.1.2 and 4.2.2 we assume that each bank exists until \( t = \infty \) unless it is liquidated.

\(^{13}\)See for instance Repullo (2004). Consequently, the higher the transaction costs the more banks are heterogeneous.

\(^{14}\)Notice that the comparison between transaction costs and returns is possible because of the normalization of payoffs and returns.
\[ D_j(r_j, r_{j-1}, r_{j+1}) = \frac{1}{n} + \frac{2r_j - r_{j+1} - r_{j-1}}{2\pi}. \]  

Henceforth, we write \( r_{j-1} \) instead of \( r_{j+1} \) and \( r_{j-1} \). The demand of deposits, \( D_j \), decreases in \( n, r_{j+1} \) and \( r_{j-1} \) and increases in \( r_j \). The higher \( \pi \) the smaller the influence of deposit rates on demand.\(^{15}\) Intuitively, the higher the distance between the banks, the less deposit rates matter as the transaction costs effect the depositors’ returns, too.

Besides, we normalize the insurance premium to zero.\(^{16}\)

### 3.1.2. Banks

Apart from offering deposit rates, banks choose the asset type. The prudent investment, indexed with \( P \), yields a relative low return, \( \gamma^L \in (0, 1) \), with a relative high probability, \( p^H \), and a return of zero otherwise. The gambling investment, indexed with \( G \), generates a relative high return, \( \gamma^H \in (0, 1) \), with a relative low probability, \( p^L \), and a zero return otherwise.\(^{17}\) The probability of project success depends on the project type, prudent or gambling, and on an exogenous state of nature which can be either high, indexed by \( H \), or low, indexed by \( L \). These state of natures represent the stability of the financial sector, where \( H \) stands for a relatively high and \( L \) for a relatively low stability.

We denote \( q \) as the probability that state \( H \) appears and \( 1 - q \) as the probability that nature reveals \( L \). In the stable case \( H \), banks are hit by a shock with probability \( 1 - v^H \), thus with probability \( v^H \) the shock does not occur. If the shock occurs, the project return is zero independent of whether a bank chose the prudent or the gambling investment. Hence, the return can be zero due to the solvency shock and/or due to project failure (probability \( 1 - p \)). Equivalently, in the relatively instable case \( L \), a shock occurs with probability \( 1 - v^L \), where \( v^H > v^L \).

Following these assumptions we can summarize the return functions \( R^G \) and \( R^P \):

\[
R^G = \begin{cases} 
\gamma^H & \text{with probability } qv^H p^L + [1 - q] v^L p^L \cr 
0 & \text{with probability } 1 - qv^H p^L - [1 - q] v^L p^L \cr
\end{cases} 
\]

\[
R^P = \begin{cases} 
\gamma^L & \text{with probability } qv^H p^L + [1 - q] v^L p^H \cr 
0 & \text{with probability } 1 - qv^H p^H - [1 - q] v^L p^H \cr
\end{cases} 
\]

In order to include moral hazard in our model we assume that the return of the gambling asset is higher than the return of the prudent asset, \( \gamma^H > \gamma^L \), while the expected return of the prudent asset is higher than the expected return of the gambling investment, \( p^H \gamma^L > p^L \gamma^H \). Although \( p^H \gamma^L > p^L \gamma^H \), banks are sometimes willing to gamble which is due to the limited

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\(^{15}\)Notice that the demand of deposits does not depend on the banks’ investment risk because deposits are covered by a deposit insurance.

\(^{16}\)The level of the insurance premium is unimportant as far as the deposit insurance is unfair which means that the insurance premium is not equal to the expected payment of the insurance to the depositors. The introduction of a fairly priced insurance premium is impossible if several banks are assured and the characteristics of the banks are private information. See Chan, Greenbaum & Thakor (1992). For the impact of deposit insurance on risk-taking, see, for instance, Matutes & Vives (2000) and Cordella & Yeyati (2002).

\(^{17}\)The index \( L \) stands for "low" and the index \( H \) stands for "high".
liability assumed throughout the paper. It enables the shareholders to pass a part of the risk to depositors. The residual of the project payoff is distributed among the banks’ shareholders. We further assume that equity capital has a perfectly elastic supply at an expected rate of return \( \delta > 0 \).

3.1.3. Regulator

As in Repullo (2004), the regulator requires a minimum amount of equity \( kD_j \), where \( k > 0 \) is constant and exogenously determined. Although we do not model the banks’ equity choice, it is easy to show that in our framework, banks choose the minimum amount of equity if equity is sufficiently costly. Otherwise the banks want to hold as much equity as possible.\(^{18}\)

Each bank receives a license from the regulator to operate in an initial period. If a bank is insolvent at the end of period \( t \), the regulator decides whether to bail it out with probability \( \beta(v) \) or to liquidate it with probability \( 1 - \beta(v) \).\(^{19}\) The regulator only takes the impact of his decision on the banks’ risk-taking into account. If a bank is bailed out, it can continue to operate because the liability claims are paid by the regulator.\(^{20}\) Otherwise it is liquidated and replaced by another bank so that the number of banks remains always constant.

Following these assumptions, the probability of surviving in the prudent case is defined by:

\[
s^P = q \left[ p^H v^H + \left[ 1 - p^H v^H \right] \beta^H \right] + \left[ 1 - q \right] \left[ p^H v^L + \left[ 1 - p^H v^L \right] \beta^L \right]
\]

and in the gambling case by

\[
s^G = q \left[ p^L v^H + \left[ 1 - p^L v^H \right] \beta^H \right] + \left[ 1 - q \right] \left[ p^L v^L + \left[ 1 - p^L v^L \right] \beta^L \right].
\]

Notice that the regulator can differentiate between \( H \) and \( L \), but he does not know whether the bank failed due to the shock or due to the project failure. In the symmetric information case, the regulator can also differentiate between the project type, whereas in the asymmetric information case, the regulator has to apply the same bailout probability for prudent and gambling banks.

3.2. Sequence of events

The time structure of the model is illustrated in figure 3.1. At the beginning of an initial period, \( t = 0 \), the regulator announces and can credibly commit to a bailout policy that is contingent on the state of nature \( v \). Thereafter, each bank raises funds - equity capital and deposits - at the beginning of the first investment period and invests them in either a prudent or a gambling asset. If the project is successful, shareholders and depositors share the payoff and the bank can continue to operate in the second period where it collects again funds from shareholders and depositors. If the project fails, depositors are paid by the regulator who enforces the bailout

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\(^{18}\)See Repullo (2004).

\(^{19}\)A bank is insolvent if the project fails since the return of the project is zero and the bank has still debt in its accounts.

\(^{20}\)There are enough shareholders in order to ensure that each bank is able to continue after being rescued by the regulator. Notice that the regulator could also bail out banks by injecting capital, but in this case the regulator becomes a shareholder which is equivalent to a (partial) nationalization.
strategy announced at the beginning of the initial period. After the second period, the game is over and the terminal value amounts to zero.

We focus on two types of symmetric and stationary equilibria: a prudent equilibrium where all banks invest in prudent assets and a gambling equilibrium where all banks choose the relative risky asset.

4. The optimal design of bank bailouts

4.1. Symmetric information

4.1.1. Short-term planning horizon

The regulator is perfectly informed about the banks’ investment risk and can apply different bailout probabilities for prudent and gambling banks. We denote the bailout probability for banks investing in prudent assets with \( \beta^H_P \) and \( \beta^P_P \) and the rescue probability for gambling banks with \( \beta^H_G \) and \( \beta^P_G \). At the beginning of each period \( t \in [1, 2] \), bank \( j \) chooses the deposit rate \( r_j \) and the investment type \( \gamma_j \) in order to maximize its value \( V_j \), thus the net present value of expected period profits.

