

Exchange Rate, Risk Premium and Factors: What Can Term Structure of Interest Rates Tell Us about the Dynamics of the Exchange Rate?

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Abstract

In this paper, I investigate the role of expectations of the current and future status of economies in determining the dynamics of exchange rate, through the channel of the risk premium for holding a currency. The unobservable risk premium, which can be properly instrumented by the bilateral latent factors obtained from the term structure of interest rates, is an important determinant of the exchange rate, as it significantly enhances the in-sample goodness of fit and out-of-sample forecast accuracy in exchange rate changes. In particular, the proposed model can beat the benchmark *naive* models in out-of-sample forecasting at short-run horizons that range from one to twelve months, as measured by the root of mean squared errors or the direction of changes. The non-linearity of the risk premium in latent factors further renders state-dependent and time-varying response of change in the exchange rate to an identified monetary policy adjustment. The above findings hold for seven out of eight advanced-economy currency pairs (AUD, CAD, GBP, JPY, NOK, NZD, SEK against USD). Once it is included in the Fama regression, the risk premium can also help in solving the UIP Puzzle, which has been detected in the cases of GBP/USD and JPY/USD.

Keywords: Exchange Rate, Term Structure of Interest Rates, Risk Premium, Forecast, Policy Analysis, UIP Puzzle

JEL Classification: E43, F31, F37, G14

1 Introduction

The failures of the uncovered interest rate parity (UIP) and macro models to explain the exchange rate (change) as documented in empirical literature indicate that the differentials of interest rates and traditional macroeconomic fundamentals play a very limited role in driving the changes in exchange rate. Then what else, if any, can potentially impact on the exchange rate?

Some recent phenomena observed in the exchange rate market may shed some light on the above question. During the European Sovereign Debt Crisis, the exchange rate of Euro to U.S. Dollar was found to fluctuate contemporaneously with the long-term bond yields of European peripheral countries. This can be taken as evidence that the long-term interest rates, or more accurately the whole spectrum of term structure of interest rates which contain information on the present and expected future stance of an economy, may contribute to the dynamics of the exchange rate of its currency.

Several recent researchers have already put their efforts in linking the foreign exchange rate and bond yields. For example, Chen and Tsang (2011) take a macro-finance approach and explicitly use the differentials of latent factors extracted from cross-country yield curves as proxies for the

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risk premium. They find strong evidence that both financial (factors) and macro variables (output gap and inflation) are important in explaining exchange rate and *ex post* excess currency return.

Another approach in this direction is to relate term structure of interest rates to the exchange rate in a non-arbitrage joint bond-currency model. The risk premium can arise endogenously in the model and play a key role in determining the exchange rate (e.g. Backus et al., 2001; Li and Yin, 2010; Sarno et. al, 2012). Their results show that model-implied exchange rate changes can closely match the observed data, and the model-implied risk premia can produce unbiased predictions for currency excess returns.

However, previous studies which link bond and currency generally possess two shortcomings: They either (1) neglect higher order term-structure factors in the currency risk premium term by assuming it as a linear function of the factors; or (2) neglect a very important property of the currency market – the efficiency of the foreign exchange market in absorbing new information – by treating the risk premium as a determinant of the future exchange rate, rather than the current period one. This is not plausible because, due to the semi-strong efficient market hypothesis, the spot exchange rate, as the price of an asset (one unit of foreign currency), should incorporate all publicly available information and change instantly to reflect the flow of new information (Frenkel and Mussa, 1985).

After taking the above shortcomings into account, the present work distinguishes itself from the previous bond-currency studies in the following ways: (1) currency risk premium is assumed (also can be proved) to depend on the first and second order of term structure factors; (2) it *partly* introduces the semi-strong market efficiency hypothesis and assumes the exchange rate instantly adjusts to reflect new information. In practice, this means that the risk premium is loaded into the current period exchange rate change rather than to act as a predictor for future changes; (3) pure out-of-sample forecast exercises are conducted, and are evaluated by using two criteria: the root of mean squared forecast errors and direction of changes; (4) it conducts monetary policy analysis through its impact on the risk premium term (expressed in factors); (5) it revisits the UIP Puzzle that results from the Fama (1984) regression, by utilizing the model-implied quantitative estimates of the unobservable risk premium.

In this paper, eight advanced-economy currency pairs (AUD, CAD, CHF, GBP, JPY, NOK, NZD, SEK against USD) are investigated with monthly observations, over the period between 1990s-2009.

The empirical model used for estimation, forecast and policy analysis is a risk-premium augmented autoregressive distributed lag (ARDL) model: The ARDL model, which includes the currency return (first difference of log exchange rate) and interest rate differential in lag terms, is augmented by a contemporaneous risk premium term that is instrumented by term structure factors in the first and second orders.

Empirical results show that:

(1) The estimated coefficients of the first and second order factors (which represent the risk premium) are jointly significant at a high critical level (1% or 5%). The inclusion of the factors (risk premium) enhances the explanatory power of the original ARDL model, if measured by the *adjusted-R²*, from 0.106 to 0.177 (lowest increase) or from 0.001 to 0.209 (highest increase).

(2) The proposed model can generate a more accurate forecast than benchmark naive models at short-term horizons ranging from one to twelve months. The pure time-t information forecasts over the 2004.06-2009.05 period¹ show that, if measured by the root of mean squared error criterion, the model can beat the random walk model at 1- to 3-period ahead horizons for GBP, NZD, NOK and SEK, 6-10 for JPY. If the same forecasts are evaluated with the changes in the right direction criterion, the model can beat the benchmark model that issues equal probability for upward and downward changes at longer horizons and for more currencies: H=1-12 for GBP, H=1-8 for SEK, H=1-5 for NZD, H=1-12 for NOK, H= 2-12 for JPY and H=1 for CAD. The accuracy in forecasting the AUD and CAD can be greatly improved once the *realized* future data are used. However, the proposed model is not very successful in predicting the CHF.

(3) Monetary policy shocks can, through their impact on the risk premium, induce time-varying and state-dependent responses of the change in exchange rate, while even the signs of responses can vary over time.

¹2005-2009.05 for NOK and CAD due to a smaller sample size

(4) The UIP Puzzle is no longer present if a risk premium term implied by the proposed model in this paper is added into the Fama (1984) regression, as evidenced by the fact that the coefficient of the interest rate differential turns from significantly negative to (in)significantly positive. Moreover, these model-implied values of risk premium are found to be better proxies for the unobservable risk premium in the UIP context than the widely used differential of spreads between long- and short-term bonds, as the puzzle still remains once the latter is added into the Fama regression. These results hold for both GBP/USD and JPY/USD, when the UIP condition is significantly violated.

The paper is organized as follows. Section 2 describes the model for exchange rate dynamics. Section 3 presents the data and regression results. Section 4 conducts forecasting and monetary policy analysis. Section 5 discusses robustness check and interprets the risk premium. Section 6 revisits the UIP Puzzle. Section 7 concludes.

2 Model on Exchange Rate Dynamics

2.1 Modeling the dynamics of exchange rate

2.1.1 A risk-adjusted UIP approach to the exchange rate

Deviating from the standard uncovered interest rate parity (UIP), I assume that (a) the investors on the foreign exchange market are risk-averse, and (b) the returns on holding one period domestic and foreign government bonds are risky. The consequent risk-adjusted UIP approach is as follows:

At time t , domestic investor checks their investment opportunities. the expected return on domestic bond market is:

$$E_t^{RA}[1 + r_{t+1}] = 1 + i_t + \lambda_t^r$$

where i_t is the domestic risk-free rate, and λ_t^r is the risk premium for holding a risky asset, namely the government bond in the present case.

Similarly, the expected return on foreign bond market is:

$$\begin{aligned} & E_t^{RA}[(1 + (e_{t+1} - e_t))(1 + r_{t+1}^*)] \\ &= 1 + E_t^{RA}(e_{t+1} - e_t) + i_t^* + \lambda_t^{r*} + E_t^{RA}[(e_{t+1} - e_t)r_{t+1}^*] \end{aligned}$$

where i_t^* is the foreign risk-free rate, and λ_t^{r*} is the risk premium for holding foreign government bond.

In equilibrium, the expected returns should be identical such that investors would be indifferent between investing in the domestic bond and the foreign bond:

$$i_t = E_t^{RA}(e_{t+1} - e_t) + i_t^* + \lambda_t$$

where $\lambda_t = \lambda_t^{r*} + E_t^{RA}[(e_{t+1} - e_t)r_{t+1}^*] - \lambda_t^r$ is the overall relative risk premium that domestic investors require to compensate the risks for investing in foreign market. This term makes the risk-averse investors value the foreign currency differently from the risk-neutral investors do in the standard UIP approach, where the equilibrium condition is: $i_t = E_t^{RN}(e_{t+1} - e_t) + i_t^*$. To illustrate the effect of the extra term λ_t , assume at time t , both domestic and foreign risk-free interest rate (i_t and i_t^*) are 1%, and domestic investor perceive future risk for the foreign investment and require a compensation of 10% (λ_t), then the expected exchange rate change will be $E_t^{RA}(e_{t+1} - e_t) = -10\%$, which means that the foreign (domestic) currency is expected to depreciate (appreciate) in the consecutive period, whereas the risk-neutral investors would expect no change at all.

2.1.2 'Efficient' foreign exchange market

The foreign exchange market is the largest and most liquid financial market in the world. Its average daily turnover exceeded \$3 Trillion as of April 2007 (Wang, 2008). Thus it is fair to assume that the exchange rate, like other financial asset prices, instantly changes to reflect new

information. Here I re-define the term 'market efficiency' of the foreign exchange market as the swift absorption of new information and immediate adjustment in the exchange rate.

At time t , domestic investors observe the current status and form expectations on both domestic and foreign economies, then perceive the relative risk for the foreign investment, and require λ_t as the risk compensation. The implication for the exchange rate is that, the *ex ante* expected exchange rate change is

$$E_t e_{t+1} - e_{t+0} = i_t - i_t^* - \lambda_t$$

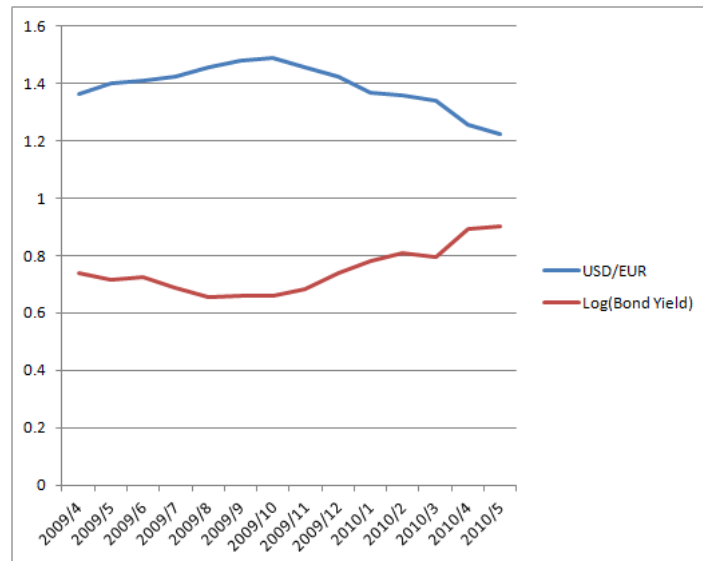
However, given that the foreign exchange market is 'efficient', the expected exchange depreciation will be materialized immediately.

$$e_{t+0} - e_{t-0} = -\lambda_t$$

This means that the exchange rate instantly changes to reflect new information and investors' new expectations. Thus the investors would *not* wait and price the perceived risk into the next period's exchange rate. Instead, they would load the risk premium λ_t into current period's fair value of a currency.

Then the expected adjustment for the next period is loaded to present one. As an illustration, the following chart shows that the exchange rate moves one on one with the new information that also reflected in the government bond yield.

Figure 1: Illustration of foreign exchange market 'efficiency'



[Referring to section 2.1.2] Immediate adjustment of the EUR/USD rate to new information. Especially, a downgrade of Greek government bond in April 2010 resulted in a contemporaneous downward jump in the EUR exchange rate, which reflects the efficiency of the foreign exchange market as defined in the text.

The earlier adjustment of the exchange rate can be illustrated by the example of Greece government debt crisis: In April 2010, the Greek government bond was downgraded to junk grade resulting in a jump in bond yields and a consequent immediate-plunge of the Euro in exchange with the US Dollar.

2.1.3 The overall change in exchange rate

To complete the model, pre-beliefs of the exchange rate $e_{t-0} - e_{t-1}$ is assumed to follow economic theory, econometrician's best forecast, or market participants believes, etc. Several models are considered in the section 2.3, here I take the random walk model for the purpose of illustration:

$$e_{t-0} - e_{t-1} = 0$$

Combine the above two equations, and let $e_t = e_{t+0}$, we get the *eventual* dynamics for the exchange rate:

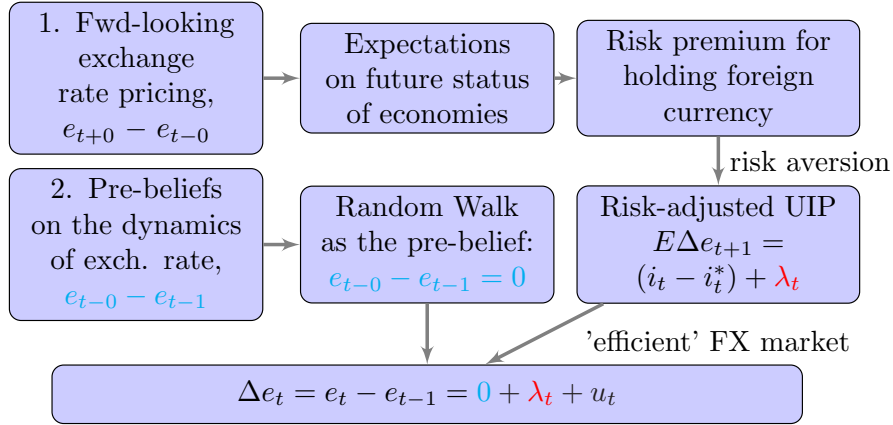
$$\Delta e_t \equiv e_t - e_{t-1} = 0 - \lambda_t + u_t$$

u_t is the residual term that contains the unexplained component of the change in exchange rate.

The above equation means that the current period excess return for holding foreign assets $\lambda_{t-1 \rightarrow t}^{ex\ post} = k \cdot \lambda_{t \rightarrow t+1}^{ex\ ante} \equiv k \cdot \lambda_t$ is determined by the risk premium required for a one-period holding a currency. Here we assume $k = 1$.

As an illustration, the following chart shows that the intuition of the overall model for the dynamics of exchange rate:

Figure 2: Illustration of intuition on the dynamics of exchange market rate



[Referring to section 2.1.3] Flow chart of modelling the dynamics of the exchange rate. The exchange rate is determined by two components: (1) the forward-looking element that reflects market's expectation on the future status of economies. (2) the pre-belief element that reflects the investors'/economists'/econometricians' beliefs on the fair value (change) of the exchange rate at time t if no new information emerges.

2.2 Determinants of the risk premium

As the risk premium for currency holding is not observable, it would be hard to link it to other variables. However, there are some efforts in both theoretical and empirical literature that try to explain the determinants of the risk premium: Utility-based asset pricing models have explicitly shown that it depends on the bilateral growths of consumption and their risks (e.g. Verdelhan, 2010; Bansal and Shaliastovich, 2008). Arbitrage-free currency-bond yield factor models that have linked the risk premium to the factors that determines both exchange rate return and bond yields (e.g. Sarno, 2012a). Empirical work also link the risk premium with macroeconomic and policy uncertainties (e.g. Martin and Urrea, 2007).

In this paper, I will mainly focus on the framework that the risk premium depends on the factors of the bond yields². But being different from Sarno et al. (2012) who estimate bond-currency factors jointly and but get ambiguous factors as their in-sample prediction of bond yield is somehow unsatisfactory, the factors used in this paper are extracted from the affine term structure model that only explaining the bond yields. The reasons are given below:

Presumably, the risk premium depends on the investors' expectations of the perspective of the two economies, which are very hard to measure. But it is possible to find proxies for these expectations: factors for the term structures of interest rates. This is because: First, intuitively the term structure is a spectrum of interest rates that covers short-term, mid-term, and long-time rates, thus it naturally incorporates the information on the current status and expected development of the economies. Second, the interest rate are the underlying pricing factors for all asset prices in an economy, all returns, intertemporal changes in asset prices, are pinned down by the interest rate. Third, the interest rates are also financial prices, and move instantly with the new information on macroeconomic variables (whereas those variables themselves are usually persistent in values and measured with lags). Thus the information picked by the term structure of interest rates may also be perfect inputs that drive changes in exchange rate. Fourth, in the affine model for the term structure of interest rates, all the interest rates as well as the time-varying market price of risk can be explicitly modeled as function of latent factors, and one can estimate the observed interest rate very well with these factors. These characters of the model allow us to trust these factors and effectively utilize the information embedded therein.

1. Affine term structure model of interest rates

For simplicity, here I only show the most relevant equations of the affine term structure model, and leave the detailed modeling in a full-text version of this paper:

The observation equations are:

$$y_t = A + BF_t + \mu_t$$

Where $y_t = [y_{1,t}, y_{2,t}, \dots, y_{n,t}]'$ is a vector of observed bond yields at various maturities, and $F_t = [f_{1,t}, f_{2,t}, f_{3,t}]'$ is a vector of latent factors and assumed to follow a VAR(1) process:

$$F_t = \tilde{c} + \tilde{\rho}F_{t-1} + v_t$$

By some assumptions of the affine model and after some algebra, we can get the closed-form expression of bond yields as a function of factors:

$$y_{n,t} = A_n + B_n' F_t$$

where the coefficients are : $A_{n+1} = A_n + B_n(\mu - \Sigma\lambda_0) + \frac{1}{2}B_n'\Sigma\Sigma'B_n - \delta_0$ and $B_{n+1} = B_n(\phi - \Sigma\lambda_1) - \delta_1$.

And the risk-free rate is given by:

$$r_t = \delta_0 + \delta_1 F_t$$

2. The currency risk premium

Due to aforementioned reasons, it is natural to assume that the risk premium for holding a foreign currency also depends on the factors that are generated from the affine term structure

²The focus on the currency-bond factor model is because that the utility-based model find itself difficult to match the data due to the high persistence of consumptions; in alternative approaches such as Marin and Urrea (2004), the way of choosing instruments for macroeconomic and policy uncertainties are somehow arbitrary, thus it is difficult to find valid instruments that are universal for all currencies.

of interest rates. Thus without lose of generosity, the risk premium is *assumed*³ as a function of domestic and foreign factors.

$$\lambda_t = H(\tilde{F}_t) = (C + D\tilde{F}_t)' \tilde{F}_t$$

Here, $\tilde{F}_t = [F', F^{*'}]'$, and $H(\tilde{F}_t)$ can be a liner function ($C \neq 0_{6 \times 1}$, and $D = 0_{6 \times 6}$) or a higher order ($C \neq 0_{6 \times 1}$, and $D \neq 0_{6 \times 6}$) function of the factors. In addition, I assume that the matrix D is block diagonal such that there is no cross product between domestic and foreign factor in the second order terms.

In the next section, I will report results for regressions that include both linear and second order terms.

2.3 Pre-beliefs on the models for the exchange rate change: $e_{t-0} - e_{t-1}$

While there is no agreement on how the exchange rate changes intertemporally in the absence of new information, Economists, econometricians, or investors may hold different believes on the 'fair value' for the exchange rate, or the change of it. In the following, I list six cases that can potentially describe the popular believes, with a focus on those of monetary economists and time-series econometricians.

- M1: Purchasing power parity model (PPP)

Suppose the Purchasing Power Parity (PPP) holds continuously, the exchange rate depends on the differential of price levels.

$$e_t = p_t - p_t^*$$

then the assumed intertemporal exchange rate change is:

$$e_{t-0} - e_{t-1} = \Delta p_t - \Delta p_t^* = \pi_t - \pi_t^*$$

- M2: Flexible price monetary model (FP)

Suppose the Purchasing Power Parity holds continuously, and prices can adjust freely. When the money demand equals money supply:

$$m_t - p_t = \alpha_1 y_t - \alpha_2 i_t$$

The change in exchange rate is given by:

$$e_{t-0} - e_{t-1} = \Delta p_t - \Delta p_t^* = (\Delta m_t - \Delta m_t^*) - \alpha_1(\Delta y_t - \Delta y_t^*) + \alpha_2(\Delta i_t - \Delta i_t^*)$$

- M3: Uncovered interest parity model (UIP)

The Uncovered Interest Parity (UIP) model argues that the expected exchange rate depreciation equals to the interest rate differential:

$$e_{t-0} - e_{t-1} = i_{t-1} - i_{t-1}^*$$

- M4: Taylor rule model (TR)

³In the international finance literature, it can be shown that the risk premium can be modeled explicitly as function of those factors at least to the second order. However, those factors are estimated to jointly match data on exchange rate returns and yield curves for both countries. According to Sarno (2012), a good estimation of the exchange rate data is at the cost of poor match of the yield curves, thus the factors are no longer easily interpreted. As the priority of this paper is to look for the driving forces of the exchange rate, other than trying to find the best statistical estimation of the *ex ante* expected excess return and the expected exchange rate change. Thus I would prefer to keeping the original source of information unaltered, thus make it is easy to make economically meaningful and consistent explanations. In the appendix, details are offered on how to model the risk premium as function the factors

Suppose the Uncovered Interest Parity (UIP) holds, and the monetary policies in both domestic and foreign countries are assumed to follow the Taylor rule, in a symmetric setup⁴:

$$i_t = \rho_1 y_t^{gap} + \rho_2 \pi_t$$

Then the change in exchange rate is given by:

$$e_{t-0} - e_{t-1} = \rho_1 (y_{t-1}^{gap} - y_{t-1}^{gap*}) + \rho_2 (\pi_{t-1} - \pi_{t-1}^*)$$

- M5: Random walk model (RM)

The random walk model argues that the best prediction of the exchange rate is the one of the last period, thus the pre-belief of exchange rate change would be zero:

$$e_{t-0} - e_{t-1} = 0$$

- M6: Autoregressive distributed lags model (ARDL)

The Autoregressive Distributed Lags (ARDL) model is also a time-series model that is free of underlying economic theory. In this model, dependent variable is a function of lag terms of its own and some exogenous variables. In this paper, following the spirit of the UIP, we assume interest rate differential is the reasonable exogenous variable that drives the changes in exchange rate together with its own lags. The ARDL(p,q) model is:

$$e_{t-0} - e_{t-1} = \beta_0 + \beta_1 \Delta e_{t-1} + \beta_2 \Delta e_{t-2} + \dots + \beta_p \Delta e_{t-p} + \gamma_1 (i_{t-1} - i_{t-1}^*) + \dots + \gamma_q (i_{t-q} - i_{t-q}^*)$$

where p is the lag order of exchange rate change, and q is the one of interest rate differential.

In the next section, I will check which of the above models can fit the data best, and then can be used as the benchmark for the pre-belief component ($e_{t-0} - e_{t-1}$) of the overall exchange rate dynamics ($e_t - e_{t-1}$).

3 Empirical Results

3.1 Data

3.1.1 Data on exchange rates

In this paper, the US is treated as the home country, thus all exchange rates are defined as the amount of the US dollars that one unit of foreign currency can convert to at the foreign exchange market. In the analysis, monthly data are used for those countries. The following is a summary of available time periods and sources of the exchange rates.

⁴For simplicity, the parameters of the Taylor rule for both countries are assumed to be the same. In a popular setup in the literature, the Taylor rule should be asymmetric; also the central bank of foreign country explicitly targets the nominal exchange rate to its PPP level. As the primary goal is not to examine how the Taylor rule components, i.e. output gap and inflation, drives the exchange rate, but how well they can explain the variation of the change in exchange rate, the assumption of symmetric rules is feasible.

Table 1: Summary of Datasets for Exchange Rate

Country	Time Period	Source
US	n.a.	n.a.
Japan	1957.01-now	International Financial Statistics, IMF
Germany/EA	n.a.	n.a.
UK	1975.01-now	Bank of England
Canada	1957.01-now	International Financial Statistics, IMF
Switzerland	1957.01-now	International Financial Statistics, IMF
Norway	1960.01-now	Norges Bank
Sweden	1957.01-now	International Financial Statistics, IMF
Australia	1969.07-now	Reserve Bank of Australia
New Zealand	1957.01-now	International Financial Statistics, IMF

3.1.2 Data on macro fundamentals

For the macroeconomic fundamentals in the pre-believed models. Money base M0 is used for the money supply whenever it is available, otherwise narrow money M1 is employed as an alternative. Monthly Industry Production is used as proxy for the output/income. The output gap is obtained by detrended output with HP filter ($\lambda=129600$). CPI is used for the price index and for calculating inflation, which is defined as 12-month difference of CPI⁵. All variables are used in logs. All variables are drawn primarily from the IMF's International Financial Statistics, and supplemented by the 'Economic Indicators' of the OECD statistics.

3.1.3 Data on term structure of interest rates

Thanks to Wright's elaborate work (2011), an international panel dataset of the term structure of interest rates is publicly available and ready-to-use. The dataset contains zero-coupon non-callable government bond yields at all maturities starting from three month up to ten years with a 3-month increment, for ten major advanced economies. In the dataset, data are reported at a monthly frequency from the starting date to 2009.05. The following table gives a brief summary of the dataset:

Table 2: Summary of Datasets for Bond Yields

Country	Time Period	Source
US	1971.09-2009.05	Gürkaynak, Sack, and Jonathan H. Wright (2007)
Japan	1987.01-2009.05	Datastream and author's calculations
Germany/EA	-	-
UK	1979.01-2009.05	Nicola Anderson and John Sleath (2001)
Canada	1986.01-2009.05	Bank of Canada and BIS database
Switzerland	1988.01-2009.05	Swiss National Bank and BIS database
Norway	1998.01-2009.05	Norges Bank and BIS database
Sweden	1993.01-2009.05	Riksbank and BIS database
Australia	1987.02-2009.05	Datastream and author's calculations
New Zealand	1990.01-2009.05	Datastream and author's calculations

⁵The RPI (retail price index) is used for UK due to its availability in a longer period of time

3.1.4 Factor generation

Factors for the term structure of interest rates: In the recent macroeconomics and finance literature, the class of Gaussian affine term structure models (e.g. Duffie and Kan, 1996; Dai and Singleton, 2002) has been becoming the workhorse in modeling the bond yields at various maturities. According to the model setup, all variables, e.g. time-varying market price of risk, bond yields and term premia are driven by the latent factors of the economy. The model is usually estimated by algorithms such as maximum likelihood and minimum chi square. In this paper, I particularly refer to the latter method that is proposed by Hamilton and Wu (2012a) due to its time-efficiency. The factors are calculated utilizing the estimated parameters.

3.2 Empirical Evidence

3.2.1 The sample for estimation

Although the data on exchange rate, bond yields and macroeconomic fundamentals are available for long periods for most currencies, it is not a good idea to use the full samples as exchange rate models generally suffer from parameter instability problems, especially risks of structural change may emerge when a long sample is used. Take a few for example: For the case of GBP, the sample starts from 1979.01, but in 1992.09 the Bank of England changed its monetary policy from pegged pound sterling in the European Exchange Rate Mechanism to an inflation-targeting interest rate setting after the turmoil of Black Wednesday. Thus the sample of 1993.01-2009.05 is used for GBP. Similarly, the JPY entered into a phase of zero interest rate⁶ in the second half of 1995, thus the functioning of the monetary policy and economic activity is supposed to be different from previous periods. Hence the 1996.01-2009.05 period is used for the JPY. For some other reasons, the CHF and CAD are investigated in the period of 1996.01-2009.05 and 1999.01-2009.05 respectively. However, the truncation of these samples does not mean the model works badly during the periods that has been left out, but it should be taken as necessary steps to avoid parameters instability across periods of time. To see this, a robustness check is implemented in section 5 for various time periods that includes the full and sub samples of each currency.

3.2.2 Model selection on the pre-believed models

In this section, I investigate which of the pre-believed models that listed in section 2.3 is the best one in terms of fitting the realized data on exchange rate changes. Then the best model will be selected as the pre-believed model for the $e_{t-0} - e_{t-1}$ as mentioned in section 2.1.3. The combination of it with the risk adjustment component $e_{t+0} - e_{t-0} = -\lambda_t$ after taking into account the new information about the future status of the economies, gives the complete model for the exchange rate change.

The $Adj. - R^2$ s of the models for each of currency are reported in table 3:

————— insert Table 3 here —————

The model selection criteria used here is the simple $Adj. - R^2$. The motivation behind this is that the primary purpose of this paper is to investigate whether the risk premium term, as a function of bond yield factors, can explain a negligible portion of the variation in exchange rate change on top of that can be maximums explained by the existing economic or time-series models, rather than to find the best model with more rigid statistics criteria. Thus the $Adj. - R^2$ can be taken as a proper criteria to meet the requirements.

As shown in table 3, the M6: ARDL model fits the data best for five out of eight currencies in selected periods. The M2: Flexible price monetary model is the second best model and it fits the data best for the rest three currencies. Simple numerical comparison would suggest that the ARDL model should be selected as the benchmark of the pre-belief on the exchange rate change, $e_{t-0} - e_{t-1}$.

⁶ A phase of zero interest rate is defined as the headline monetary policy rate, 3-month interest rate, goes and has been stayed below 1%.

There are several other reasons for the preference of the ARDL models v.s. the Flexible price model: First, data on macroeconomic fundamentals are released with time lags, and often subject to adjustment in the future. Thus when it comes to the forecast, the availability of these data in real time and also the quality of them are not guaranteed. Second, the fundamentals are found to offer little contribution to a more accurate forecast for the change in exchange rate when comparing to a naive benchmark random walk model (e.g. see Engle, 2005). Third, when the bond yield factors are added into the pre-believed models, the model based on Flexible price model are found to underperform the one based on the ARDL model, especially for the cases of JPY and CHF⁷. Fourth, the macroeconomic fundamentals are barely significant once included in the regression together with the contemporaneous factors, as will be examined by a robustness check exercise in section 5.