The game is solved by backward induction starting with the investment decision of each bank in period 2. As the terminal value of bank \( j \) at the end of the second investment period is supposed to be zero, we can describe bank \( j \)’s maximization problem in the second period as:

\[
V_j = \max_{r_j, \gamma_j} \left[ -kD(r_j, r_{-j}) + \delta\Pi(r_j, r_{-j}) \right].
\]

Notice that the number of banks is exogenous. Hence if the regulator liquidates a bank, the license is passed to another one.
where \( \Pi (r_j, r_{-j}) \) is the expected period profit defined by:

\[
\Pi (r_j, r_{-j}) = p (\gamma_j) \left[ qv_H + [1 - q] v_L \right] \left[ \gamma_j - r_j + k_j (1 + \gamma_j) \right] D (r_j, r_{-j}). \tag{4.1}
\]

\( \gamma_j - r_j \) is the net payoff on deposits and \( k (1 + \gamma_j) \) the payoff on equity.

We receive \( \Pi^P (r_j, r_{-j}) \) through replacing \( p (\gamma_j) \) by \( p^H \) and \( \gamma_j \) by \( \gamma^L \). Equivalently, we receive \( \Pi^G (r_j, r_{-j}) \) through replacing \( p (\gamma_j) \) by \( p^L \) and \( \gamma_j \) by \( \gamma^H \).

In each period the game consists of two stages: First the banks decide about the deposit rate and then about the project type. We start with the latter one, the investment decision. Banks invest in the prudent asset in both periods if a one-sided deviation from the prudent investment path (prudent investment in both periods) is not beneficial. This is true if the net present value obtaining from investing in the gambling asset in the first period and in the prudent asset in the second period is smaller than the value receiving from choosing the prudent asset in both periods \( t = 1, 2 \):

\[
\max_{r_j} \left[ -kD_j (r_j, r_{-j}) + \delta \Pi^P_j (r_j, r_{-j}) + \delta s^G V_j^P \right] \leq \left[ 1 + \delta s^P \right] V_j^P, \tag{4.2}
\]

with

\[
\bar{s}^P = q \left[ p^H v_H + [1 - p^H v_H] \beta^P \right] + [1 - q] \left[ p^H v_L + [1 - p^H v_L] \beta^P \right], \tag{4.3}
\]

and

\[
\bar{s}^G = q \left[ p^L v_H + [1 - p^L v_H] \beta^G \right] + [1 - q] \left[ p^L v_L + [1 - p^L v_L] \beta^G \right].
\]

Bank \( j \) faces the following trade-off: The gambling asset yields a higher return in case of success, but the probability of success as well as the probability of surviving is smaller than in case of the prudent asset. Therefore, by choosing the gambling asset in period one, bank \( j \) does not only risk the failure of the project in this period, but also the possibility to generate a payoff in the second period.

Similarly, we have a gambling equilibrium if banks do not have an incentive to behave prudently (at least in one period) though all other banks gamble, that is if the following inequation holds:

\[
\max_{r_j} \left[ -kD_j (r_j, r_{-j}) + \delta \Pi^P_j (r_j, r_{-j}) + \delta s^G V_j^G \right] \leq \left[ 1 + \delta s^G \right] V_j^G. \tag{4.3}
\]

To solve inequations (4.2) and (4.3), we have to determine the bank values \( V_j^P \) and \( V_j^G \).

Suppose that condition (4.2) holds and bank \( j \) prefers to invest in the prudent asset. Then bank \( j \)'s maximization problem in the second period equals:

\[
V_j^P = \max_{r_j} \left[ -kD_j (r_j, r_{-j}) + \delta \Pi^P_j (r_j, r_{-j}) \right], \tag{4.4}
\]

and the optimal deposit rate amounts to:

\[
\gamma^L = \frac{\pi}{n} + k \left[ 1 + \gamma^L - \frac{1}{\delta p^H [qv_H + [1 - q] v_L]} \right] =: r_j^P. \tag{4.5}
\]

\( \text{Notice that } \gamma_j - r_j + k_j (1 + \gamma_j) = (1 + \gamma^L) (1 + k) - (1 + r_j). \)
Obviously, $r^P_j$ is increasing in $\gamma^L$, $p^H$, and $n$ and decreasing in $\pi$, $\delta$ and $k$. Not surprisingly, depositors receive the return on equity and liability less the shareholders’ opportunity costs $\frac{1}{p^H (q v^H + [1 - q] v^L)}$. Consequently, shareholders only benefit beyond their opportunity costs if competition is imperfect.

Inserting $r^P_j$ in (4.4) and solving for $V^p_j$ leads to the bank value

$$V^p_j = \delta p^H [q v^H + [1 - q] v^L] \frac{\pi}{n^2}. \quad (4.6)$$

Equivalently, the optimal deposit rate in case of the gambling investment amounts to:

$$\gamma^H - \frac{\pi}{n} + k \left[ 1 + \gamma^H - \frac{1}{\delta p^L (q v^H + [1 - q] v^L)} \right] = r^G_j. \quad (4.7)$$

which leads to the bank value

$$V^G_j = \delta p^L [q v^H + [1 - q] v^L] \frac{\pi}{n^2}. \quad (4.8)$$

$V^p_j$ and $V^G_j$ are decreasing in $\pi$ and decreasing in the number of banks $n$, thus the market power for each bank. This is due to the fact that higher competition leads to higher returns on deposits (4.5 and 4.7) and to lower expected profits. However, the bank value does not depend on $k$ because a higher capital requirement is completely compensated by a lower deposit rate. Perfect competition, $n \to \infty$, increases the deposit rates in such that the bank value converges to zero.

**Proposition 1.** The equilibrium conditions are the following:

- **Banks invest in prudent assets if**

  $$\frac{\pi}{n} > \frac{r^G_j - r^P_j}{2 \left[ 1 - \delta s^G - \delta s^P \right]} =: m^G_j. \quad (4.1)$$

- **Banks invest either in a prudent or a gambling asset if**

  $$m^P := \frac{r^G_j - r^P_j}{2 \left[ \sqrt{1 + \delta s^G + \delta s^P} \frac{p^H}{p^L} - 1 \right]} \leq \frac{\pi}{n} \leq m^G_j. \quad (4.2)$$

- **Banks invest in gambling assets if**

  $$\frac{\pi}{n} < m^P_j. \quad (4.3)$$

---

24 Apart from a decrease of deposit rates, higher competition, thus a higher number of banks, might reduce loan rates, too (Hauswald & Marquez (2005)). From a welfare point of view, an increase in deposit rates might be desirable as higher deposit rates lead to more coordination failures and bank runs (Goldstein & Pauzner (2005)).

25 This results from the fact that banks compete in price, thus we have imperfect Bertrand competition where the "firms" do not receive a payoff beyond their costs. Banks only receive the costs of capital and a surplus $\frac{\pi}{n}$ which results from the monopolistic environment.

26 Notice that, due to the symmetry assumption, this holds for every bank $j$, thus we can omit the bank index.
The optimal bailout policy is given by

\[ \beta_s^* = \begin{cases} 
0 & : \gamma = \gamma^H \\
1 & : \gamma = \gamma^L 
\end{cases} \]

Proof. See appendix. ■

A sufficiently large number of banks, thus a sufficiently low market power, mitigates excessive risk-taking. In line with the recent literature, a more competitive deposit market increases deposit rates and thus decreases banks’ profits which generates higher risk level (Hellmann et al. (2000), Repullo (2004) and Boyd & De Nicoló (2005)). Competition matters more the higher the difference of returns, \( \gamma^H - \gamma^L \), since the benefit from moral hazard increases.

This becomes more perspicuous by recalling the effects resulting from a one-sided deviation. If bank \( j \) deviated from the prudent path, the margin in the deviating period would reach \( (k = 0) \):

\[ \gamma^H - \frac{r_G + r_P}{2} = \frac{\gamma^H - \gamma^L}{2} + \frac{\pi}{n} \]

instead of \( \frac{\pi}{n} \). For sufficiently low values of \( n \) the margin \( \frac{\pi}{n} \) is too high in order to receive additionally \( \frac{\gamma^H - \gamma^L}{2} \) at the expense of a higher default risk. Then it is not profitable to deviate from the situation where all banks choose the low-risk asset. In this case a bailout policy, such as \( \beta_s^* \), which increases the expected franchise value, is not necessary since banks already behave prudently.