Now, combining the ARDL model and the risk premium adjustment, $e_{t+0} - e_{t-0}$, the ultimate model used in this paper that describes the dynamics of exchange rate change is:

$$\Delta e_t = \beta_0 + \beta_1 \Delta e_{t-1} + \dots + \beta_p \Delta e_{t-p} + \gamma_1 (i_{t-1} - i_{t-1}^*) + \dots + \gamma_q (i_{t-q} - i_{t-q}^*) + \underbrace{(C + D\tilde{F}_t)' \tilde{F}_t}_{\lambda_t} + u_t \quad (1)$$

3.2.3 The first step when using the ARDL model: Testing a long-run relationship

Before running the regression, it is necessary to test whether there is a long-run relationship between the exchange rate change and the interest rates differential in the ARDL model. If there is one, it indicates that in equilibrium the Δe_t is expected to change at a constant value in response to one unit change in $i_{t-1} - i_{t-1}^*$ in the long-run. If a long-run relationship is detected, it would be more proper to use the error correction (EC) representation of the ARDL model⁸.

After estimating the ARDL model, the Wald test can be used for testing the null hypothesis:

$$H_0 : \quad LRC = \frac{\gamma_1 + \dots + \gamma_q}{1 - \beta_1 - \beta_2 - \dots - \beta_p} = 0$$

Where the LRC stands for the long-run coefficient for a potential long-run relationship $E[\Delta e_t] = LRC \cdot E[i_{t-1} - i_{t-1}^*]$. Results for tests of the null suggest that 1) in general, there is no long-run relationship in the truncated part of the sample for all of the currencies. 2) When full samples are considered, long-run relationships are found to exist for the GBP and JPY. Thus in the following, unless otherwise stated, only the original ARDL model will be used for estimation, forecast and policy analysis.

3.2.4 Regression results and the relevance of risk premium.

In this paper, there are two important questions to be answered:

- a) Whether the risk premium that based on expectations on the future status of economies (embodied in bond yield factors) is relevant for the dynamics of the exchange rate?
- b) Analytically, whether the risk premium term is a linear function of the factors or it should also include the higher (second) order terms?

To answer the first question, one would check in the pre-assumed function $\lambda_t = (C + D\tilde{F}_t)' \tilde{F}_t$ ⁹, whether the estimates of vector C or the joint estimates of vector C and matrix D is significantly different from zero.

To answer the second question, one would check whether the goodness of fit is sufficiently enhanced by inclusion of the second order factor terms in comparison with the case of only first order is considered, conditional on the significance of the vector C or/and joint significance of the

⁷For the case of JPY (CHF), only 10.0% (9.73%) of variation in exchange rate change can be explained by the Flexible Price + factor model, whereas the counterpart explained by the ARDL + factor model is 20.88% (20.59%). The numbers for the NOK is moderate: 12.37% v.s. 14.32%.

⁸More details for the EC representation of the ARDL model are shown in the Appendix

⁹For simplicity, the interaction between factors across countries in matrix D are suppressed.

vector C and matrix D . The measurements of goodness of fit are the *adjusted- R^2* and Akaike Information Criteria, *AIC*. The former measures how much variation in the dependent variable can be explained by the independent variables. The higher the *adjusted- R^2* , the better the model fits the data. While the latter measures the information loss of a given model when it is used to represent the date generating process of the independent variable, and it make a balance of the goodness of fit and the complexity of the model. For this measurement, the lower a value is, the better the model fits the data.

In the following, I provide a set of results that allow one to assess how well the *ARDL + factor* models fit the data.

———— insert Table 4 here ————

———— insert Table 5 here ————

———— insert Table 6 here ————

———— insert Table 7 here ————

As shown in the above tables, for all of the currencies, the null hypothesis that of $C = 0_{6 \times 1}$ ($D = 0_{6 \times 6}$) for the *ARDL + $F^{1^{st}}$* (*ARDL + $F^{1^{st}, 2^{nd}}$*) is strongly rejected. This indicates that the factors, either in a linear form or in a non-linear one, are joint significant. This further suggests that our presumed risk premium indeed determines the dynamics of exchange rate.

It also worth noting that the goodness of fit is greatly enhanced by including the factors (risk premium) into the regression. The variation of exchange rate that can be explained by the non-linear factor model ranges from 17.73% to 35.33%. This is sufficiently higher than that has been reported in a non-linear approach to explain the UIP Puzzle by Sarno, et. al. (2006), which ranges from 3.7% to 17%.

The above evidence suggests that the risk premium, expressed in a function of bond yield factors, does influence the dynamics of the exchange rate, and contribute enormously to the variation of the exchange rate change. Especially, this seems to be a common feature for almost all the currencies. Thus we can take it as strong evidence that the risk premium should not be neglected in exchange rate models. And this view will be further supported by the forecast performance of the models with risk premium terms.

When it comes to the question that whether the risk premium should be in a linear or non-linear form of the factors, it is obvious for most currencies that the second order terms of factors should be included, as 1) they are jointly significant, 2) greatly improve the goodness of fit as measured by both *adjusted- R^2* and *AIC*, in comparison with the one which only first order terms are included. One exception is the GBP, as the *AIC* for the non-linear model is than that of the linear model, but as the two values are very close, and the joint significance of the second order terms are very strong, I take these results as evidences that there is no clear cut for the case of GBP during the reported period. But to keep its model consistent with other currencies, the performance of non-linear model is still examined in the following sections.

The non-linearity of the risk premium in the factors indicates that the way factors could affect exchange rate dynamics is state dependent. This is also the rational underlying the pre-assumption $\lambda_t = (C + D\tilde{F}_t)' \tilde{F}_t$, where the $(C + D\tilde{F}_t)'$ is effectively a time-varying coefficient of the factor vector \tilde{F}_t . I will discuss this further in the policy analysis section of the next section.

4 Forecasts and Policy Analysis

4.1 Exchang rage forecasting

The primary purpose of this paper is to improve our understanding of the behavior of the exchange rate and to test the assumption that its risk premium component should be non-linear in latent

factors. As shown in the section 3, non-linear model is found to fit the data better than the non-factor as well as its linear counterparts. However, as it is documented in studies by Shinn (2012) and Engle (2013), while evaluating the models of exchange rates by their in-sample fit remains to be valuable, ever since the study of Messe and Rogoff (1983), the 'Golden Rule' to do so has been shifted towards their usefulness in forecasting exchange rates.

Following this guideline, in this section I will check the performance of the proposed non-linear model in forecast accuracy, in comparison with its linear counterparts, and naive benchmark models. If this model can also be used as predictive models for the exchange rate changes/returns, it would shed further light on its ability to explain exchange rate movements over time.

Before proceeding further, it may prove worthwhile to emphasize that I focus on the pure time-out-of-sample forecast as the basis of judging the relative advantages of the models¹⁰. That being said, unless otherwise stated, non contemporaneous values of the right-hand-side variables are used to predict future exchange rates.

4.1.1 Models for Forecasting

The models used for forecasting and comparison are again the same as the ones used in regressions:

The best performance time series model, *ARDL*:

$$\Delta e_t = \beta_0^0 + \beta_1^0 \Delta e_{t-1} + \cdots + \beta_p^0 \Delta e_{t-p} + \gamma_1^0 (i_{t-1} - i_{t-1}^*) + \cdots + \gamma_q^0 (i_{t-q} - i_{t-q}^*) + u_t^0$$

ARDL model with linear factors, *ARDL* + F^{1st} :

$$\Delta e_t = \beta_0^1 + \beta_1^1 \Delta e_{t-1} + \cdots + \beta_p^1 \Delta e_{t-p} + \gamma_1^1 (i_{t-1} - i_{t-1}^*) + \cdots + \gamma_q^1 (i_{t-q} - i_{t-q}^*) + C^1 \tilde{F}_t + u_t^1$$

ARDL model with non-linear factors, *ARDL* + $F^{1st, 2^{nd}}$:

$$\Delta e_t = \beta_0^{1,2} + \beta_1^{1,2} \Delta e_{t-1} + \cdots + \beta_p^{1,2} \Delta e_{t-p} + \gamma_1^{1,2} (i_{t-1} - i_{t-1}^*) + \cdots + \gamma_q^{1,2} (i_{t-q} - i_{t-q}^*) + (C^{1,2} + D^{1,2} \tilde{F}_t)' \tilde{F}_t + u_t^{1,2}$$

The naive¹¹ benchmark Random Walk model, *RM*:

$$\Delta e_t = \eta_t^{RM}$$

Although the exchange rate change depends on contemporaneous value of factors, it is still possible to make forecast as the dynamics of factors is exogenously given by $F_t = \tilde{c} + \tilde{\rho} F_{t-1} + v_t$, and $F_t^* = \tilde{c}^* + \tilde{\rho}^* F_{t-1}^* + v_t^*$ and it can be shown that the second order terms of factors satisfies $f_{i,t} \cdot f_{j,t} = F_{t-1}' Q_{ij} F_{t-1}$, and $f_{i,t}^* \cdot f_{j,t}^* = F_{t-1}^{*'} Q_{ij}^* F_{t-1}^*$. The interest rates follows $r_t = \delta_0 + \delta_1 F_t$ and $r_t^* = \delta_0^* + \delta_1^* F_t^*$. Based on the estimates of coefficients α , β s, γ s and C s and D s, and forecasts of factors in the first and second order, one can obtain the one-period ahead forecast on exchange rate changes. Accumulating one-period ahead forecasts for certain horizons, one may further obtain longer period ahead forecasts.

One-period ahead forecast experiment is carried out for all currency pairs. The convention of implementing 'rolling regressions' is adopted here¹². Being consistent with the estimation exercises in section 3, the first estimation window starts from 1990.01, 1993.01, 1996.01, 1998.01 and 1999.01 respectively, and ends at 2004.05¹³, thus the size of estimation window varies across currencies. The estimation window rolls forward until the last period of 2009.05. Thus the forecast period

¹⁰ The time- $(t+h)$ are used as complementary evidence once necessary

¹¹ Naive model may refer to difference meanings: (1) no-change model, such as the Random Walk model, which will be used as the benchmark when evaluating forecast accuracy by the criterion of RMSE. (2) equal-chance of upward and downward change, which will be used as the benchmark when evaluating forecast accuracy by the criterion of direction of changes.

¹² Exceptions are for the case of SEK and CAD, in which cases the parameter instability of the rolling regression might be problematic

¹³ For NOK and CAD, it ends at 2005.05 to guarantee a thumb rule that the estimation window takes 2/3 of full sample size, and the forecast window takes the rest 1/3. Thus there are 48 (36) period of forecast at 1- (12) horizons.

for the 1- (12-) period ahead horizon is 60 (48) months. The forecast setup for each currency is summarized in the following table:

Table': Summary of the one-step ahead forecast setup

	report	window_size	rolling	Horizon	Forecast period	Forecast_size
AUD	90.01-09.05	173	1	1-12	2004.06-2009.05	60
NZD	90.01-09.05	173	1	1-12	2004.06-2009.05	60
GBP	93.01-09.05	137	1	1-12	2004.06-2009.05	60
SEK	93.01-09.05	Growing	0 ¹⁴	1-12	2004.06-2009.05	60
CHF	96.01-09.05	101	1	1-12	2004.06-2009.05	60
JPY	96.01-09.05	101	1	1-12	2004.06-2009.05	60
NOK	98.01-09.05	90	1	1-12	2005.06-2009.05	48
CAD	99.01-09.05	Growing	0	1-12	2005.06-2009.05	48

Note: The regime of forecasting considered here is the pure time-t information forecast $\widehat{\Delta e_{t+h}}|I_t$.

I evaluate predictive performance in two ways. First, it is the root of mean squared errors. A smaller value indicates a better performance of the model. Inferences are based on the Clark and West (2005) test. Second, the direction of change statistics, which is the probability of correct predictions of direction which is computed as divide its value by the total number of predictions. A value above 50% indicates a better performance in forecasting than a naive model that have predicts even probability in downward or upward changes. The Diebold and Mariano (1995) test will be used for inferences.

4.1.2 Forecast Comparison

The following tables report the forecast results for all currencies at horizons from one month to twelve months. The numbers in bold indicate the best forecast at each horizon.

———— insert Table 8 here ————

———— insert Table 9 here ————

———— insert Table 10 here ————

———— insert Table 11 here ————

———— insert Table 12 here ————

- The RMSE criterion

As is shown in the left panel of each table, where the forecast accuracy is measured by the RMSE, the $ARDL + F^{1^{st}, 2^{nd}}$ model can beat the random walk in the following cases: H= 1-3 for GBP, H=1 for NZD, H=6-10 for JPY, H=1 for NOK and H= 1-2 for SEK; for cases of AUD, CAD and CHF, $ARDL + F^{1^{st}, 2^{nd}}$ cannot outperform the random walk model.

However, for the cases of GBP and NZD at H=1-4, the $ARDL + F^{1^{st}, 2^{nd}}$ model are not the best forecast model, instead, the $ARDL + F^{1^{st}}$ model is. This indicate that the $ARDL$ augmented with first order factors has the least deviation from the true values as compared with other models. It also suggests that the second order terms *may* bring extra noises that drive the forecast value deviate from the true value *further* than the first order terms do, which reduces the creditability of the $ARDL + F^{1^{st}, 2^{nd}}$ model. Nevertheless, it is implausible to draw a conclusion based on above evidence. The reasons are discussed below.

As has been pointed by Cheung et al. (2005), the RMSE measurement of accuracy has its own shortcomings: It only measures the distance that forecasts deviate from the true values, but

ignores whether the forecasts are in the right direction. Studies with simulation exercise have already shown that even if forecasts are in the wrong direction for *all* periods, it can still beat the benchmark random walk model if measured by the RMSE criterion.

To avoid drawing improper conclusions based on spurious evidence, I also check the accuracy of forecast with an economic measure, which is the probability of changes in the right direction.

- The direction of change criterion

As is shown in the right panel of each table, if the forecast accuracy is measured by the direction of change, the $ARDL + F^{1^{st}, 2^{nd}}$ model can beat the benchmark naive model that issues equal probability on upward and downward changes, in the following cases: H=1-12 for GBP, H=1-8 for SEK, H=1-4 for AUD, H=1-5 for NZD, H= 2-12 for JPY, H=2, 5, 9 for CHF, H=1-12 for NOK and H=1 for CAD.

One may see that the $ARDL + F^{1^{st}, 2^{nd}}$ model can beat the benchmark naive model at a greater success: in a broader range of horizons and for a bigger set of currencies. In particular, for most of the cases the $ARDL + F^{1^{st}, 2^{nd}}$ model acts as the best forecast model. The only exceptions are at the H=1-2 for GBP, H=1 for NZD and JPY, H= 1-3 for CAD, where the $ARDL + F^{1^{st}}$ model is the best forecast model but the marginal increment is moderate in comparison with the $ARDL + F^{1^{st}, 2^{nd}}$ model.

Discussion: The forecast performance of the $ARDL + F^{1^{st}, 2^{nd}}$ model for the AUD, CAD and CHF are unexpectedly poor. The reasons might be: 1) the forecasts of the factors deviate from their true values to a large extent, or 2) this model is not a good one for these currencies, or 3) if the model is a valid one, its parameters may be highly unstable. To see whether it is because of the reason 2) or 3), I use the observed value of the factors, i.e. utilizing time- $(t + h)$ information, and repeat the forecast exercises. The results turn out that, as shown in table 12, if measured by the RMSE, the $ARDL + F^{1^{st}, 2^{nd}}$ model outperforms the RM model at H=1-10 horizons for AUD; and if measured by the change in right direction, the $ARDL + F^{1^{st}, 2^{nd}}$ model outperforms the RM model at H=1-12 horizons for AUD, H=4, 5-12 for the CAD. And in most of the cases, the $ARDL + F^{1^{st}, 2^{nd}}$ model is the best forecasting model. Although the factor models still perform poorly in forecasting for the case of CAD if measured by the RMSE criterion, they are able to outperform the time series model, ARDL, and the benchmark naive model if measured by the changes in the right direction criterion.

Based on above observations, we can conclude that for the cases of AUD and CAD, the mild forecast performance is very likely due to the poor forecast of the bond yield factors, rather than the failure of the model itself.

However, the forecast performance of $ARDL + F^{1^{st}, 2^{nd}}$ and $ARDL + F^{1^{st}}$ model for the CHF are very poor even if the time- $(t + h)$ information is used. This indicates that either the factor models are not proper models for the CHF in forecasting or the parameters are highly unstable even if this can be partly relieved by the rolling estimation.

4.1.3 Sub-conclusion for forecast exercises

The above forecast exercises show evidence that based on the pure time- t out-of-sample forecast and supplemented by the time- $(t + h)$ information forecast, the factor models ($ARDL + F^{1^{st}}$ and $ARDL + F^{1^{st}, 2^{nd}}$) in general outperforms the time series model $ARDL$ and naive models at short horizons that ranges from one month to twelve month for multiple currencies, as measured by the RMSE. The evidence is even stronger if the direction of change metrics is used for evaluating the accuracy of forecasts. Under this criterion, the $ARDL + F^{1^{st}, 2^{nd}}$ model is found to significantly outperform the $ARDL + F^{1^{st}}$ model for most of the cases. This indicates that the second order terms when combined with the first order ones brings important information about the direction of changes, despite the fact that it may also bring extra deviation from the true values. We can further interpret this finding as an evidence indicating that the RMSE criterion alone is not a good measure of forecast accuracy as it overlooks an important feature of forecasts, which is whether the change is in the right direction.

Above findings suggest that all the factors, especially in their second order, play an important role in capturing the actual dynamics of the exchange rate, as firstly evidenced by the goodness

of fit of the factor models, and then further strengthened by their forecast accuracy. This justifies our motivation of considering nonlinearity of the risk premium in factors.

4.2 Policy Analysis

It is of great interest to learn how the exchange rate responds to a monetary policy adjustment. In the present context, it is possible to conduct such a policy analysis as the 3-month government bond yields for the advanced economies, 1) are nearly the same as the 3-month money market interest rates, and the latter are usually taken as proxies that represent the monetary policies, and 2) can be expressed in terms of the latent factors *analytically* thanks to the convenience of the arbitrage-free affine terms structure model for the bond yields. Accordingly, as will be shown below, a monetary policy adjustment can be transformed to (or expressed in) shocks to the latent factors. That being said, an unexpected monetary adjustment is treated as actually being induced by the same underlying source of shocks¹⁵ as the unexpected adjustments in latent factors are, but it loads the shocks only in a certain way. The Cholesky decomposition is employed for identifying the shocks on short-, mid- and long-term interest rates, of which the short-term one is relevant for monetary policy analysis.

4.2.1 Transforming policy shocks to factor shocks

Recall that in the affine term structure models, yields can be expressed as $y_t = A + BF_t + \mu_t$. In such models, the policy rates ($y_t = (i_t^{3m}, i_t^{12m}, i_t^{10y})'$) can be assumed to be observed without measurement errors; and the factors follow a VAR(1) process: $F_t = \tilde{c} + \tilde{\rho}F_{t-1} + v_t$, where \tilde{c} is assumed to be zero. The above two assumptions are standard setups for the estimation of affine models (e.g. see Hamilton and Wu, 2010).

Substituting factors with yields in the VAR(1) model for factors, we get: $B^{-1}(y_t - A) = \tilde{\rho}B^{-1}(y_{t-1} - A) + v_t$.

Rearranging above equation, we have: $y_t = B(A - \tilde{\rho}B^{-1}A) + B\tilde{\rho}B^{-1}y_{t-1} + Bv_t$

This is a VAR(1) model for the yields, with error term $\eta_t = Bv_t$, and $\text{var}(\eta_t) = B'B$.

Given that the interest rates in vector y_t are short-, mid- and long-term bond yields, it is possible to identify the shocks with a Cholesky decomposition: $\eta_t = C\epsilon_t$, where C is a lower triangular matrix and satisfies $C'C = \text{var}(\eta_t)$; ϵ_t is the identified shock vector for short-, mid- and long-term rates, respectively. The intuition behind this arrangement is that contemporaneously, longer maturity bond yields tend to be affected by the shorter maturity bond yields, but not *vice versa*¹⁶.

By $Bv_t = C\epsilon_t$, we can get $v_t = B^{-1}C\epsilon_t$. This equation shows the way that how a shock on the short-/mid-/long-term interest rate (element in ϵ_t) can be transformed into shocks to the factors, which further allows us to make policy analysis through those factors.

4.2.2 Counter-factual study on monetary policy shocks

In the following, I carry out a counter-factual perturbation study in which, at each point of time, I allow a further one standard deviation change in the short term interest rates (3-month) on top of the one that has already realized in the data, and see how much further change in the exchange rate will be induced. This exercise is similar to but not exactly the same as the impulse response analysis¹⁷.

The responses are calculated based on equation 1:

$$\Delta e_t = \beta_0 + \beta_1 \Delta e_{t-1} + \dots + \beta_p \Delta e_{t-p} + \gamma_1 (i_{t-1} - i_{t-1}^*) + \dots + \gamma_q (i_{t-q} - i_{t-q}^*) + \underbrace{(C + D\tilde{F}_t)' \tilde{F}_t}_{\lambda_t} + u_t$$

¹⁵ All economic shocks that may affect the whole spectrum of bond yields.

¹⁶ However, the yield on longer maturity bond can still impact the its shorter maturity counterpart with a time lag, as they follow a VAR(1) process.

¹⁷ As for the impulse response, a shock is defined as the deviation from its expected equilibrium value. In the present context, I investigate the effect of a perturbation, which is defined as deviation from its realized value. The latter is similar as a policy-rate elasticity of exchange rate.

Given the presence of non-linear factor terms, the exchange rate change responses to a policy perturbation (analogous to a shock) tends to be state-dependent and time-varying. As an illustration, given a shock vector v_t to the factors, the $t = 0$ change in the second order term $\Delta(f_{i,t|v_t} \cdot f_{j,t|v_t})$ depends on the realized state variables $f_{i,t}$ and $f_{j,t}$:

$$\Delta(f_{i,t|v_t} \cdot f_{j,t|v_t}) = (f_{i,t} + \nu_{i,t}) \cdot (f_{j,t} + \nu_{j,t}) - f_{i,t} \cdot f_{j,t} = (f_{i,t} \cdot u_{j,t} + f_{j,t} \cdot u_{i,t}) + \nu_{i,t} \cdot \nu_{j,t}$$

This is the central difference between the non-linear and linear analyses, as the latter¹⁸ generates homogenous initial responses for all periods in real time, while the former gives different responses at each point of time, depending on the state of the economy that reflected by the bond yield factors, F_t .

4.2.3 Results

The **concurrent** responses (similar to the elasticity) of exchange rate change following a one-standard-error monetary policy adjustment are calculated for each point of time in history. The monetary policy is proxied with the 3-month government bond yield. In the following, Figure 3 and 4 depict the exchange-rate-change responses to a one standard-deviation change in the U.S. and foreign monetary policy rate¹⁹.

Exchange rate responses to extra US monetary policy adjustment

———— insert Figure 3 here ————

Exchange rate responses to extra foreign monetary policy adjustment

———— insert Figure 4 here ————

As shown in the above figures, due to the existence of non-linear factor terms, the instant response of exchange rate change to a monetary policy 'shock', tends to be time-varying. For example, a one-standard-error increase in the domestic (U.S.) monetary policy rate on top of its realized value does not induce homogenous responses in exchange rate changes over time. In contrast, it may cause an instant appreciation or depreciation of the domestic currency (USD) depending on the state of economy (which is reflected by the bond yield factors). The same results apply for the responses to foreign monetary policy adjustments. The figure 3 and 4 show the cases for the seven advanced-economy currencies against the USD.

5 Robustness check and interpretation of the risk premium

5.1 Robustness check

5.1.1 Robustness at different time horizons

To check whether the risk premium, in the second order of factors, is significant only at the reported sample periods or it holds more generally, I redo the regressions for all currencies at four alternative sample periods: 1) Full Sample, which takes all available period into account. 2) Early Days, which start from the very beginning of a sample and stops around the mid of the sample. 3) Common, which includes all the common periods for all currencies, i.e. 1998.01-2009.05. 4) No-Crisis, which contains all the common periods but excludes the recent financial crisis period. The results are reported in the following table, together with the ones for reported periods.

———— insert Table 13 here ————

¹⁸Conventional linear analysis such as VAR or VECM.

¹⁹Here I treat the interest rates across countries as independent to each other. In an extended version of this paper, the interaction between interest rates will be considered explicitly.

There are two criteria for the robustness check: (i) significance of the risk premium (expressed in factors) in determining the exchange rate change, and (ii) gains in goodness of fit, in comparison with the *ARDL* model, as measured by the *AIC*²⁰.

In the Full Sample case, most currencies meet these two criteria and get a 'y', which indicates that the above two criteria are satisfied, except for the CHF, JPY and CAD. An 'n' for the CHF, JPY and CAD means that the gains in goodness of fit are negligible for those currencies in their full samples, thus factors are redundant in these cases. However, if one splits the full samples and check the resulted sub-samples named as Early Days and the Reported periods, she can see that for CHF and CAD, the two criteria are again met separately in each of the sub-samples. This indicates that the redundancy of factors in the full sample cases may be arise from the instability of parameters across sub-samples. In contrast, we still get an '~n' for the JPY on Early Days, which indicates that it is only a recent phenomenon for the proposed $ARDL + F^{1st,2nd}$ model to well explain the dynamics of JPY.

In the Common sample between 1998.01 and 2009.05 for all currencies, we can see that the proposed $ARDL + F^{1st,2nd}$ model works very well for most currencies. Less strong evidence is obtained for the AUD, NZD and CAD. A '~y' for these currencies shows that the goodness of fit as measured by the *AIC* for the $ARDL + F^{1st,2nd}$ model is significantly better than the one for *ARDL* model, but *not* better than its counterpart for the $ARDL + F^{1st}$ model. This indicates that a risk premium, which includes the first order factors *only*, can perform as good as the one counts the factors in second order, thus it may *not* be necessary to go to the second order at this period of time for these currencies.

Furthermore, conditional on the Common sample for all currencies, I explicitly exclude the recent financial crisis period during 2008.02-2009.05, to check whether the above findings are merely a result of crisis-time phenomenon or they hold in a more general sense. Except for the AUD and NOK, results for the other currencies are still robust. Less satisfactorily, a 'y-' for the NOK means that the although the two criteria are met, the goodness of fit as measured by the *AIC* for the $ARDL + F^{1st,2nd}$ model is just as good as the *ARDL* model. In this sense, the inclusion of the risk premium to the second order only brings moderate gain in fitting the data. However, when the *AIC* is complemented by the *adj. - R²*, the gain is found to be substantial.

For the AUD, neither $ARDL + F^{1st,2nd}$ nor $ARDL + F^{1st}$ works well at the no-crisis time and in early days, which indicates that the factor models can properly capture the dynamics of AUD *only* when the recent crisis periods are included.

To sum up, we can conclude that despite of a few exceptions, the $ARDL + F^{1st,2nd}$ model fits the data best and the second order factors in the risk premium term are proven not to be redundant, as suggested by the multi-currency and multi-period evidence.

5.1.2 Robustness under model specification with macroeconomic fundamentals

In section 3.2.2, the *ARDL* model is selected as the best-fit pre-believed exchange rate model when comparing to monetary economic models for the exchange rate. Accordingly, all the conventional macro fundamentals are absent in the working model of this paper, the $ARDL + F^{1st,2nd}$ model. A natural question to ask is that, whether the significance of the factors is still robust once these macro fundamentals are included? Or put differently, whether the impacts of factors on exchange rate are merely perfect substitutes for the ones of macro fundamentals? In this section, I will check the robustness of the reported results by putting back all the relevant macroeconomic variables that have been used in section 3.2.2.

The results are reported in the following tables.

————— insert Table 14 here —————

————— insert Table 15 here —————

²⁰ Here I use the *AIC* as the measurement of goodness of fit, because it also punishes the attempt to add more regressors into the model. Thus it is more informative than the *adj. - R²*, as the latter generally increases when more regressors are included in a regression.

———— insert Table 16 here ————

———— insert Table 17 here ————

As one can see, for all of the currencies the significance of factors is still very *strong* even if the macroeconomic fundamentals are included in the regression. But the marginal gain in goodness of fit from the inclusion of the fundamentals differs across currencies: for the AUD, GBP, NOK and NZD, no gain at all; for the CAD, CHF, JPY and SEK, the marginal gains are moderate. It is worth noting that for the latter four currencies, the macroeconomic fundamentals are jointly significant, which suggests that these variables should not be jointly excluded in a fully specified model for the exchange rate.

As for the significance of an individual macroeconomic fundamental, a mixed picture is obtained: for the differential of money supplies, it is relevant for the JPY and CAD; for the contemporaneous interest rate differential, it is significant in the cases of AUD, CHF and SEK; for the Taylor rule components, the lagged output gap differential is relevant for the NZD and JPY, while the lagged inflation differential is important for the CAD and JPY.

Above results suggests that macroeconomic fundamentals, either individually or jointly, still play a role in explaining the dynamics of exchange rate. However, as the primary goal of this paper is to investigate whether the information embedded in the term structure of interests rates are relevant for the exchange rate dynamics, rather than to find the best specification of a exchange rate model; Moreover, also the values and significance of the coefficients of the factor terms are found to *barely* change after including the macro fundamentals, thus it is safe to stick to the working model - $ARDL + F^{1st, 2nd}$ - in the whole paper, even if the model can be mis-specified and the coefficients of factors, to a subtle extent, can be biasedly estimated. The main results reported so far can still hold even when the macro variables are putting back. In the following section, the $\Delta(i_t - i_t^*)$ will be brought back when necessary for certain currencies such as the AUD, CHF and SEK.

5.2 Interpretation of the risk premium

In the previous section, we have seen plenty of evidence that suggests the risk premium, which is non-linear in factors, plays an important role in determining the dynamics of exchange rate. However, even though the existence of a link between the risk premium and factors has been verified, we still have limited understanding of what exactly influences the risk premium, as the factors are merely statistical variables and have no economic meanings. Some researchers, e.g. Chen and Tsang (2013), Dewachter and Iyrio (2006) show that the 'level' factor can be linked to inflation expectations, and the 'curvature' factor can be connected with output growth expectations or business cycle. But those variables themselves are weak explanatory variables for the risk premium and the dynamics of exchange rate, as evidenced in the previous section, which is also in line with the failure of monetary exchange rate models that have been documented in many empirical studies. In this section, I will take an alternative approach, which utilizes the intrinsic connection between factors and macro economic variables, to understand the economic determinants of the currency risk premium.

5.2.1 Transforming factors to expectations and uncertainties:

One special feature that attracted little attention in previous currency-bond studies is that, latent factors can be further 'transformed' to economically meaningful variables which reflect market expectations regarding the current and future status of the economy. Those variables, such as expected short-term rate and term premium can be obtained in a closed-form of bond yields and factors after the dynamics of latent factors has been estimated.