If all banks invest in gambling assets, it is not worth to deviate and choose prudent asset in a sufficiently competitive regime. By deviating from the gambling investment, bank \( j \) offers a lower deposit rate and receives the following margin \( (k = 0) \):

\[ \gamma^L - \frac{r_G + r_P}{2} = \frac{\gamma^L - \gamma^H}{2} + \frac{\pi}{n} \]

instead of \( \frac{\pi}{n} \). One can immediately see that this margin can be even negative if the number of banks is high enough since \( \gamma^L < \gamma^H \). Indeed if the intermediation margin is negative, it cannot be profitable for the banks to behave prudently, independent of the announced bailout policy.

Unless the deposit market is sufficiently competitive or incompitive, the regulator can reach a risk shift through punishing banks invested in gambling assets, thus set \( \beta^G = 0 \), and subsidizing prudent banks, thus set \( \beta^P = 1 \). The change in the investment risk results from the fact that the regulator aggravates the opportunity costs of higher project risk; an increase in the bailout probability raises the expected value of the banks. However, as the regulator can discriminate between prudent and gambling banks, this policy increases the bank value without inducing moral hazard. As the regulator can directly punish the gambling investment, he does not have to differentiate between \( H \) and \( L \) where he accounts for less information.

4.1.2. Long-term planning horizon

We modify the time horizon in such that the banks operate in \( t = 1, 2, ..., \infty \) except that they fail and the regulator liquidates them. By contrast to section 4.1.1, we do not have a last period and thus we cannot apply backward induction. However, we assume that the maximization problem remains the same in each investment period and we only consider stationary equilibria.
Then we know that bank \( j \) chooses the same project risk and deposit rate in each period \( t \) which means that we can restrict our attention on the equilibrium in one representative period \( t \).

Suppose that bank \( j \) chooses the prudent investment. Bank \( j \)'s maximization problem is:

\[
\tilde{V}_j^P = \max_{r_j} \left[ -kD(r_j, r_{-j}) + \delta \Pi_j^P (r_j, r_{-j}) + \delta \tilde{s}^P \tilde{V}_j^P \right].
\] (4.9)

Bank \( j \)'s deposit rate is the same as in the short-term horizon case. Inserting (4.5) in (4.9) and solving for \( \tilde{V}_j^P \) reveals:

\[
\tilde{V}_j^P = \frac{\delta pH [quH + (1 - q) vL] \pi}{1 - \delta \tilde{s}^P}.
\] (4.10)

Now the bailout policy does not only have a moral-hazard-effect, but also a value effect. An increase of \( \beta^P \) or/and \( \beta^G \) increases \( \tilde{V}_j^P \) which in turn increases the opportunity costs of an investment with higher probability of failure.

Equivalently, if all banks invest in the gambling asset, their maximization problem can be described as:

\[
\tilde{V}_j^G = \max_{r_j} \left[ -kD(r_j, r_{-j}) + \delta \Pi_j^G (r_j, r_{-j}) + \delta \tilde{s}^G \tilde{V}_j^G \right]
\] (4.11)

and the bank value \( \tilde{V}_j^G \) amounts fo:

\[
\tilde{V}_j^G = \frac{\delta pL [quH + (1 - q) vL] \pi}{1 - \delta \tilde{s}^G}.
\] (4.12)

**Proposition 2.** The equilibrium conditions are the following:

- Banks invest in prudent assets if

\[
\frac{\pi}{n} > \frac{r^G - r^P}{2 \left[ 1 - \sqrt{\frac{1 - \delta \tilde{s}^P}{1 - \delta \tilde{s}^G}} \frac{p^P}{p^L} \right]} =: \tilde{m}^G.
\]

- Banks invest either in the prudent or the gambling asset if

\[
\tilde{m}^P := \frac{r^G - r^P}{2 \left[ \sqrt{\frac{p^P}{p^L} \frac{1 - \delta \tilde{s}^G}{1 - \delta \tilde{s}^P}} - 1 \right]} \leq \frac{\pi}{n} \leq \tilde{m}^G.
\]

- Banks invest in the gambling asset if

\[
\frac{\pi}{n} < \tilde{m}^P.
\]

**Proof.** For the derivation of the equilibrium conditions, consult the appendix!  

The planning horizon of the banks does not change the optimal bailout policy in the symmetric information case. Due to the fact that the regulator has perfect information about the investment risk he can punish banks investing in gambling assets in each period \( t \). Intuitively,
if the regulator differentiates between prudent and gambling banks he accounts for more information than in case of differentiating between stable and instable periods. Therefore, the latter differentiation cannot improve welfare.

Figure (4.1) illustrates the effect of the bailout strategy $\beta^*_S$. If banks are never rescued they behave prudently in case of $\bar{\pi}_n \geq m^r_P(k)$ and gamble in case of $\bar{\pi}_n \leq m^r_P(k)$. The bailout strategy $\beta^*_S$ leads to a decrease of the straight lines and therefore lowers the gambling incentives. However, the risk-reducing-effect is limited to an intermediate number of banks which means that the bailout policy is only effective in reducing risk if the number of banks is neither too large nor too low.

![Figure 4.1: First-best bailout policy versus “never rescue”](image)

4.2. Asymmetric information

4.2.1. Short-term planning horizon

In this section, we assume that the information of the project risk is asymmetric in such that only banks have information about their project risk. The regulator cannot react to the banks’ risk behaviour and has to apply the same bailout policy for each type of project, thus the bailout probability is denoted with $\beta$ instead of $\beta^G$ and $\beta^P$. However, the regulator can still differentiate between $H$ and $L$. While the optimal interest rate remains the same as in the symmetric case, the investment decision changes.

**Proposition 3.** We have to differentiate between three cases:

$^{28}$The index "r" stands for "never rescued".

14
Banks invest in the prudent asset if
\[ \frac{\pi}{n} > \frac{r^G - r^P}{2 \left[ 1 - \sqrt{1 + \delta s^G - \delta s^P} \frac{P^L}{P^H} \right]} =: \hat{m}^G. \]

Banks invest in either the prudent or the gambling asset if
\[ \hat{m}^P := \frac{r^G - r^P}{2 \left[ \sqrt{1 - \delta s^G + \delta s^P} \frac{P^H}{P^L} - 1 \right]} \leq \frac{\pi}{n} \leq \hat{m}^G. \]

Banks gamble if
\[ \frac{\pi}{n} < \hat{m}^P. \]

The optimal bailout policy is \( \beta^H = \beta^L = 0 \)

**Proof.** We can take the results from the symmetric information case (short-term planning horizon) and replace \( s^G \) and \( s^P \) by \( s^G \) defined in (3.3) and \( s^P \) defined in (3.2) in order to receive \( \hat{m}^G \) and \( \hat{m}^P \). Furthermore, we immediately see that an increase of \( \beta^H \) or \( \beta^L \) decreases \( \delta s^P - \delta s^G \) which in turn increases \( \hat{m}^G \) and \( \hat{m}^P \). As a result, the regulator does not have an adequate bailout policy at hand in order to induce banks to behave more prudent. By contrast, in times of financial stability and instability, an increase in the bailout probability increases the banks’ project risk, thus the range where the gambling equilibrium exists. The reason is that in a short-term planning horizon the bailout policy increases moral hazard without increasing the bank value. As bailouts only induce gambling behaviour, the regulator should liquidate insolvent banks with probability one.

### 4.2.2. Long-term planning horizon

Each bank has a long-term planning horizon. Equivalently to the symmetric information case (long-term planning horizon), we do not have a last period and thus we cannot apply backward induction. Again, we assume that the maximization problem remains the same in each investment period and we only consider the equilibrium outcome which is an equilibrium in every stage of the game.

**Proposition 4.** The equilibrium conditions are the following:

- **Banks invest in prudent assets if**
  \[ \frac{\pi}{n} > \frac{r^G - r^P}{2 \left[ 1 - \sqrt{1 - \delta s^G - \delta s^P} \frac{P^L}{P^H} \right]} =: \hat{m}^G. \]

- **Banks invest in either the gambling or the prudent asset if**
  \[ \hat{m}^P := \frac{r^G - r^P}{2 \left[ \sqrt{1 - \delta s^G + \delta s^P} \frac{P^H}{P^L} - 1 \right]} \leq \frac{\pi}{n} \leq \hat{m}^G. \]
Banks gamble if

\[ \frac{\pi}{n} < \hat{m}_P. \]

The optimal bailout policy \( \hat{\beta}^* \) is \( \beta^L = 1 \) and

\[
\beta^H = \begin{cases} 
1 - \frac{v^L[1 - \delta]}{s_q[\nu^H - \nu^L]} & \text{if } \frac{v^H}{\nu^H} \geq \frac{1 - \delta + \delta q}{\delta q} \\
1 & \text{if } \frac{v^H}{\nu^H} < \frac{1 - \delta + \delta q}{\delta q}
\end{cases}
\]

**Proof.** We can take the result from the symmetric information case (long-term planning horizon) and replace \( \hat{s}_G \) and \( \hat{s}_P \) by \( s^G \) defined in (3.3) and \( s^P \) defined in (3.2). The derivation of the optimal bailout policy is provided in the appendix. ■

We know from the symmetric information case (long-term planning horizon) that an increase in the bailout probability \( \beta \) increases the probability of surviving, \( \hat{s}_P (s^G) \), which increases the bank value. A higher bank value, in turn, increases ceteris paribus the opportunity costs of the gambling investment because the loss in case of a failure increases (value effect). However, an increase of the bailout probability decreases the impact of the project risk on the probability of surviving, too, because both the gambling and the prudent value increases. This induces the banks ceteris paribus to increase their risk (moral-hazard-effect). Under symmetric information the regulator can differentiate between prudent and gambling projects and punish gambling banks. By choosing \( \beta^P = 1 \) for prudent banks and \( \beta^G = 0 \) for gambling banks, he can increase \( \nu^P \) (value effect) without increasing \( \nu^G \) (moral-hazard-effect). Therefore, the bank has an incentive to switch the projects except for sufficiently tough competition.

Asymmetric information forecloses the discrimination between "good" (P) and "bad" (G) projects. However, the regulator can discriminate between "good" and "bad" circumstances. Obviously (proposition 4), if \( v^H \) is sufficiently low, thus sufficiently close to \( v^L \), banks should always rescued with probability one independent of the financial market’s stability. This raises from the fact that in case of sufficiently low values of \( v^H \), the value-effect outweighs the moral-hazard-effect which is in line with the findings of Cordella & Yeyati (2003). By contrast, if \( v^H \) increases and reaches the threshold \( v^L \left[1 + \frac{1 - \delta}{s_q}\right] \), the regulator should reduce its bailout probability in case of \( H \), but still rescue banks with probability one in case of \( L \). The reason is that \( \beta^L \) is relatively more important for the prudent asset than for the gambling asset whereas \( \beta^H \) is relatively more important for the gambling than for the prudent asset. Differently speaking, the prudent asset has a comparatively higher benefit from a high value of \( \beta^L \) and the gambling asset from a high value of \( \beta^H \). Therefore, the gambling asset is punished more by the combination of a low \( \beta^H \) and a high \( \beta^L \) than the prudent asset. However, \( \beta^H \) should increase in \( v^H \) because an increase of \( v^H \) increases the bank value and thus makes a deviation from the prudent asset more expensive. Therefore, the regulator can increase the bailout probability in \( v^H \) and the moral-hazard-effect is partly compensated by the value effect. Due to the fact that the moral-hazard-effect is only partly compensated and not completely outweighed - this is the case for \( v^H < v^L \left[1 + \frac{1 - \delta}{s_q}\right] \) - the regulator should not rescue with probability one, but construct ambiguity, thus rescue with probability \( \beta^H \in (0, 1) \). Figure (4.2) illustrates the optimal bailout policy.

This solution is different to Cordella & Yeyati (2003) who find that the regulator should liquidate any bank in case of a sufficiently low \( v \). However, we receive the same solution as Cordella & Yeyati (2003), if we consider \( v \) as being continuous, with \( v \in [0,1] \).
Figure 4.2: Optimal Bailout Policy

To see this, assume that \( v \) is a stochastic iid variable with the density function \( f(v) \) and the distribution function \( F(v) \), with \( F(1) = 1 \). Both functions are known ex ante by all players. Following these assumptions, we can summarize the return functions \( \bar{R}^G \) and \( \bar{R}^P \):

\[
\bar{R}^G = \begin{cases} 
\gamma^H & \text{with probability } p^L(v) \\
0 & \text{with probability } 1 - p^L(v)
\end{cases}
\]

\[
\bar{R}^P = \begin{cases} 
\gamma^L & \text{with probability } p^H(v) \\
0 & \text{with probability } 1 - p^H(v)
\end{cases}
\]

Of course, the regulator cannot differentiate between prudent and gambling assets, but he can react to a continuous state of nature, thus \( \beta : v \mapsto [0, 1] \).

**Proposition 5.** The optimal bailout policy is given by:

\[
\bar{\beta}^*(v) = \begin{cases} 
0 & v > \overline{v} \\
1 & v \leq \overline{v}
\end{cases},
\]

with

\[
\delta \int_{\overline{v}}^{1} vf(v) dv
\]

\[
\frac{\int_{\overline{v}}^{1} vf(v) dv}{\overline{v}} =: \overline{\nu}.
\]

\[
\int_{0}^{1} f(v) dv
\]

17
Proof. For the derivation of the equilibrium conditions and the optimal bailout policy, consult the appendix!

It is easy to show that a bailout policy reduces the area where banks gamble if \( \text{Cov}(v, \beta(v)) < 0 \) (consult proof of proposition 5 in the appendix!) holds which means: An increase in \( v \) has to induce a decrease in the bailout probability. Following the optimal bailout strategy \( \beta^* \), a regulator rescues with probability one in times of financial instability, \( v \leq \overline{v} \), and liquidates otherwise with probability one. This raises from the fact that in case of a sufficiently low \( v \) the value effect outweighs the moral-hazard-effect and consequently the regulator should rescue all banks with probability one (and vice versa) which is in line with Cordella & Yeyati (2003).

By contrast to Cordella & Yeyati (2003), we suggest a more differentiated view with regard to the effectiveness of the bailout strategy \( \beta^* \). For an intermediate number of banks, \( n(k) \leq n(k) \leq \overline{n}(k) \), the bailout strategy \( \beta^* \) can refrain banks from excessive risk-taking behaviour since it raises the franchise value and hence increases incentives to stay solvent. If the number of banks is sufficiently low, \( n(k) < \overline{n}(k) \), banks already behave prudently and consequently the bailout policy \( \beta^* \) does not influence the investment decision. In case of \( n(k) > \overline{n}(k) \), banks gamble independent of whether \( \beta^* \) increases their franchise value because the value effect does not compensate for the low franchise value resulting from the high number of banks. As a result, the effectiveness of the bailout policy \( \beta^* \) to prevent moral hazard depends on the competitiveness of the banking sector.

5. The impact of funding instruments on banks’ risk behaviour

5.1. The bank levy

We remain within the long-term planning horizon framework (as in section 4.2.2). Assume that banks have to pay a levy on deposits at the beginning of each investment period. The levy rate is denoted with \( \theta \). Hence each bank has to generate \( \theta D_j \) beyond the capital reserves, however, only \( [1 + k] D_j \) can be invested. Due to the fact that both the levy and the bailout probability do not depend on banks’ project risk we can determine the deposit rate and the bank value generally.

The objective function of bank \( j \) is given by:

\[
\hat{V}_j = \max_{r_j} \left[ - (k + \theta) D_j (r_j, r_{-j}) + \delta \Pi_j (r_j, r_{-j}) + \delta \hat{s} \hat{V}_j \right],
\]

(5.1)

where \( \hat{s} \) is the probability of surviving defined by

\[
\hat{s} = \int_0^1 \left[ v p + (1 - v p) \beta(v) \right] f(v) dv
\]

and the expected period profit \( \hat{\Pi}_j (r_j, r_{-j}) \) amounts to:

\[
\hat{\Pi}_j (r_j, r_{-j}) = \mu p \left( \gamma_j \right) \left[ \gamma_j - r_j + k \left( 1 + \gamma_j \right) \right] D (r_j, r_{-j}),
\]

(5.2)
with \( \mu = \int_0^1 v f(v) dv \). The value of bank \( j \) is the sum of the discounted net expected profit \( \hat{\Pi}_j \) and the discounted expected future value \( \delta \hat{s}V_j \), less the capital reserves \( kD_j \) and the bank levy \( \theta D_j \).