After applying some simple algebra, the expected short-term risk-free rate can be obtained:

$$E[r_{t+i}] = \delta_0 + \delta_1 E_t[F_{t+i}] = \delta_0 + f(\delta_0, \delta_1, \tilde{c}, \tilde{\rho}) + \delta_1 \tilde{\rho}^i F_t$$

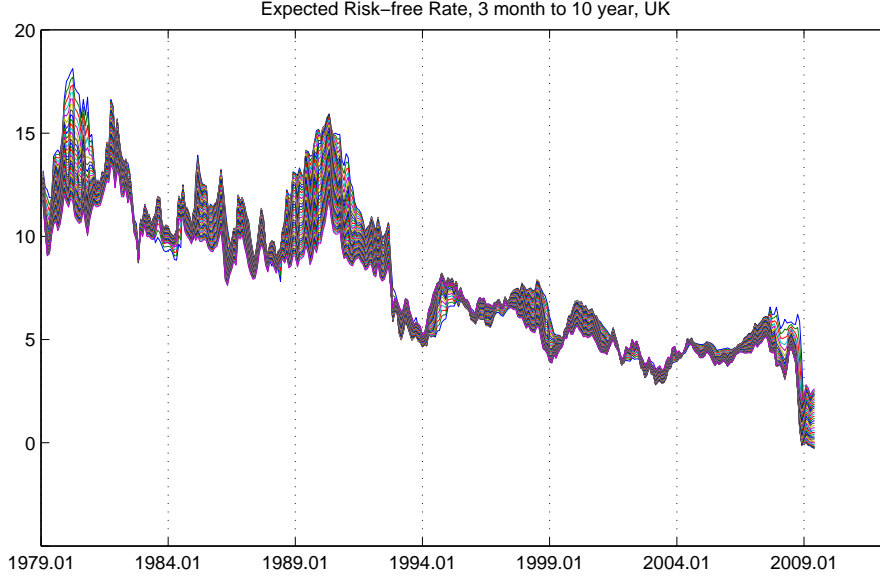
where the $f(\delta_0, \delta_1, \tilde{c}, \tilde{\rho})$ is a constant and is a function of the structural parameters. The term premium for holding longer term bonds is given by:

$$\tau_{n,t} = y_{nt} - \frac{1}{n} E_t \sum_{i=1}^n r_{t+i}$$

Where y_{nt} is the observed yield for bond that matures at $t + n$.

The following figure depicts the estimated risk-free rates for the UK range from 3 month to 10 year in the future, at each point of time from 1979.01 to 2009.05.

Figure 5: Distribution of expected risk-free rates of the UK



[Referring to section 5.2.1] Table 5: Expected short-term interest rates for the UK. At each point of time from 1979.01 to 2009.05, there are 40 expected rates that range from 3 month to 10 year in the future. The rates are calculated from the latent term structure factors.

5.2.2 Model-implied expectations and uncertainties vs. the exchange rate dynamics

As one can see from the figure 5, at a certain point of time, values of expected short-term rates vary with forecast horizons. Most likely, both the level and the volatility of these expectations may capture information about investors' perception of the perspective of economic activity. For example, harmonized expected rates (less volatile) may have different implications for the exchange rate compared with dispersed rates (more volatile). The term premia may also convey a certain type of information about the future.

In the following, I treat the means and variances of the expected risk-free rate and term premium for both domestic and foreign countries as determinants of the currency risk premium, then put them into the *ARDL* model and check whether they can impact the dynamics of exchange rate.

Redefine $\lambda_t = \tilde{C}'\tilde{R} + \tilde{D}'\tilde{\Sigma}$, where $\tilde{R} = [\bar{r}_{t,...,t+m|t}^*, \bar{r}_{t,...,t+m|t}, \bar{t}p_{t,...,t+m|t}^*, \bar{t}p_{t,...,t+m|t}]'$ is a vector that contains the means of expected risk-free rate and term premium for both domestic and foreign countries, $\tilde{\Sigma} = [\sigma_r^*, \sigma_r, \sigma_{tp}^*, \sigma_{tp}]'$ is the corresponding variance vector, \tilde{C} and \tilde{D} are matrices of coefficients.

$$\Delta e_t = \beta_0 + \beta_1 \Delta e_{t-1} + \dots + \beta_p \Delta e_{t-p} + \gamma_1 (i_{t-1} - i_{t-1}^*) + \dots + \gamma_q (i_{t-q} - i_{t-q}^*) + \underbrace{(\tilde{C}'\tilde{R} + \tilde{D}'\tilde{\Sigma})}_{\lambda_t} + u_t \quad (2)$$

The regression results of *equation 2* are given as follows:

————— insert Table 18 here —————
 ————— insert Table 19 here —————
 ————— insert Table 20 here —————
 ————— insert Table 21 here —————

As shown above, the risk premium that expressed in the factor-transformed variables still significantly influences the dynamics of exchange rate: for AUD, JPY and SEK (define as **G1** currencies), the means of expected risk-free rate and term premium for both domestic and/or foreign countries are found to be important; for the GBP, NOK and CHF (define as **G2** currencies), both the means and the variances are found to be key determinants; but for the NZD and CAD (define as **G3** currencies), those variables are not *jointly* relevant.

When one looks into the G1 and G2 currencies and asks how the first moments of expectations influence the value of a currency, she may find that 1) the signs of the coefficient of the same variable for domestic and foreign countries are opposite to each other, which, very intuitively, means the same variable has the opposite impact on the exchange rate. This feature can allow us to conveniently focus only on one side of the exchange rate determinants. 2) the means of expected risk-free rate and term premium posses the same sign. This is somewhat surprising: For example, as reflected in the positive coefficient of the $\bar{r}_{t,\dots,t+m|t}^*$, *ceteris paribus*, a higher mean of expected foreign risk-free rate in the future reduces the risk premium for hold its currency, resulting in a stronger foreign currency (in short: higher interest rate, stronger currency). But it is odd for the mean of foreign term premium for holding foreign government bond, $\bar{tp}_{t,\dots,t+m|t}^*$, plays the same role as it is generally perceived as a measure of riskiness of a country's economy which indicates that a higher term premium should correspond to a weaker currency (e.g. see Chen and Tasng, 2011).

When it comes to the second moment of the expectations, the effects do not follow a common rule. However, there is a general rule that the higher the variance of expected foreign (domestic) risk-free rate, the stronger the foreign (domestic) currency, as reflected by the positive (negative) coefficient of the $\sigma_r^*(\sigma_r)$. The exceptions are the JPY and CHF, in which a higher variance of expected JPY (CHF) risk-free rate is related to a weaker Yen (Franc). One possible explanation to this finding is that the observed risk-free rate of the JPY and CHF are nearly always lower than the one of the USD in the post 1996 period investigated in this paper, while the observed risk-free rates of the other currencies are higher than the one of the USD in most of time in history. This difference may cause the domestic (the U.S.) investors to hold different believes on the perceived higher uncertainties of the expected foreign risk-free rate: It is taken as a sign of lower risk for holding higher interest-rate currencies (GBP, AUD, etc.), but higher risk for holding lower interest-rate currencies (JPY and CHF). These believes in turn results in stronger and weaker foreign currencies, respectively.

Among these currencies, the values of GBP and NOK in G2, CAD in G3²¹ are found to be negatively correlated with their variances of the bond term premium, σ_{tp}^* . This can be interpreted as that when the investors in foreign bond market hold dispersed opinion on how much more compensation (term premia) should be charged on top of the risk-free rate over the horizons up to 10 years, then the domestic²² investors will take this as a strong signal that the foreign economy is under risk in the future, and then price the foreign currency at a lower value. This finding suggests that it is the variance (uncertainty) of the bond term premium, rather than the level or mean of it, that is perceived as a sign of riskiness of the perspective of an economy by foreign exchange market investors, as evidenced by the GBP, NOK and CAD.

²¹ For the case of CAD, the individual coefficients of the variances are mostly significant despite of the weak joint significance

²² Also foreign investors hold the same believes as the domestic investors as we assume symmetric information and believes among investors.

6 Implications

What I have obtained so far can be understood in two ways: First, the risk premium is an important determinant of the exchange rate dynamics, thus should not be omitted in a dynamic exchange rate model. Second, it is possible to quantify the risk premium, which is usually unobservable, with the help of bond yield factors or factor-transformed economic variables.

Accordingly, these two findings can have two implications: 1) Substituting the dynamic exchange rate component (e.g. UIP) in multivariate empirical models or now open economic models (esp. in the DSGE context) with the $ARDL + F^{1st, 2nd}$ model or augment it by adding a risk premium term in the form factors, combining this with the dynamics of factors, one may obtain more realistic moments, forecasts, or impulse responses of the exchange rate and related macroeconomic variables. 2) To utilize the numerical estimate of the risk premium and test exchange rate models that otherwise can hardly be tested if the risk premium is unobservable or unmeasurable.

In the following, I will focus on the second implication and test whether the UIP Puzzle can originate from the omission of a risk premium term, which is suggested by many studies of exchange rate. Due to its complexity, the first implication will be discussed in an extended version of this paper or very likely, a separate paper.

Risk premium and the UIP Puzzle

6.1 Risk-premium augmented Fama regression

Despite its popularity in the international macroeconomic/finance literature, empirically the UIP is severely violated as the estimate of β in the regression $\Delta e_t = \alpha + \beta(i_{t-1} - i_{t-1}^*) + u_t$ (In the present paper, I refer this equation as the Fama regression) deviates significantly from unity and usually has a negative value. This is documented as the UIP Puzzle in the literature. Among many other hypotheses, Fama (1984) argues that this puzzle may be caused by the omission of a time-varying risk premium term that led to a biased estimate of β . In this section, I will focus on the risk-premium solution to the UIP Puzzle²³, and check whether the Puzzle can be mitigated by the inclusion of a time-varying risk premium term. Three proxies that can represent the unobservable risk premium, $\hat{\lambda}_t$, are considered:

(1)

$$\hat{\lambda}_t^{tp} = tp_t - tp_t^*$$

i.e. the bilateral differential of bond term premia between domestic and foreign country. Where the term premium is defined as the spread between long- and short-term government bond yields, $tp_t = i_t^{10y} - i_t^{3m}$.

(2)

$$\hat{\lambda}_t^F = (\hat{C} + \hat{D}\tilde{F}_t)' \tilde{F}_t$$

i.e. the estimate of risk premium that implied by the $ARDL + F^{1st, 2nd}$ model, which is a function of factors from the terms structure of interest rates.

(3)

$$\hat{\lambda}_t^V = \hat{C}'\tilde{R} + \hat{D}'\tilde{\Sigma}$$

i.e. the estimate of risk premium that implied by the $ARDL + \tilde{R}, \tilde{\Sigma}$ model, which is a function of mean and variance of expected future short-term rates and term premia.

The implied risk premia $\hat{\lambda}_t^F$ and $\hat{\lambda}_t^V$ are constructed using the coefficient estimates obtained from the regressions in previous sections.

These risk premia can be used in an augmented version of Fama regression:

$$\Delta e_t = \alpha' + \beta'(i_{t-1} - i_{t-1}^*) + \gamma'\hat{\lambda}_t + u_t' \quad (3)$$

²³ Alternative solutions could be irrational or biased expectation of future exchange rate

Then one can check whether the risk premium can help in solving the UIP Puzzle by testing whether the $\hat{\beta}'$ is still significantly negative.

6.2 Results

Among the currencies considered in this paper, only the GBP and JPY are found to significantly violate the UIP condition ²⁴, as evidenced by the significant and negative estimate of β . Thus in the following, I will only focus on the cases of GBP and JPY.

As shown in the following table for the case of GBP, the interest rate differential, $i_{t-1} - i_{t-1}^*$, is negatively related to the change in exchange rate, Δe_t , for the period of 1979.01-2009.05. However, once the risk premium estimate, $\hat{\lambda}_t^F$ ($\hat{\lambda}_t^V$), that obtained from the $ARDL + F^{1st, 2nd}$ model ($ARDL + \tilde{R}, \tilde{\Sigma}$ model) is added to the Fama regression, the coefficient estimate of the interest rate differential, $\hat{\beta}'$, is no longer negative, but turns to insignificantly positive. This can be taken as an evidence that the UIP Puzzle can possibly be solved with the help of a risk premium term.

However, when the currency risk premium is proxied with the differential of term premia, $tp_t - tp_t^*$, the $\hat{\beta}'$ is found to still possess a negative sign. This indicates that the $tp_t - tp_t^*$ may not be a good proxy for the currency risk premium. Or put differently, it fails to capture the information on the relative riskiness for currency holding. This is somehow surprising, because the $tp_t - tp_t^*$ is frequently used in the international economics literature as a proxy for the relative cross-country riskiness.

Table 22: Comparison of Fama and risk-premium augmented Fama regression, GBP

Δe_t	UIP	UIP + λ_t^{tp}	UIP + λ_t^F	UIP + λ_t^V
c	-4.636251*	-4.583003	6.886364***	2.430201
$i_{t-1} - i_{t-1}^*$	-1.761350**	-1.396426	0.538926	0.615436
$\hat{\lambda}_t$		0.583589	0.990817***	1.023784***
$adj - R^2$	0.009248	0.006809	0.159325	0.087091
AIC	10.04270	10.04788	9.881162	9.963594

[Referring to section 6.2] Fama regression (UIP) vs risk-premium augmented Fama regression for the GBP over the period from 1979.01 to 2009.05. The first column reports the results for the Fama regression. The rest columns report the results for risk-premium augmented Fama regression, in which a risk premium term is added to the Fama regression. Details of proxies for the risk premium λ_t^{tp} , λ_t^F and λ_t^V can be found in section 6.1.

Similar results are obtained for JPY over the period of 1985.01-2009.05. One difference with the case of GBP is that, the risk premium term, $\hat{\lambda}_t$, is calculated using the factors to the first order or only the first moments (means) of the expected risk-free rates and term premia, i.e. the $\hat{\lambda}_t$ is constructed out of the $ARDL + F^{1st}$ model or the $ARDL + \tilde{R}$ model for JPY. The reason for omitting the second order factors or the second moment of the variables is that, for the JPY during this period of time, these terms barely contribute to explaining the variation of the exchange rate²⁵.

²⁴While for the other currencies, negative values of $\hat{\beta}$ can also be obtained, but they are not significantly different from zero. Thus these cases are not considered as significant violation of the UIP condition.

²⁵That is to say, the $ARDL + F^{1st}$ ($ARDL + \tilde{R}$) model performs as good as the $ARDL + F^{1st, 2nd}$ ($ARDL + \tilde{R}, \tilde{\Sigma}$) model in fitting the exchange rate data, as measured by the AIC . Thus it is not necessary to keep the higher order (moment) terms in the regression.

Table 23: Comparison of Fama and risk-premium augmented Fama regression, GBP

Δe_t	UIP	UIP + λ_t^{tp}	UIP + $\lambda_t^{F^{1st}}$	UIP + $\hat{\lambda}_t^V$
c	11.35734***	5.856104	-44.97101**	15.90314***
$i_{t-1} - i_{t-1}^*$	-2.656034**	-1.097689	8.895920**	7.476455*
$\hat{\lambda}_t$		2.494553	0.652328***	0.624643**
$adj - R^2$	0.016855	0.015353	0.048130	0.043769
AIC	10.14960	10.15453	10.12067	10.12524

[Referring to section 6.2] Fama regression (UIP) vs risk-premium augmented Fama regression for the JPY over the period from 1985.01 to 2009.05. The inputs of the table are the same as the ones for the GBP. The only difference is the the estimated risk premium in the third and fourth columns are calculated using the factors to the first order and only the first moments (means) of the expected risk-free rates and term premia. Result and conclusions are similar as the ones for GBP.