The optimal deposit rate for bank \( j \) amounts to:

\[
\gamma - \frac{\pi}{n} - \frac{\theta}{\delta \mu p} + k \left( 1 + \gamma - \frac{1}{\delta \mu p} \right) =: r^*_j. \quad (5.3)
\]

It decreases in the levy rate \( \theta \), the transaction costs \( \pi \) and the capital requirement \( k \) and increases in the project return \( \gamma \), the number of banks \( n \) and the discount factor \( \delta \). The intuition is that the levy and the capital requirement are passed to depositors which reduces their return. Due to the fact that the levy is paid effectively by the depositors the bank value does not depend on \( \theta \).

**Proposition 6.** An increase in \( \theta \) decreases the range where banks choose the gambling asset.

**Proof.** See appendix.

Accordingly, an increase of the levy rate \( \theta \) leads to a shift of the straight lines \( m^G_\tau(k) \) and \( m^P_\tau(k) \) towards the origin (see figure 5.1). Shareholders are not effected by the bank levy as it is completely passed to the depositors. However, the higher the levy the more shareholders risk to lose and thus the more they are interested in staying solvent. In comparison to the capital ratio \( k \), the shareholders do not benefit from a return point of view, though the stabilizing mechanism is the same.

### 5.2. The profit tax surcharge

At the end of each investment period banks have to pay a profit tax which is denoted by \( \tau \). Accordingly, each bank can only retain \( 1 - \tau \) of the profit. If we omitted capital requirements, the tax rate would not change the banks’ investment decision. However, capital requirements induce the banks to adjust their investment decisions to the tax policy.

To see this, we determine the bank values. Bank \( j \)'s maximization problem can be described as:

\[
V^*_j = \max_{r_j} \left[ -kD_j (r_j, r_{-j}) + (1 - \tau) \delta \hat{\Pi}_j (r_j, r_{-j}) + \delta \hat{s}V^*_j \right]. \quad (5.4)
\]

and the optimal deposit rate amounts to:

\[
\gamma - \frac{\pi}{n} + k \left[ 1 + \gamma - \frac{1}{(1 - \tau) \delta \mu p} \right] =: r^*_j. \quad (5.5)
\]

According to (5.5), an increase in \( \tau \) decreases the deposit rates. However, the tax rate does not effect \( r^*_j \) for \( k = 0 \).

\( ^{30} \)For a definition of \( m^G_\tau(k) \) and \( m^P_\tau(k) \) consult the appendix.

\( ^{31} \)The index "\( \tau \)" stands for "tax".
In comparison with (4.7) and (4.5), depositors receive less because shareholders require a compensation for the decrease of the expected equity return resulting from the implementation of the tax. Inserting (5.5) in (5.4) leads to the following bank value $V_j^\tau$:

$$V_j^\tau = (1 - \tau) \frac{\delta \mu \rho}{1 - \delta \sigma n^2}. \quad (5.6)$$

Obviously, the bank value decreases if the tax ratio increases. By contrast to the levy, depositors and shareholders share the tax payment because shareholders have to pay the tax on their net return $\gamma - r$.

**Proposition 7.** An increase in the tax rate $t$ decreases the range where all banks gamble. The risk-reducing effect of a profit tax increases in the minimum capital requirement $k$.

**Proof.** See appendix. ■

For $k = 0$, the profit tax does not have any effect on the equilibrium conditions. The tax rate reduces the payoff level, but it does not effect the decision between gambling and prudent behaviour. In the previous section we argued that a decrease in market power decreases banks’ franchise value and thus induces gambling behaviour. This is due to the fact that the return of success in case of gambling behaviour increases relatively to the loss in case of failure since payoffs are lower. The same argumentation does not hold in case of a profit tax. Although the expected period profit decreases if the tax rate increases, the return of success in case of gambling behaviour does not increase relatively because these returns are taxed, too.

By contrast, for $k > 0$ the profit tax matters because the equity return increases relatively to the return on deposits. Due to the fact that the tax on equity return is compensated by lower deposit rates, shareholders risk to lose relatively more if the minimum ratio of equity increases. Hence, the higher the equity ratio the more it matters since it is the part of shareholders’ payoff which does not depend on the tax rate. Consequently, although the bank value decreases, risk incentives decrease because return on equity relatively increases.

Figure 5.1 illustrates the risk-reducing-effect of a profit tax and a bank levy for $\tau = \theta$.\textsuperscript{32} It shows that a bank levy depending on the level of liabilities is more effective than a profit tax if the regulator is only concerned about the investment risk. However as deposit rates decrease, the burden of the levy is completely passed to the depositors.\textsuperscript{33}

**5.3. Time varying bailout funding**

In order to avoid that the charge is passed to depositors, we consider a third way of funding bank bailouts which suggests that bailed out banks should be (partially) nationalised. The degree of nationalisation should depend on the state of nature, thus $\nu^L$ and $\nu^H$. Therefore, we interpret $\beta$ as the degree of nationalisation, where $\beta = 1$ means that the regulator bails out an insolvent

\textsuperscript{32}Notice that $m(k)$ and $m(k)$ illustrate the reference case where neither a bank levy nor a profit tax exists. Recall that we normalized the payoffs and returns which allows a comparison between the effectiveness of a bank levy and a profit tax.

\textsuperscript{33}The US model may provide an alternative as it only burdens large banks. Since small banks do not have to pay the fee they may have an advantage on the deposit market which makes it difficult for large banks to shift the burden to depositors. See European Commission (2010).
bank, but he does not buy any shares of the rescued bank, and $\beta = 0$ is equivalent to a purchase of all shares, thus a nationalization of insolvent banks.

From section 4.2.2, we know that the depositors are not punished by this measurement as $\beta$ does not affect the deposit rates (4.5 and 4.7). In order to prevent moral hazard, insolvent banks should not be punished by a partial nationalisation in times of financial instability, $v^H$, whereas banks being insolvent in times of $v^L$ should either be nationalised partially or the regulator should not acquire any shares.\footnote{The nationalisations of banks in Norway in the course of the crisis in 1990 provides an example for the benefits of raising funds through acquiring the banks’ future value (Sandal (2004)).}

Although this policy generates funds and reduces moral hazard without charging depositors, it raises the question whether benefits from bailouts in times of $v^H$ are sufficient to finance bailouts in times of $v^L$. In order to increase the benefits, it might be necessary to punish banks in times of $v^L$, too, which dampens the effectiveness of this policy to reduce moral hazard among banks. Accordingly, the regulator faces the trade-off between avoiding moral hazard and receiving funds for future bailouts. In order to avoid moral hazard, the regulator should at least apply a higher degree of bank nationalization in case of $v^H$ than in case of $v^L$.

Besides, partial nationalizations are not as efficient as a bank levy or a profit tax surcharge. If competition is sufficiently high or sufficiently low, banks gamble or behave prudent independent of whether the regulator follows the strategy of nationalizing banks or not. By contrast, the regulator can adjust his tax and levy rate to the competitiveness of the deposit market, though at the expense of the depositors.

To conclude, the presented strategy can generate funds for future bailouts, avoid that the charge is passed to depositors and decrease moral hazard. While the depositors are never punished, the regulator faces a trade-off between increasing the fund level and decreasing moral

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**Figure 5.1: Bank levy versus profit tax**

\[ \]
hazard. Nevertheless, even if the regulator avoids partial nationalizations and introduces a levy on deposits or a profit tax surcharge, he faces a trade-off: An increase in the levy rate (tax rate) increases range where banks behave more prudent and (might) increase the fund level, it decreases the wealth of the banks’ customers. Consequently, a combination of these instruments might be a suitable policy implication for the regulators as partial nationalizations can prevent moral hazard without burdening depositors and thus enable the regulators to reduce their levy rate or/and tax rate.

6. Conclusion

We determined the optimal bailout policy for a regulator who tries to prevent excessive risk-taking among banks. Furthermore, we analysed the effect of three funding possibilities on banks’ risk behaviour: a bank levy on deposits, a profit tax surcharge and nationalizations of insolvent banks. The paper accounts for competition on deposit market by using the imperfect competition framework à la Salop (1979). A regulator can credibly commit to an ex ante announced bailout policy and differentiate between degrees of financial market stability and the banks’ planning horizon. The banks fund their assets with secured deposits and equity capital and invest them in either a prudent or a gambling project.