In conclusion, the above results for the GBP and JPY suggest that the UIP Puzzle is very likely caused by improperly omitting a risk premium term in the Fama regression. And once the risk premium term is included, the Puzzle is no longer in existence. One pre-condition for potentially solving the UIP Puzzle is that a valid proxy must be selected for the unobservable currency risk premium, that being said, it should properly capture the information on the relative riskiness for holding domestic (foreign) currency.

6.3 Discussion: the relation between $i_{t-1} - i_{t-1}^*$ and Δe_t revisited

Above results indicate that the augmented UIP (equation 3) is a better model than the UIP in explaining the actual dynamics of exchange rate and its structural relationship with the interest rate differentials, and the positive value of the $\hat{\beta}'$ indicates that higher interest rate currency tends to depreciate once the currency risk premium is controlled, which is in line with what the original UIP condition predicts.

However, a crucial question to be answered is that, whether the above finding is in line or in contradiction with the *hard* empirical evidence—'higher interest rate currency tends to appreciate'? Which is suggested by the negative value of $\hat{\beta}$ from the original Fama regression (see table 22 and 23). The negative $\hat{\beta}$ should be interpreted as a negative estimate of unconditional correlation²⁶ between interest rate differential, $i_{t-1} - i_{t-1}^*$, and the currency depreciation rate Δe_t .

Table 24: Correlation between interest rate differentials and estimates of risk premia

$Corr.$	GBP (79.01-09.05)			JPY (85.01-09.05)		
	$\hat{\lambda}_t^{tp}$	$\hat{\lambda}_t^F$	$\hat{\lambda}_t^{V(F)}$	$\hat{\lambda}_t^{tp}$	$\hat{\lambda}_t^F$	$\hat{\lambda}_t^{V(F)}$
$i_{t-1} - i_{t-1}^*$	-0.7964	-0.3441	-0.4624	-0.8870	-0.9578	-0.9525
$i_t - i_t^*$	-0.8423	-0.4092	-0.5244	-0.8811	-0.9743	-0.9707

An answer to this question based on the augmented UIP (equation 3) could be that, the correlation between interest rate differential, $i_{t-1} - i_{t-1}^*$, with the currency depreciation rate Δe_t , may arise through two different channels. First, the direct and structural channel of the differential in returns on one-period holding of currencies, $i_{t-1} - i_{t-1}^*$; Second, the indirect channel—through its correlation with the risk premium term, $\hat{\lambda}_t$, which may be just a statistical relationship between the two without any structural economic connection²⁷. As shown below, the $\hat{\lambda}_t^F$ and $\hat{\lambda}_t^V$ are found to be negatively correlated with the $i_{t-1} - i_{t-1}^*$, for both GBP and JPY.

²⁶Which is a measure of reduced-form rather than a structural-form of linear relationship between two variables.

²⁷One possible explanation is that the $i_{t-1} - i_{t-1}^*$ is highly correlated with $i_t - i_t^*$ (0.9469 for GBP and 0.9919 for JPY), and the $i_t - i_t^*$ is negatively correlated with the $\hat{\lambda}_t$. The latter correlation is actually a linear projection of the non-linear correlation between $i_t - i_t^*$ and $\hat{\lambda}_t$, as suggested by this paper, the $\hat{\lambda}_t$ is non-linear in factors (also in interest rates).

Thus the augmented-UIP implied correlation between the $i_{t-1} - i_{t-1}^*$ and Δe_t is a combination of a positive structural correlation and a negative statistical one. When the negative one dominates the positive one, the overall correlation tends to be negative. Thus from a technical perspective, a positive value of $\hat{\beta}'$ is not in contradiction with the negative value of $\hat{\beta}$, because they measure a fraction and the overall correlation between the $i_{t-1} - i_{t-1}^*$ and Δe_t , respectively.

7 Concluding Remarks

The objective of the paper has been to investigate whether the expected status of economies plays a role in explaining the dynamics of exchange rates, with a focus on the forward-looking information that is embedded in the term structure of interest rates, through a risk premium channel for holding domestic (foreign) currency. The unobservable risk premium is in particular assumed to be linked non-linearly to bond yield factors, and serves as a determinant of the exchange rate dynamics.

I have applied this setup to nominal dollar exchange rates with the eight advanced-economy currencies that are defined as the AUD, CAD, CHF, GBP, JPY, NOK, NZD and SEK against USD, between the 1990s and 2009. The empirical results generally show the outstanding performance of the proposed model in terms of in-sample goodness of fit and out-of-sample forecast accuracy in the exchange rate changes, which in turn reflect the importance of the risk premium in explaining the dynamics of the exchange rate, and also its non-linearity in bond yield factors. In particular, I find evidence that the exchange rate is predictable at the short-term horizons, ranging from one month to twelve months for all the currencies studied in this paper except the CHF. Although the best forecast horizons may vary across currencies, the proposed model can basically generate better forecast than the benchmark models that either claim no-change in exchange rate value or equal probability of change in direction.

Another result worth noting is that, due to its non-linearity in factors via the risk premium channel, the exchange rate reacts to monetary policy shocks in a time-varying and state-dependent manner. I show evidence that given a certain amount of monetary policy shock, the concurrent response of the exchange rate change will no longer be homogenous as is the case with linear models, but may change over time in its amount and even its direction, depending on the perceived present and future status of the economies.

An important implication of being able to quantify the unobservable risk premium is that one can utilize the model-implied quantity of risk premium to test exchange rate models that has been otherwise impossible. I find empirical evidence for the GBP and JPY, in which cases the UIP condition is significantly violated, so adding a risk premium term into the Fama regression can help in solving the UIP Puzzle. This in turn suggests that, everything else being equal, the higher interest rate currency tends to depreciate to some extent. However, this does not contradict the empirical findings that high interest rate currency tends to appreciate, as the interest rate differential is a relatively weak determinant of the exchange rate dynamics, and its negative correlation (not necessarily causal) with the risk premium is the underlying reason for the stylized but puzzling findings.

Still, many open questions and directions for future research remain. Firstly, the information about expectations that are embedded in other variables may also be relevant for the exchange rate dynamics and thus can be included in an exchange rate model. Such variables could include commodity price, policy uncertainty or financial soundness and may further be allowed to interact with term structure factors or even treated as determinants of bond yields. Secondly, using a Bayesian model averaging approach to combine the proposed model with other well-performing models, one can obtain a best forecast model for a specific exchange rate. Thirdly, this may prove to be a fruitful research area if one substitutes the dynamic exchange rate component (e.g UIP) in multivariate empirical models or now open economic models (especially in the DSGE context) with the proposed model in this paper, in order to obtain more realistic moments, forecasts, or impulse responses for the exchange rate and related macroeconomic variables.

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Tables and Figures

Table 3: Evaluation on how well pre-believed models can fit the data

	Models					
	M1 (PPP)	M2 (FP)	M3 (UIP)	M4 (TR)	M5 (RW)	M6 (ARDL)
AUD						
90.01-09.05	0.002847	0.030202	-0.004322	0.010868	-	0.031807 (3,1)
CAD						
99.01-09.05	0.009290	0.092047	0.063539	0.039803	-	0.179898 (6,5)
CHF						
96.01-09.05	-0.005115	0.040054	0.009539	0.025011	-	0.019749(8,3)
						0.205917*
GBP						
93.01-06.04 ²⁸	-0.001037	0.030668	-0.003049	-0.005328	-	0.055758 (7,2)
JPY						
96.01-09.05	0.015983	0.063908	0.002030	-0.003745	-	0.000779 (6,5)
						0.208859*
NOK						
98.01-09.05	0.018394	0.033015	-0.006952	-0.008798	-	0.013185 (1,1)
NZD						
90.01-09.05	0.021886	0.010411	-0.004327	0.014445	-	0.130208 (3,1)
SEK						
97.01-09.05 ²⁹	0.024112	0.040760	-0.000447	0.000592	-	0.150437 (2,1)

[Referring to section 3.2.2] Model selection of the pre-believed models on exchange rate changes, $e_{t-0} - e_{t-1}$. The M1-M6 are defined in section 2.3. For each currency, $Adj. - R^2$ s are report for all models. (p,q) in the last column for M6 indicates the lag orders in the ARDL model that selected according to the AIC . The numbers in bold represent the highest value of $Adj. - R^2$ for each currency. For CHF and JPY, the number with a * is the $Adj. - R^2$ after including the factors in the regression, the ARDL+factor model has a higher $Adj. - R^2$ than its counterparts.

Table 4: Regression results of linear and non-linear models for the exchange rate. GBP and SEK

	GBP (93.01-09.05)				SEK (93.01-09.05)		
Δe_t	<i>ARDL</i>	<i>ARDL</i> + F^{1st}	<i>ARDL</i> + $F^{1st, 2nd}$		<i>ARDL</i>	<i>ARDL</i> + F^{1st}	<i>ARDL</i> + $F^{1st, 2nd}$
<i>constant</i>	5.597804	-8.181222	-14.49341*		-0.206939	-25.03893*	-49.62301***
Δe_{t-1}	0.062807	-0.096913	-0.158929***		0.442030***	0.373680***	0.278751***
Δe_{t-2}					-0.172505**	-0.191793***	-0.261047***
Δe_{t-3}							
Δe_{t-4}							
Δe_{t-5}							
Δe_{t-6}							
Δe_{t-7}							
Δe_{t-8}							
$i_{t-1} - i_{t-1}^*$	3.365628**	9.746557**	12.94162**		-9.641009	-0.301176	-11.04477
$i_{t-2} - i_{t-2}^*$					9.287479	3.620617	12.75431
$i_{t-3} - i_{t-3}^*$							
$i_{t-4} - i_{t-4}^*$							
$i_{t-5} - i_{t-5}^*$							
$f_{1,t}^*$		3.270911***	4.093740*			2.660618**	7.518157***
$f_{2,t}^*$		7.008977***	4.170401**			2.687876***	4.857086**
$f_{3,t}^*$		-3.626806***	0.704236			-0.411540	-1.730068
$f_{1,t}$		4.481000**	-3.269649			5.494941**	-7.572669
$f_{2,t}$		-3.940776***	-7.816412*			-3.925754**	-9.500255***
$f_{3,t}$		-4.513902**	-12.49997**			-1.519771	-8.983435**
$f_{1,t}^* \cdot f_{1,t}$			0.027558				-0.236658
$f_{1,t}^* \cdot f_{2,t}$			0.532879				-0.085270
$f_{1,t}^* \cdot f_{3,t}$			-0.433638				0.050497
$f_{2,t}^* \cdot f_{2,t}$			-0.063122				-0.170399
$f_{2,t}^* \cdot f_{3,t}$			1.091648**				0.025376
$f_{3,t}^* \cdot f_{3,t}$			-0.352737				0.026191
$f_{1,t} \cdot f_{1,t}$			-2.414802*				-2.175842
$f_{1,t} \cdot f_{2,t}$			-1.090505				-2.725851***
$f_{1,t} \cdot f_{3,t}$			-1.936353*				-2.457503*
$f_{2,t} \cdot f_{2,t}$			-0.349361				0.152370
$f_{2,t} \cdot f_{3,t}$			-1.209779***				-1.405457***
$f_{3,t} \cdot f_{3,t}$			-0.473133				-0.853322*
<i>Adj. - R²</i>	0.014807	0.154524	0.188009		0.162717	0.208140	0.260814
<i>AIC</i>	9.577392	9.453952	9.469410		9.598535	9.571922	9.558223
Prob χ_{1st}^2	-	0.0000***	0.0017 ***		-	0.0505***	0.0001***
Prob χ_{2nd}^2	-	-	0.0000***		-	-	0.0000***
Prob $\chi_{1st, 2nd}^2$	-	-	0.0000***		-	-	0.0000***

[Referring to section 3.2.4] Comparison of models on how well they can fit the data, for the GBP and SEK during the period of 1993.01-2009.05. The notations used in the tables are *ARDL*: Autoregressive distributed lags model discussed in section 2.3; *ARDL* + F^{1st} : *ARDL* model augmented with factors in linear form; *ARDL* + $F^{1st, 2nd}$: *ARDL* model augmented with factor in non-linear form (including the second order factors). The *Adj. - R²* and *AIC* are reported for each model and each currency. Especially, the nulls on the $C = 0_{6 \times 1}$, $D = 0_{6 \times 6}$, C and D are jointly zero are tested respectively, probabilities of the χ^2 statistics are reported. The '***', '**', '*' denote significance at 1%, 5% and 10% level respectively. All the inferences are adjusted with the Newey-West autocorrelation and heteroskedasticity consistent standard errors. The lag orders of the *ARDL* model are selected by the *AIC*.

Table 5: Regression results of linear and non-linear models for the exchange rate. AUD and NZD

Δe_t	AUD (90.01-09.05)				NZD (90.01-09.05)		
	$ARDL$	$ARDL + F^{1^{st}}$	$ARDL + F^{1^{st}, 2^{nd}}$		$ARDL$	$ARDL + F^{1^{st}}$	$ARDL + F^{1^{st}, 2^{nd}}$
<i>constant</i>	0.495515	-25.98079**	-9.012043		0.456397	-2.480006	-9.903086
Δe_{t-1}	0.182373*	0.107767	0.020697		0.339334***	0.284310***	0.190668***
Δe_{t-2}	-0.083594	-0.131261**	-0.141496				
Δe_{t-3}	0.146119***	0.082473	0.025153**				
Δe_{t-4}							
Δe_{t-5}							
Δe_{t-6}							
Δe_{t-7}							
Δe_{t-8}							
$i_{t-1} - i_{t-1}^*$	0.153410	0.400601	4.021980		0.113527	0.941270	-3.291492
$i_{t-2} - i_{t-2}^*$							
$i_{t-3} - i_{t-3}^*$							
$i_{t-4} - i_{t-4}^*$							
$i_{t-5} - i_{t-5}^*$							
$f_{1,t}^*$		1.220156	0.063879			3.034814*	1.963514
$f_{2,t}^*$		-10.46787**	-19.53336			1.237132	2.208666
$f_{3,t}^*$		-6.311557	-16.36752**			-1.791633	-5.420379***
$f_{1,t}$		8.545139**	4.477018**			4.472535**	0.699866
$f_{2,t}$		-5.376314**	-1.689460			-0.543845	-1.381435
$f_{3,t}$		1.211274	-1.367353			-0.150251	-3.379949
$f_{1,t}^* \cdot f_{1,t}^*$			-0.147412				0.194297
$f_{1,t}^* \cdot f_{2,t}^*$			-0.577908				-0.223674
$f_{1,t}^* \cdot f_{3,t}^*$			-0.064977				0.084312
$f_{2,t}^* \cdot f_{2,t}^*$			-4.105370***				-0.382260**
$f_{2,t}^* \cdot f_{3,t}^*$			-5.037975***				0.716588**
$f_{3,t}^* \cdot f_{3,t}^*$			-1.412484***				0.079214
$f_{1,t} \cdot f_{1,t}$			1.696218				0.189727
$f_{1,t} \cdot f_{2,t}$			-1.884283***				-1.995192***
$f_{1,t} \cdot f_{3,t}$			1.670596				0.409905
$f_{2,t} \cdot f_{2,t}$			0.774623**				0.131581
$f_{2,t} \cdot f_{3,t}$			0.096388				-1.061442***
$f_{3,t} \cdot f_{3,t}$			-0.182368				-0.482494
<i>Adj. - R²</i>	0.031807	0.104885	0.214802		0.105866	0.128891	0.177333
<i>AIC</i>	10.08084	10.02720	9.943615		9.646137	9.645118	9.635847
Prob $\chi^2_{1^{st}}$	-	0.0010 ***	0.0325 **		-	0.0472 **	0.0003 **
Prob $\chi^2_{2^{nd}}$	-	-	0.0000 ***		-	-	0.0001 ***
Prob $\chi^2_{1^{st}, 2^{nd}}$	-	-	0.0000 ***		-	-	0.0000 ***

[Referring to section 3.2.4] Comparison of models on how well they can fit the data, for the AUD and NZD during the period of 1990.01-2009.05. The notations used here are the same as in the previous table.

Table 6: Regression results of linear and non-linear models for the exchange rate. JPY and CHF

	JPY (96.01-09.05)				CHF (96.01-09.05)		
Δe_t	<i>ARDL</i>	<i>ARDL</i> + F^{1st}	<i>ARDL</i> + $F^{1st, 2nd}$		<i>ARDL</i>	<i>ARDL</i> + F^{1st}	<i>ARDL</i> + $F^{1st, 2nd}$
<i>constant</i>	13.85854*	-121.1385***	-52.20926		8.031115	-83.15247***	-82.39075***
Δe_{t-1}	-0.029807	-0.101853	-0.193388***		-0.028994	-0.165121	-0.300112**
Δe_{t-2}	0.052889	0.003827	-0.067417		-0.033358	-0.161828**	-0.258148***
Δe_{t-3}	0.051886	0.021780	-0.037383		-0.002256	-0.153842**	-0.257753***
Δe_{t-4}	-0.126041	-0.111242	-0.165177**		-0.158438**	-0.291547***	-0.396227***
Δe_{t-5}	-0.141809*	-0.152784*	-0.259870***		0.018951	-0.092576	-0.159252*
Δe_{t-6}	-0.064520	-0.075488	-0.167027**		-0.058431	-0.115879	-0.179641*
Δe_{t-7}					-0.094092	-0.132712	-0.179531**
Δe_{t-8}					-0.059452	-0.080268	-0.167273**
$i_{t-1} - i_{t-1}^*$	-9.630588	44.89281*	35.47422*		4.285005	22.00541	8.868533
$i_{t-2} - i_{t-2}^*$	23.60070	5.797371	16.46784		-38.05468	-42.19050	-32.33669
$i_{t-3} - i_{t-3}^*$	-25.51819	-25.96010	-32.99534		30.00714	38.54862**	28.35007
$i_{t-4} - i_{t-4}^*$	25.93667	32.16958	41.19106				
$i_{t-5} - i_{t-5}^*$	-17.95750	-31.65515*	-42.98444*				
$f_{1,t}^*$		-0.531379	-4.033030			2.084810	8.407364****
$f_{2,t}^*$		20.48985***	15.08973*			-2.549035	1.650923
$f_{3,t}^*$		-2.082991	-7.516744			1.762246	0.058317
$f_{1,t}$		-1.303154	-16.00488*			10.04627***	-15.12762*
$f_{2,t}$		-3.162448	6.227034			-7.958053***	-22.60410***
$f_{3,t}$		-18.77178***	-20.36388***			-8.498777**	-23.82422***
$f_{1,t}^* \cdot f_{1,t}$			-0.583863**				-0.133832
$f_{1,t}^* \cdot f_{2,t}$			-0.699953				-0.047795
$f_{1,t}^* \cdot f_{3,t}$			-6.755596***				1.314841*
$f_{2,t}^* \cdot f_{2,t}$			5.604472				0.745474
$f_{2,t}^* \cdot f_{3,t}$			-1.572984				1.908858
$f_{3,t}^* \cdot f_{3,t}$			-3.972497				0.232905
$f_{1,t} \cdot f_{1,t}$			-2.902119				0.429247
$f_{1,t} \cdot f_{2,t}$			-4.063325***				-6.051722***
$f_{1,t} \cdot f_{3,t}$			-2.550657				0.194466
$f_{2,t} \cdot f_{2,t}$			0.731746				-0.722317
$f_{2,t} \cdot f_{3,t}$			-0.478346				-3.976959***
$f_{3,t} \cdot f_{3,t}$			-1.215976*				-0.935704
<i>Adj. - R²</i>	0.000779	0.133813	0.208859		0.019749	0.117667	0.205917
<i>AIC</i>	10.13415	10.02471	9.995506		10.14179	10.06998	10.02602
Prob χ_{1st}^2	-	0.0000 ***	0.0116 **		-	0.0007 ***	0.0000 ***
Prob χ_{2nd}^2	-	-	0.0000 ***		-	-	0.0000 ***
Prob $\chi_{1st, 2nd}^2$	-	-	0.0000 ***		-	-	0.0000 ***

[Referring to section 3.2.4] Comparison of models on how well they can fit the data, for the JPY and CHF during the period of 1996.01-2009.05. The notations used here are the same as in the previous table.

Table 7: Regression results of linear and non-linear models for the exchange rate. NOK and CAD

	NOK (98.01-09.05)				CAD (99.01-09.05)		
Δe_t	<i>ARDL</i>	<i>ARDL</i> + F^{1st}	<i>ARDL</i> + $F^{1st, 2nd}$		<i>ARDL</i>	<i>ARDL</i> + F^{1st}	<i>ARDL</i> + $F^{1st, 2nd}$
<i>constant</i>	1.632000	-3.828102	-37.57228		2.285305	-82.81479***	-116.6104***
Δe_{t-1}	0.165972	0.061508	-0.098720		0.307091***	0.266602***	0.080569
Δe_{t-2}					0.057781	0.024263	-0.080258
Δe_{t-3}					-0.119025	-0.138867	-0.227276**
Δe_{t-4}					0.239679**	0.208942**	0.093451
Δe_{t-5}					-0.014012	0.018919	0.027341
Δe_{t-6}					-0.249555**	-0.256982***	-0.211037**
Δe_{t-7}							
Δe_{t-8}							
$i_{t-1} - i_{t-1}^*$	0.272399	-5.784154	-4.018731		1.161302	11.67200	4.789903
$i_{t-2} - i_{t-2}^*$					-17.99161	-18.94077	-15.21391
$i_{t-3} - i_{t-3}^*$					23.36692*	25.26767*	29.69881**
$i_{t-4} - i_{t-4}^*$					21.97614**	20.13731	22.42479*
$i_{t-5} - i_{t-5}^*$					-29.29930***	-35.11951***	-45.57523***
$f_{1,t}^*$		-2.272233	-5.305696			5.348997***	9.151849*
$f_{2,t}^*$		-5.509398**	-2.565921			8.252400***	15.79980**
$f_{3,t}^*$		0.879457	2.502539			-0.768644	-12.03708*
$f_{1,t}$		9.566488***	-10.71442			4.870551*	1.576491
$f_{2,t}$		-0.808787	-7.723462			-9.146808***	-20.39541**
$f_{3,t}$		6.677506	-7.351171			-3.182890	-8.469554
$f_{1,t}^* \cdot f_{1,t}$			0.296081				0.129863
$f_{1,t}^* \cdot f_{2,t}$			1.545800*				-0.292719
$f_{1,t}^* \cdot f_{3,t}$			0.967347				-0.892027
$f_{2,t}^* \cdot f_{2,t}$			-0.104119				1.994429**
$f_{2,t}^* \cdot f_{3,t}$			-1.976031*				-3.373804**
$f_{3,t}^* \cdot f_{3,t}$			1.456566**				3.230163***
$f_{1,t} \cdot f_{1,t}$			-2.162639				-6.520114***
$f_{1,t} \cdot f_{2,t}$			-4.216904***				0.455725
$f_{1,t} \cdot f_{3,t}$			-2.493055				-6.356443***
$f_{2,t} \cdot f_{2,t}$			-0.025452				-0.869032
$f_{2,t} \cdot f_{3,t}$			-2.264375**				0.226405
$f_{3,t} \cdot f_{3,t}$			-1.427416*				-2.455694***
<i>Adj. - R²</i>	0.013185	0.074797	0.143231		0.179898	0.241420	0.353113
<i>AIC</i>	10.10530	10.08262	10.08251		9.187586	9.151046	9.064817
Prob χ_{1st}^2	-	0.0261**	0.2768		-	0.0251**	0.0037 ***
Prob χ_{2nd}^2	-	-	0.0477 **		-	-	0.0019 ***
Prob $\chi_{1st, 2nd}^2$	-	-	0.0086 ***		-	-	0.0020 ***

[Referring to section 3.2.4] Comparison of models on how well they can fit the data, for the NOK and CAD during the period of 1998.01-2009.05 and 1999.01-2009.05. The notations used here are the same as in the previous table.

Table 8: Forecast evaluation and comparison. GBP and SEK

GBP Forecasts, $\widehat{\Delta e_{t+h}} I_t$										
RMSE	ARDL	$\dots + F^{1^{st}}$	$\dots + F^{1^{st}, 2^{nd}}$	RW		Direct.	ARDL	$\dots + F^{1^{st}}$	$\dots + F^{1^{st}, 2^{nd}}$	Naive
1	0.0291	0.026	<i>0.0262</i>	0.0289		1	0.5333	0.5833	<i>0.55</i>	0.5
2	0.0468	0.0408	<i>0.0425</i>	0.0458		2	0.5085	0.5932	<i>0.5763</i>	0.5
3	0.0641	0.0569	<i>0.06</i>	0.0615		3	0.431	0.6034	0.7069	0.5
4	0.0805	0.073	0.0781	0.076		4	0.386	0.6316	0.6842	0.5
5	0.097	0.0907	0.0995	0.0902		5	0.3393	0.625	0.6786	0.5
6	0.1108	0.1071	0.1066	0.1011		6	0.2909	0.5818	0.7091	0.5
7	0.1235	0.1207	0.1231	0.111		7	0.2593	0.5556	0.6852	0.5
8	0.1344	0.1289	0.128	0.119		8	0.2075	0.4906	0.6792	0.5
9	0.1438	0.1391	0.1391	0.1255		9	0.1731	0.4231	0.6731	0.5
10	0.1519	0.1479	0.1512	0.1308		10	0.1765	0.3529	0.6275	0.5
11	0.1573	0.1532	0.1603	0.1343		11	0.16	0.34	0.6	0.5
12	0.1627	0.1578	0.1682	0.1379		12	0.1224	0.2449	0.551	0.5

SEK Forecasts, $\widehat{\Delta e_{t+h}} I_t$										
RMSE	<i>ARDL</i>	$\dots + F^{1^{st}}$	$\dots + F^{1^{st}, 2^{nd}}$	<i>RW</i>		Direct.	<i>ARDL</i>	$\dots + F^{1^{st}}$	$\dots + F^{1^{st}, 2^{nd}}$	<i>Naive</i>
1	0.0302	0.0296	0.0289	0.0323		1	0.5862	0.6207	0.6379	0.5
2	0.0567	0.0559	0.0541	0.0546		2	0.5088	0.5263	0.5789	0.5
4	0.075	0.0753	0.0728	0.0708		4	0.5	0.4821	0.5893	0.5
4	0.0892	0.0914	0.0877	0.0854		4	0.4545	0.5818	0.6909	0.5
5	0.1048	0.1103	0.1097	0.1008		5	0.4444	0.5741	0.6667	0.5
6	0.1206	0.1311	0.1357	0.1149		6	0.3962	0.434	0.6038	0.5
7	0.1345	0.1499	0.156	0.1264		7	0.3462	0.3269	0.5962	0.5
8	0.145	0.1652	0.1721	0.1343		8	0.3529	0.2941	0.5294	0.5
9	0.1525	0.1791	0.1914	0.1389		9	0.28	0.22	0.48	0.5
10	0.1577	0.1898	0.2081	0.1415		10	0.2653	0.1429	0.4082	0.5
11	0.1607	0.197	0.2201	0.1424		11	0.3333	0.1458	0.3333	0.5
12	0.1645	0.2029	0.2297	0.1429		12	0.3404	0.1277	0.3191	0.5

[Referring to section 4.1.2] Evaluation and comparison of models on how well they can forecast the exchange rate change/return, for the GBP and SEK during the sample period of 1993.01-2009.05. Rolling estimation for the GBP with a window size of 137. Recursive estimation for the SEK. Forecast window 2004.06-2009.05. The notations used in the tables are *ARDL*: Autoregressive distributed lags model discussed in section 4.1.1; $\dots + F^{1^{st}} \equiv ARDL + F^{1^{st}}$: ARDL model augmented with factors in linear form; $\dots + F^{1^{st}, 2^{nd}} \equiv ARDL + F^{1^{st}, 2^{nd}}$: ARDL model augmented with factor in non-linear form (including the second order factors). *RM*: Random walk model; *Naive*: Model predicts equal probability of change in direction. 1-12 in the left column are the respective forecast horizons. Numbers in **bold** indicate the best forecasts at each horizon. **RMSE**: root of mean squared forecast errors. **Direct.**: the probability of predicted changes in the right direction.

Table 9: Forecast evaluation and comparison. AUD and NZD

AUD Forecasts, $\widehat{\Delta e_{t+h}} I_t$										
RMSE	ARDL	... + $F^{1^{st}}$... + $F^{1^{st}, 2^{nd}}$	RW		Direct...	ARDL	... + $F^{1^{st}}$... + $F^{1^{st}, 2^{nd}}$	Naive
1	0.0437	0.046	0.0506	0.0423		1	0.5167	0.5833	0.55	0.5
2	0.0724	0.0726	0.077	0.0658		2	0.4407	0.4915	0.5254	0.5
3	0.0911	0.0892	0.0935	0.0828		3	0.5	0.3448	0.569	0.5
4	0.1119	0.1083	0.1098	0.0989		4	0.4211	0.3684	0.5263	0.5
5	0.1309	0.1275	0.1264	0.1129		5	0.375	0.3571	0.4464	0.5
6	0.1506	0.1455	0.1385	0.1262		6	0.3455	0.2364	0.3455	0.5
7	0.1638	0.1628	0.1547	0.1336		7	0.3519	0.1852	0.3148	0.5
8	0.1706	0.175	0.1689	0.1393		8	0.3396	0.2453	0.283	0.5
9	0.1802	0.1862	0.1813	0.1435		9	0.3077	0.1731	0.2308	0.5
10	0.1862	0.1933	0.1916	0.1447		10	0.3137	0.2157	0.1961	0.5
11	0.1913	0.1992	0.2021	0.1461		11	0.36	0.26	0.2	0.5
12	0.1953	0.2038	0.21	0.1474		12	0.3265	0.2653	0.2041	0.5

NZD Forecasts, $\widehat{\Delta e_{t+h}} I_t$										
RMSE	ARDL	... + $F^{1^{st}}$... + $F^{1^{st}, 2^{nd}}$	RW		Direct.	ARDL	... + $F^{1^{st}}$... + $F^{1^{st}, 2^{nd}}$	Naive
1	0.0325	0.032	0.0324	0.0341		1	0.6	0.6667	0.6667	0.5
2	0.0577	0.0563	0.0578	0.0571		2	0.661	0.6441	0.7119	0.5
3	0.0759	0.0735	0.0759	0.0736		3	0.5345	0.6207	0.7069	0.5
4	0.0944	0.0898	0.0923	0.0904		4	0.4561	0.4912	0.5965	0.5
5	0.1171	0.1109	0.1157	0.1084		5	0.4107	0.4821	0.5536	0.5
6	0.1372	0.1297	0.1387	0.1233		6	0.3818	0.4364	0.5091	0.5
7	0.1547	0.1445	0.153	0.1357		7	0.4074	0.463	0.537	0.5
8	0.1706	0.1597	0.1705	0.146		8	0.3774	0.4151	0.5094	0.5
9	0.1844	0.1735	0.1872	0.1543		9	0.3462	0.3654	0.4615	0.5
10	0.1958	0.1838	0.2015	0.1604		10	0.3529	0.3529	0.4118	0.5
11	0.2037	0.1919	0.213	0.1641		11	0.32	0.34	0.4	0.5
12	0.2105	0.1994	0.2227	0.1671		12	0.3061	0.3673	0.4082	0.5

[Referring to section 4.1.2] Evaluation and comparison of models on how well they can forecast the exchange rate change/return, for the AUD and NZD during the sample period of 1990.01-2009.05. Rolling estimation for the AUD and NZD with a window size of 173. Forecast window 2004.06-2009.05. The notations used here are the same as in the previous table.