As in Repullo (2004), a sufficiently large number of banks leads to an equilibrium where all banks gamble whereas a prudent equilibrium exists if the number of banks is sufficiently low. Our results suggest that banks with short-term planning horizon increase their risk level independent of the bailout policy. This is in line with the standard literature which reveals that bank bailouts reduce the impact of the banks’ risk decision on their probability of surviving (enforce moral hazard). By contrast, banks with a sufficiently long-term planning horizon respond to the bailout policy and might switch from the gambling to the prudent investment if the regulator announces to bail out banks with probability one in times of financial instability. However, even in times of rather stable financial markets, the regulator should bail out insolvent banks with probability one or construct ambiguity, thus rescue with probability higher than zero and lower than one. The effectiveness of this bailout policy depends on the competitiveness of the deposit market. If the number of banks is sufficiently large or sufficiently low, banks gamble or behave prudently independent of the bailout policy.

Furthermore, the paper reveals that both a levy on deposits and a profit tax surcharge can induce banks to switch from gambling to prudent assets. While a bank levy is completely passed to depositors, a profit tax surcharge is partially borne by banks’ shareholders. Consequently, a bank levy is relatively more effective to decrease banks’ risk level. As a third possibility to finance bank bailouts, we suggest that failed banks can be (partially) nationalized which was often done in the past, for instance in Norway during the crisis of the 1990s or in Germany and the US during the recent crisis. The case of Norway shows that nationalizations might be beneficial from a funding point of view. From a moral hazard point of view, the regulator should not nationalize insolvent banks in times of financial instability. However, this policy might counteract the strategy to increase enough funds for bailouts in the future. Even in rather stable times, he should only partially nationalize them. We suggest that a combination of these instruments might be an adequate policy implication for the regulator.

We are aware that our analysis is based on several assumptions. Firstly, we assumed that banks only compete on the deposit market in an imperfect competitive framework. Secondly,
there remain other funding instruments such as a levy on unsecured deposits. Thirdly, the number of banks might only be one indicator for the degree of competition and we are conscious that there is still a debate in the literature about its adequacy. Nevertheless, our paper offers some insights in the consequences of different bailout policies and the effectiveness of three funding alternatives.
Appendix

Proof of Proposition 1

A prudent bank $j$’s maximization problem in the second period can be written as:

$$
V^P_j = \max_{r_j} \left[-k D (r_j, r_{-j}) + \delta P^H \left[ qv^H + [1 - q] v^L \right] \left[ \gamma^L - r_j + k \left( 1 + \gamma^L \right) \right] D (r_j, r_{-j}) \right]
$$

which leads to the optimal deposit rate

$$
\gamma^L + k \left[ 1 + \gamma^L - \frac{1}{\delta P^H \left[ qv^H + [1 - q] v^L \right]} \right] - \frac{\pi}{n} =: r^P.
$$

Inserting the deposit rate in the bank value function and solving for $V^P$ leads to:

$$
V^P_j = \delta P^H \left[ qv^H + [1 - q] v^L \right] \frac{\pi}{n^2}.
$$

A gambling bank $j$’s maximization problem in the second period can be written as:

$$
V^G_j = \max_{r_j} \left[-k D (r_j, r_{-j}) + \delta P^L \left[ qv^H + [1 - q] v^L \right] \left[ \gamma^H - r_j + k \left( 1 + \gamma^H \right) \right] D (r_j, r_{-j}) \right]
$$

which leads to the optimal deposit rate

$$
\gamma^H + k \left[ 1 + \gamma^H - \frac{1}{\delta P^L \left[ qv^H + [1 - q] v^L \right]} \right] - \frac{\pi}{n} =: r^G.
$$

Inserting the deposit rate in the bank value function and solving for $V^G$ leads to:

$$
V^G_j = \delta P^L \left[ qv^H + [1 - q] v^L \right] \frac{\pi}{n^2}.
$$

We have a gambling equilibrium if the following inequation holds:

$$
\max_{r_j} \left[-k D (r_j, r_{-G}) + \delta P^P_j \gamma^P + \delta P^G_j \gamma^G \right] \leq \left[ 1 + \delta P^G_j \right] V^G_j
$$

Now it remains to determine the investment decision explicitly. By differentiating the value functions for the deviating bank $j$ given by the left side of the inequations (4.3) and (4.2), we reveal the following deposit rate:

$$
\frac{r^P + r^G}{2} = r_j =: \bar{r}_j.
$$

Accordingly, if a bank $j$ deviates from either the prudent or the gambling path, it offers the deposit rate $\bar{r}_j$. Inserting the optimal deposit rate in the necessary condition for a gambling equilibrium and solving for $\frac{\pi}{n}$, we receive the following solution (we can omit again one solution as $D_j > 0$ must hold):

$$
\frac{\pi}{n} \leq \frac{r^G - r^P}{2 \left[ 1 - \sqrt{[1 - \delta P^G \delta P^G] \frac{\mu}{2n}} \right]} =: m^G.
$$
Equivalently, we obtain the conditions for a prudent equilibrium:

$$\frac{\pi}{n} \geq \frac{r^G - r^P}{2 \sqrt{\frac{p^H}{p^P} \left[ 1 + \delta_s^P - \delta_s^G \right] - 1}} =: m^P.$$ 

Simple rearrangements reveal that \(m^P < m^G\).

It is easy to show that an increase in \(P\) increases \(m^G\) and \(m^P\) while an increase in \(G\) decreases \(m^P\) and \(m^G\). As the discrimination between gambling and prudent projects accounts for more information than the discrimination between financial stability and instability, it cannot be worth to follow a strategy that accounts only for the latter one, thus \(H\) and \(L\) instead of \(P\) and \(G\).

Proof of Proposition 2

We know the bank values in the prudent and the gambling case from the main text, recall (4.12) and (4.10). The necessary condition for a gambling equilibrium is (notice that the optimal deposit rate for the deviating bank does not change):

$$-kD \left(r^*_j, r_{-G}\right) + \delta P^H \left[qu^H + [1 - q]u^L\right] \left[\gamma^L - r^*_j + k \left(1 + \gamma^L\right)\right] D \left(r^*_j, r_{-G}\right) + \delta s^P \hat{V}^G_j \leq \hat{V}^G_j$$

which can be simplified to (notice that \(D_j > 0\) must hold):

$$\frac{\pi}{n} \leq \frac{r^G - r^P}{2 \left[1 - \sqrt{\frac{1 - \delta s^P}{1 - \delta s^G}} \frac{p^L}{p^H}\right]} =: \tilde{m}^G.$$  

Equivalently, the necessary condition for a prudent equilibrium is:

$$-kD \left(r^*_j, r_{-G}\right) + \delta P^L \left[qu^H + [1 - q]u^L\right] \left[\gamma^H - r^*_j + k \left(1 + \gamma^H\right)\right] D \left(r^*_j, r_{-G}\right) + \delta s^G \hat{V}^P_j \leq \hat{V}^P_j.$$  

Solving the inequation we have:

$$\frac{\pi}{n} \geq \frac{r^G - r^P}{2 \left[\sqrt{\frac{p^H}{p^P} \left[1 - \delta s^G\right]} - 1\right]} =: \tilde{m}^P.$$  

It is easy to show that \(\tilde{m}^P < \tilde{m}^G\). Obviously, \(\tilde{m}^G\) and \(\tilde{m}^P\) increase in \(\beta^P\) and decrease in \(\beta^G\).

Proof of Proposition 4

To determine the optimal bailout policy, we consider the following function \(f\) which is the unique term in \(\tilde{m}^G\) and \(\tilde{m}^P\) depending on the bailout probability:

$$f(\beta^H, \beta^L) = \frac{p^H \left[1 - \delta s^G\right]}{p^L \left[1 - \delta s^P\right]}$$
or equivalently
\[
f(\beta^H, \beta^L) = 1 + \frac{[p^H - p^L]}{p^L} \frac{[1 - \delta q \beta^H - \delta [1 - q] \beta^L]}{[1 - \delta q \beta^H - \delta [1 - q] \beta^L] - \delta qp^H p^L v_H [1 - \beta^H] - \delta [1 - q] p^H p^L v_L [1 - \beta^L]}
\]

We search for the bailout probabilities \(\beta^H\) and \(\beta^L\) which maximize the function \(f\), thus decrease \(\hat{\kappa}^H\) and \(\hat{\kappa}^G\). For \(\beta^L = 0\), the derivative of \(f\) with respect to \(\beta^H\) equals to
\[
\frac{\partial f}{\partial \beta^H} = \frac{\delta \left[\delta q p^H v_H - \delta [1 - q] p^H v_L\right] - \left[-\delta q + \delta q p^H v_H\right]}{\left[\delta q \beta^H - \delta [1 - q] \beta^L\right] - \delta qp^H v_H [1 - \beta^H] - \delta [1 - q] p^H v_L [1 - \beta^L]} > 0
\]
which means that an increase of \(\beta^L\) increases \(f\). The same is true for the derivative of \(f\) with respect to \(\beta^L\) at point \(\beta^H = 0\):
\[
\frac{\partial f}{\partial \beta^L} = \frac{\left[-\delta [1 - q] \left[-\delta q p^H v_H - \delta [1 - q] p^H v_L\right] - \left[-\delta [1 - q] + \delta [1 - q] p^H v_L\right]\right]}{\left[\delta q \beta^H - \delta [1 - q] \beta^L\right] - \delta qp^H v_H [1 - \beta^H] - \delta [1 - q] p^H v_L [1 - \beta^L]} > 0.
\]

By differentiating the function \(f\) with respect to \(\beta^H\) and \(\beta^L\), we receive the optimal bailout probabilities \(\beta^L^*\) and \(\beta^H^*\) which amount to:
\[
\beta^L^* = \frac{v^H [1 - \delta q]}{[1 - q] [v^H - v^L]} - \frac{v^L}{v^H - v^L}
\]
and
\[
\beta^H^* = 1 - \frac{v^L [1 - \delta]}{\delta q [v^H - v^L]}
\]

It is easy to check that
\[
\frac{v^H [1 - \delta q]}{[1 - q] [v^H - v^L]} > \frac{v^L}{v^H - v^L} > 1 \quad \text{for} \quad v^H > v^L.
\]
As the derivative of \(f\) with respect to \(\beta^H\) is positive at \(\beta^L^* = 0\), \(\beta^L^*\) is a maximum turning point and therefore the optimal bailout probability is \(\beta^L^* = 1.35\).

We know that \(\beta^H^*\) leads to the fact that the derivative of \(f\) with respect to \(\beta^L\) becomes zero. Therefore, the sign of \(\frac{\partial f}{\partial \beta^L}\) changes at point \(\beta^H^*\). In case of
\[
\frac{v^H}{v^L} < \frac{1 - \delta + \delta q}{\delta q},
\]
we have \(\beta^H^* < 0\). However, we know that \(\beta^H \in [0, 1]\) and \(\frac{\partial f}{\partial \beta^L} > 0\) if \(\beta^H = 0\). Consequently, \(\beta^H^*\) must be a minimal turning point and the optimal solution is \(\beta^H = 1\). By contrast, if
\[
\frac{v^H}{v^L} \geq \frac{1 - \delta + \delta q}{\delta q},
\]
we know that \(\beta^H \in [0, 1]\) and \(\frac{\partial f}{\partial \beta^L} < 0\) if \(\beta^H = 0\). Consequently, \(\beta^H^*\) must be a minimal turning point and the optimal solution is \(\beta^H = 1\). By contrast, if
\[
\frac{v^H}{v^L} \geq \frac{1 - \delta + \delta q}{\delta q},
\]

\footnote{This simple way of solving the maximization problem is possible because \(\beta^H\) does not influence the sign of \(\frac{\partial f}{\partial \beta^L}\) \((\beta^H\) only effects the denominator of \(\frac{\partial f}{\partial \beta^L}\). Similarly, \(\beta^L\) does not effect the sign of \(\frac{\partial f}{\partial \beta^L}\).}
\[ \beta^{H*} \geq 0, \text{ thus } \beta^{H*} \text{ is a maximum turning point and the optimal solution is} \]
\[ \beta^{H*} = 1 - \frac{v^L [1 - \delta]}{\delta q [v^H - v^L]}, \]
which is always smaller than one.

**Proof of Proposition 5**

We can derive the equilibrium conditions on the same way as in the case where nature can only reveal \( v^H \) and \( v^L \). Let us define where \( \mu = \int_0^1 \nu f(v) dv \). The maximization problem of the bank in the prudent case can be described as:

\[ \hat{V}_j^P = \max_{r_j} \left[ -k D_j (r_j, r_{-j}) + \delta \hat{\Pi}_j^P (r_j, r_{-j}) + \delta \hat{s}^P \hat{V}_j^P \right], \tag{6.1} \]

where

\[ \hat{\Pi}_j^P (r_j, r_{-j}) = \mu p^H [\gamma^L - r_j + k (1 + \gamma^L)] D_j (r_j, r_{-j}) \]

and

\[ \hat{s}^P = \int_0^1 [v p^H + (1 - v p^H) \beta (v)] f(v) dv. \]

Inserting the optimal deposit rate \( \hat{r}_j^P \) with

\[ \hat{r}_j^P := \gamma^L - \frac{\pi}{n} + k \left[ 1 + \gamma^L - \frac{1}{\mu \delta p^H} \right] \]

in (6.1) reveals the optimal bank value in the prudent case

\[ \hat{V}_j^P = \frac{\delta \mu p^H \pi}{1 - \delta \hat{s}^P n \hat{r}_j^P}. \tag{6.2} \]

Equivalently, the maximization problem for banks investing in the gambling asset can be described as:

\[ \hat{V}_j^G = \max_{r_j} \left[ -k D_j (r_j, r_{-j}) + \delta \hat{\Pi}_j^G (r_j, r_{-j}) + \delta \hat{s}^G \hat{V}_j^G \right], \tag{6.3} \]

where

\[ \hat{\Pi}_j^G (r_j, r_{-j}) = \mu p^L [\gamma^H - r_j + k (1 + \gamma^H)] D_j (r_j, r_{-j}) \]

and

\[ \hat{s}^G = \int_0^1 [v p^L + (1 - v p^L) \beta (v)] f(v) dv. \]

Inserting the optimal deposit rate \( \hat{r}_j^G \) with

\[ \hat{r}_j^G := \gamma^H - \frac{\pi}{n} + k \left[ 1 + \gamma^H - \frac{1}{\mu \delta p^L} \right] \]
in (6.3) reveals the optimal bank value in the gambling case

\[
\hat{V}_j^G = \frac{\delta \mu p^L}{1 - \delta \hat{G}} \frac{\pi}{n^2}.
\]  

(6.4)

We have a gambling equilibrium if:

\[
\max_{r_j} \left[ -kD_j (r_j, r_{-j}) + \delta \hat{P}_j (r_j, r_{-j}) + \delta \hat{G} \hat{V}_j \right] \leq \hat{V}_j^G.
\]

Solving the inequation with respect to \( \frac{\pi}{n} \), we receive (we can omit again one solution as \( D_j > 0 \) must hold)

\[
\frac{\pi}{n} \leq \frac{\hat{V}_j^G - \hat{V}_j^P}{2 \left[ 1 - \sqrt{\frac{\mu^L (1 - \delta \hat{G})}{\mu^L (1 - \delta \hat{G}^G)}} \right]} =: \overline{m}^G.
\]

Equivalently, we have a prudent equilibrium if:

\[
\max_{r_j} \left[ -kD_j (r_j, r_{-j}) + \delta \hat{G}_j (r_j, r_{-j}) + \delta \hat{G} \hat{V}_j \right] \leq \hat{V}_j^P.
\]

Solving the inequation with respect to \( \frac{\pi}{n} \), we receive

\[
\frac{\pi}{n} \geq \frac{\hat{V}_j^G - \hat{V}_j^P}{2 \left( \sqrt{\frac{\mu^L (1 - \delta \hat{G}^G)}{\mu^L (1 - \delta \hat{G})}} - 1 \right)} =: \overline{m}^P.
\]

It is easy to show that \( \overline{m}^P < \overline{m}^G \).

The necessary condition for a bailout policy which reduces risk level, thus which leads to a shift of the straight lines \( \overline{m}^G \) and \( \overline{m}^G \) towards the origin is:

\[
\mu \int_0^1 \beta (v) f (v) dv > \delta \int_0^1 v \beta (v) f (v) dv.
\]

It is easy to show that \( \mu \int_0^1 \beta (v) f (v) dv > \delta \int_0^1 v \beta (v) f (v) dv \) can only be true if \( \text{Cov}(v, \beta(v)) < 0 \).

In order to determine the optimal bailout policy we substitute:

\[
\int_0^1 v \beta (v) f (v) dv = \lim_{k=1}^n \sum v_{\xi_k} \beta (\xi_k) [f (x_k) - f (x_{k-1})]
\]

and

\[
\int_0^1 \beta (v) f (v) dv = \lim_{k=1}^n \sum \beta (\xi_k) [f (x_k) - f (x_{k-1})].
\]
Then we define the square root term in $\overline{m}^G$ (and $\overline{m}^P$ equivalently) by
\[
q(\beta) = \frac{p^L \left(1 - \delta \mu p^H - \delta \sum_{k=1}^n \beta(\xi_k)[f(x_k) - f(x_{k-1})] + \delta p^H \sum_{k=1}^n u_{\xi_k} \beta(\xi_k)[f(x_k) - f(x_{k-1})]\right)}{p^H \left(1 - \delta \mu p^L - \delta \sum_{k=1}^n \beta(\xi_k)[f(x_k) - f(x_{k-1})] + \delta p^L \sum_{k=1}^n u_{\xi_k} \beta(\xi_k)[f(x_k) - f(x_{k-1})]\right)},
\]
and differentiate $q(\beta)$ with respect to $\beta$ which leads to:
\[
\frac{\partial q}{\partial \beta} = \begin{cases}
> 0; & v_{\xi_k} > \left[\frac{\delta \mu - \delta \sum_{k=1}^n u_{\xi_k} \beta(\xi_k)[f(x_k) - f(x_{k-1})]}{1 - \delta \mu p^H + \delta \sum_{k=1}^n \beta(\xi_k)[f(x_k) - f(x_{k-1})]}\right] \\
\leq 0; & v_{\xi_k} \leq \left[\frac{\delta \mu - \delta \sum_{k=1}^n u_{\xi_k} \beta(\xi_k)[f(x_k) - f(x_{k-1})]}{1 - \delta \mu p^L + \delta \sum_{k=1}^n \beta(\xi_k)[f(x_k) - f(x_{k-1})]}\right].
\end{cases}
\]

In the continuous case the threshold level $\overline{\nu}$ amounts to:
\[
\overline{\nu} := \frac{1}{\nu} \int_0^\infty \frac{\delta f(v) dv}{1 - \delta \int_0^\infty f(v) dv}.
\]

**Proof of Proposition 6**

We can derive the prudent and the gambling equilibrium conditions on the same way as in proposition 1. From the main text we know that the optimal deposit rate if all banks invest in the same asset (recall 5.3). We define the deposit rate in the prudent case (all banks invest in the prudent asset) by $r_j^P$, with
\[
\gamma^L_{j} - \frac{\pi}{n} - \frac{\theta}{\delta \mu p^L} + k \left(1 + \gamma^L_{j} - \frac{1}{\delta \mu p^H}\right) =: r_j^P \quad (6.5)
\]
and the deposit rate in the gambling case (all bank invest in gambling assets) by $r_j^G$, with
\[
\gamma^H_{j} - \frac{\pi}{n} - \frac{\theta}{\delta \mu p^L} + k \left(1 + \gamma^H_{j} - \frac{1}{\delta \mu p^H}\right) =: r_j^G. \quad (6.6)
\]

If the regulator introduces a bank levy, banks gamble if
\[
\max_{r_j} \left[-(k + \theta) D_j(r_j, r_{-j}) + \delta\overline{\nu}^P_j(r_j, r_{-j}) + \delta\tilde{\nu}^G_j \right] \leq \tilde{V}_j^G,
\]
where $\hat{V}_j^G$ is the same as in (6.4). Solving the inequation with respect to $\frac{\pi}{n}$ leads to (we can omit again one solution as $D_j > 0$ must hold):

$$\frac{\pi}{n} \leq \frac{r_j^G - r_j^P}{2\left[1 - \sqrt{\frac{\mu_H(1-\delta s^P)}{\mu_H(1-\delta s^G)}}\right]} := m^G_i(k).$$

Equivalently, banks invest in the prudent asset if

$$\max_{r_j} \left[-(k + \theta)D_j(r_j, r_{-j}) + \delta \hat{\Pi}_j^G(r_j, r_{-j}) + \delta s \hat{V}_j^P \right] \leq \hat{V}_j^P.$$

where as they behave prudently in case of

$$\frac{\pi}{n} \geq \frac{r_j^G - r_j^P}{2\left[\sqrt{\frac{\mu_H(1-\delta s^P)}{\mu_H(1-\delta s^G)}} - 1\right]} := m^P_i(k).$$

It is easy to show that $m^P_i(k) < m^G_i(k)$.

By recalling (6.6) and (6.5), we immediately see that an increase of $\theta$ decreases $m^G_i(k)$ and $m^P_i(k)$ and the range, where banks behave prudently, increases.

**Proof of Proposition 7**

We can derive the equilibrium condition in the same way as the equilibrium conditions in proposition 1. From the main text we already know the optimal deposit rate if all banks invest in the same asset, recall (5.5).

We define $r_j^G$ as the optimal deposit rate if all banks invest in the gambling asset, where

$$\gamma^H - \frac{\pi}{n} + k \left[1 + \gamma^H - \frac{1}{(1 - \tau) \delta \mu H} \right] := r_j^G.$$

Equivalently, we define $r_j^P$ as the optimal deposit rate if all banks invest in the prudent asset, where

$$\gamma^L - \frac{\pi}{n} + k \left[1 + \gamma^L - \frac{1}{(1 - \tau) \delta \mu H} \right] := r_j^P.$$

If the regulator introduces a profit tax, banks behave prudently in case of

$$\max_{r_j} \left[-kD_j(r_j, r_{-j}) + (1 - \tau) \delta \hat{\Pi}_j^G(r_j, r_{-j}) + \delta \hat{s}^G V_j^{\tau P} \right] \leq V_j^{\tau P},$$

where $V_j^{\tau P}$ is similar to (5.6) if we replace $p$ by $p_H$ and $s$ by $s^P$. Solving the inequation with respect to $\frac{\pi}{n}$ leads to:

$$\frac{\pi}{n} \geq \frac{r_j^G - r_j^P}{2\left[\sqrt{\frac{\mu_H(1-\delta s^G)}{\mu_H(1-\delta s^P)}} - 1\right]} := m^P_i(k).$$

Equivalently, we have a gambling equilibrium if:

$$\max_{r_j} \left[-kD_j(r_j, r_{-j}) + (1 - \tau) \delta \hat{\Pi}_j^P(r_j, r_{-j}) + \delta \hat{s}^P V_j^{\tau G} \right] \leq V_j^{\tau G},$$

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where $V^G_j$ is similar to (5.6) if we replace $p$ by $p^L$ and $s$ by $s^G$. Solving the inequation with respect to $\frac{\pi}{n}$ leads to (we can omit again one solution as $D_j > 0$ must hold):

$$\frac{\pi}{n} \leq \frac{r^G_j - r^P_j}{2 \left[ 1 - \sqrt{\frac{p^L_j (1 - \delta^L_j)}{p^G_j (1 - \delta^G_j)}} \right]} =: m^G_\tau(k).$$

It is easy to show that $m^P_\tau(k) < m^G_\tau(k)$.

Notice that $r^G_j$ and $r^P_j$ depend on $\tau$. An increase of $\tau$ decreases $m^P_\tau(k)$ and $m^G_\tau(k)$ and the range, where banks gamble, decreases.
References


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