Table 10: Forecast evaluation and comparison. JPY and CHF

JPY Forecasts, $\widehat{\Delta e_{t+h}} I_t$										
RMSE	ARDL	$\dots + F^{1^{st}}$	$\dots + F^{2^{nd}}$	RW		Direct.	ARDL	$\dots + F^{1^{st}}$	$\dots + F^{2^{nd}}$	Naive
1	0.0379	0.0398	0.0394	0.0274		1	0.4167	0.55	0.4833	0.5
2	0.0478	0.0541	0.0537	0.0404		2	0.3898	0.4746	0.5424	0.5
3	0.0564	0.06	0.0629	0.0517		3	0.4828	0.5172	0.6379	0.5
4	0.0637	0.0657	0.0684	0.0591		4	0.5439	0.5439	0.6316	0.5
5	0.069	0.0662	0.0701	0.0646		5	0.4821	0.6071	0.6964	0.5
6	0.0727	0.0662	0.0636	0.0679		6	0.5091	0.6727	0.7636	0.5
7	0.0761	0.0696	0.0624	0.0713		7	0.5	0.7778	0.8704	0.5
8	0.0807	0.0757	0.0669	0.0749		8	0.4906	0.7736	0.8491	0.5
9	0.0859	0.083	0.0703	0.0785		9	0.5192	0.8077	0.8846	0.5
10	0.0921	0.0934	0.0783	0.0817		10	0.4902	0.8039	0.8235	0.5
11	0.1012	0.1006	0.091	0.0867		11	0.48	0.8	0.78	0.5
12	0.1104	0.1065	0.102	0.0919		12	0.3878	0.7551	0.7551	0.5

CHF Forecasts, $\widehat{\Delta e_{t+h}} I_t$										
RMSE	ARDL	$\dots + F^{1^{st}}$	$\dots + F^{1^{st}, 2^{nd}}$	RW		Direct.	ARDL	$\dots + F^{1^{st}}$	$\dots + F^{1^{st}, 2^{nd}}$	Naive
1	0.0364	0.038	0.0394	0.0339		1	0.4333	0.45	0.4333	0.5
2	0.0505	0.056	0.057	0.0439		2	0.5932	0.5254	0.5763	0.5
3	0.0604	0.0711	0.0734	0.0509		3	0.5	0.3448	0.4138	0.5
4	0.0724	0.0846	0.0859	0.0603		4	0.5439	0.4386	0.5263	0.5
5	0.0819	0.0945	0.0949	0.0675		5	0.5	0.4286	0.5357	0.5
6	0.0915	0.1038	0.1039	0.075		6	0.4909	0.4364	0.5091	0.5
7	0.1006	0.1143	0.1103	0.0804		7	0.4259	0.3519	0.5185	0.5
8	0.1082	0.124	0.1199	0.0838		8	0.3962	0.3019	0.4717	0.5
9	0.1127	0.1311	0.126	0.0854		9	0.4423	0.3462	0.5385	0.5
10	0.1181	0.1395	0.1336	0.0877		10	0.3529	0.2941	0.4902	0.5
11	0.1221	0.1458	0.1432	0.0893		11	0.3	0.24	0.44	0.5
12	0.1249	0.1496	0.1488	0.0911		12	0.3469	0.2653	0.4898	0.5

[Referring to section 4.1.2] Evaluation and comparison of models on how well they can forecast the exchange rate change/return, for the JPY and CHF during the sample period of 1996.01-2009.05. Rolling estimation with a window size of 101. Forecast window 2004.06-2009.05. The notations used in the tables: *ARDL*: Autoregressive distributed lags model discussed in section 4.1.1; $\dots + F^{1^{st}} \equiv ARDL + F^{1^{st}}$: ARDL model augmented with factors in linear form; $\dots + F^{1^{st}, 2^{nd}} \equiv ARDL + F^{1^{st}, 2^{nd}}$: ARDL model augmented with factor in non-linear form (including the second order factors)^a. *RM*: Random walk model; *Naive*: Model predicts equal probability of change in direction. 1-12 in the left column are the respective forecast horizons. Numbers in bold indicate the best forecasts at each horizon. **RMSE**: root of mean squared forecast errors. **Direct.**: the probability of predicted changes in the right direction.

^aNote that the forecasts by $ARDL + F^{2^{nd}}$ model are reported for JPY

Table 11: Forecast evaluation and comparison. NOK and CAD

NOK Forecasts, $\widehat{\Delta e_{t+h}} I_t$										
RMSE	ARDL	... + $F^{1^{st}}$... + $F^{1^{st}, 2^{nd}}$	RW		Direct.	ARDL	... + $F^{1^{st}}$... + $F^{1^{st}, 2^{nd}}$	Naive
1	0.0357	0.0367	0.0338	0.0353		1	0.4894	0.4681	0.5532	0.5
2	0.0627	0.0651	0.0600	0.0595		2	0.5	0.5	0.6087	0.5
4	0.0865	0.0912	0.0871	0.0799		4	0.4667	<i>0.5556</i>	0.7111	0.5
4	0.1072	0.1115	0.1125	0.0972		4	0.5	<i>0.5909</i>	0.6818	0.5
5	0.1264	0.1309	0.1383	0.1123		5	0.4884	<i>0.5581</i>	0.6744	0.5
6	0.1424	0.1485	0.1645	0.1237		6	0.4524	<i>0.5476</i>	0.6429	0.5
7	0.1566	0.1552	0.1819	0.1327		7	0.4146	<i>0.5366</i>	0.7073	0.5
8	0.1696	0.1483	0.1856	0.1405		8	0.35	0.5	0.7500	0.5
9	0.1808	0.1595	0.2018	0.1463		9	0.3077	0.4359	0.7436	0.5
10	0.1887	0.1696	0.2247	0.1499		10	0.2368	0.2895	0.7368	0.5
11	0.1946	0.1778	0.2471	0.1525		11	0.2432	0.2162	0.7297	0.5
12	0.1995	0.1863	0.2698	0.1548		12	0.2222	0.25	0.7222	0.5

CAD Forecasts, $\widehat{\Delta e_{t+h}} I_t$										
RMSE	ARDL	... + $F^{1^{st}}$... + $F^{1^{st}, 2^{nd}}$	RW		Direct.	ARDL	... + $F^{1^{st}}$... + $F^{1^{st}, 2^{nd}}$	Naive
1	0.0265	0.0279	0.0321	0.0275		1	<i>0.6042</i>	0.6458	<i>0.6042</i>	0.5
2	0.0459	0.0486	0.0581	0.0437		2	0.4894	0.5957	0.4681	0.5
3	0.0614	0.0675	0.0811	0.0552		3	0.3696	0.5435	0.413	0.5
4	0.075	0.0834	0.0996	0.0663		4	0.3556	0.4889	0.4222	0.5
5	0.0905	0.1	0.1181	0.0787		5	0.2727	0.5	0.4545	0.5
6	0.1071	0.1218	0.1406	0.0902		6	0.2326	0.3721	0.4186	0.5
7	0.119	0.1394	0.1615	0.0994		7	0.1429	0.381	0.4048	0.5
8	0.1273	0.1453	0.1662	0.1067		8	0.1463	0.4146	0.4146	0.5
9	0.1352	0.1553	0.1852	0.1124		9	0.15	0.4	0.45	0.5
10	0.1424	0.1658	0.2064	0.1171		10	0.2051	0.359	0.3846	0.5
11	0.1482	0.1742	0.229	0.1212		11	0.1579	0.3158	0.4211	0.5
12	0.153	0.1803	0.2493	0.125		12	0.1892	0.2703	0.4865	0.5

[Referring to section 4.1.2] Evaluation and comparison of models on how well they can forecast the exchange rate change/return, for the NOK and CAD during the sample periods of 1998.01-2009.05 and 1999.01-2009.05. Rolling estimation for the NOK with a window size of 89. Recursive estimation for the CAD. Forecast window 2004.06-2009.05. The notations used here are the same as in the previous table.

Table 12: Forecast evaluation and comparison. AUD and CAD

AUD Forecasts, $\widehat{\Delta e_{t+h}}|I_{t+h}$

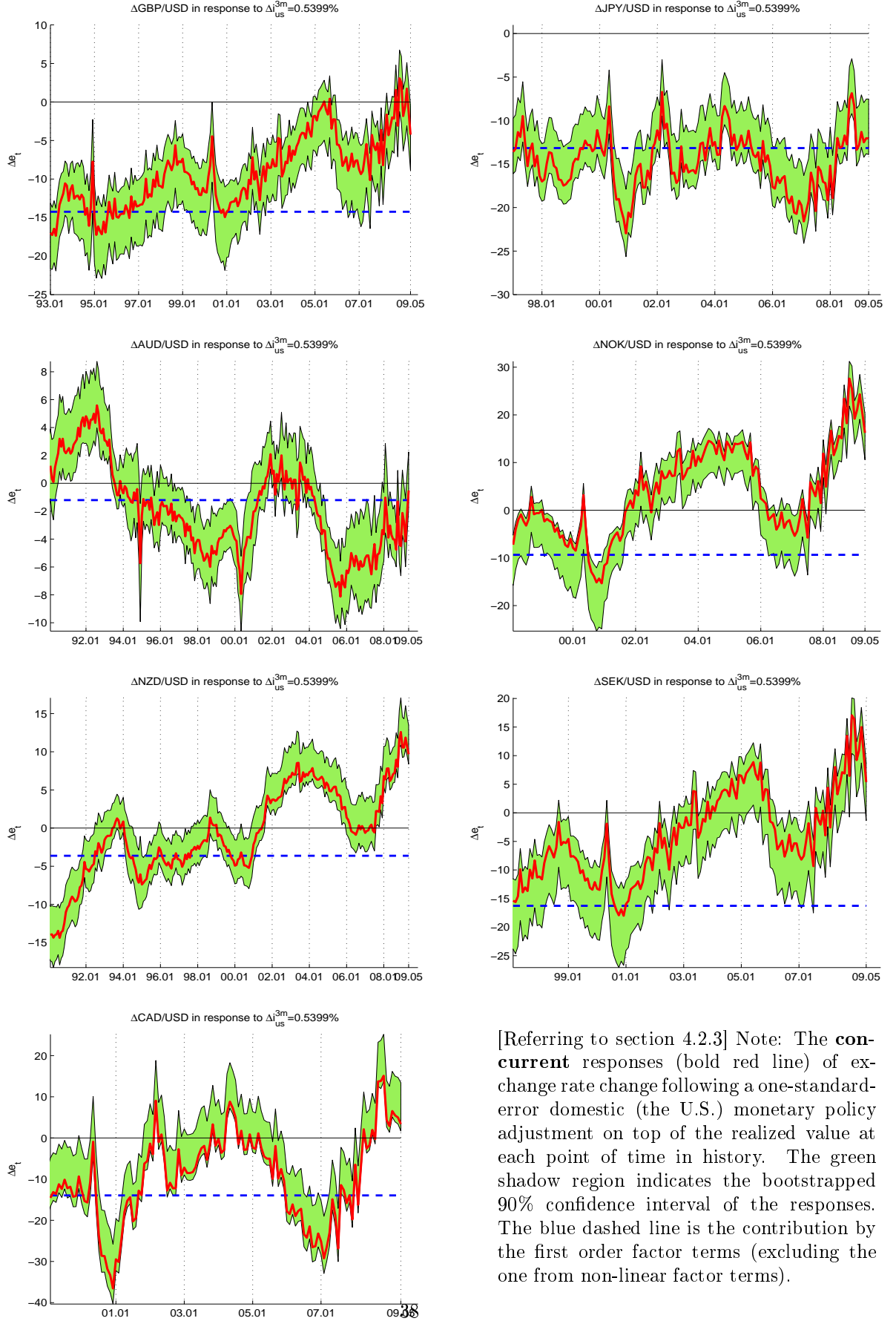
RMSE	ARDL	... + $F^{1^{st}}$... + $F^{1^{st}, 2^{nd}}$	RW		Direct.	ARDL	... + $F^{1^{st}}$... + $F^{1^{st}, 2^{nd}}$	Naive
1	0.0437	0.038	<i>0.0414</i>	0.0423		1	0.5167	0.6333	0.6667	0.5
2	0.0729	0.0595	<i>0.0599</i>	0.0658		2	0.5254	0.5932	0.6441	0.5
3	0.0925	0.0744	<i>0.0748</i>	0.0828		3	0.4483	0.6207	0.7069	0.5
4	0.1141	0.0916	0.0874	0.0989		4	0.4737	0.6667	0.8421	0.5
5	0.1342	0.1078	0.1014	0.1129		5	0.3929	0.6607	0.8036	0.5
6	0.1547	0.1244	0.1202	0.1262		6	0.4	0.5818	0.8	0.5
7	0.1688	0.137	0.1394	0.1336		7	0.4074	0.5185	0.8333	0.5
8	0.1773	0.1467	0.1214	0.1393		8	0.4151	0.5094	0.7925	0.5
9	0.1882	0.1574	0.1268	0.1435		9	0.3846	0.4808	0.8077	0.5
10	0.1961	0.1666	0.1335	0.1447		10	0.3922	0.451	0.8039	0.5
11	0.2026	0.1751	0.1471	0.1461		11	0.42	0.48	0.8	0.5
12	0.2076	0.1834	0.1619	0.1474		12	0.4286	0.5306	0.7551	0.5

CAD Forecasts, $\widehat{\Delta e_{t+h}}|I_{t+h}$

RMSE	ARDL	... + $F^{1^{st}}$... + $F^{1^{st}, 2^{nd}}$	RW		Direct.	ARDL	... + $F^{1^{st}}$... + $F^{1^{st}, 2^{nd}}$	Naive
1	0.0265	0.0273	0.0322	0.0275		1	0.6042	0.6458	0.4583	0.5
2	0.0401	0.0433	0.0585	0.0437		2	0.6596	0.6596	0.4894	0.5
3	0.0543	0.0601	0.0872	0.0552		3	0.587	0.6304	0.5217	0.5
4	0.0663	0.0767	0.1211	0.0663		4	0.4667	0.5556	0.4889	0.5
5	0.0785	0.0947	0.1541	0.0787		5	0.5	0.5909	0.5455	0.5
6	0.0896	0.1123	0.1826	0.0902		6	0.4884	0.5814	0.5349	0.5
7	0.1002	0.1304	0.2092	0.0994		7	0.5	0.5952	0.5714	0.5
8	0.1087	0.148	0.2284	0.1067		8	0.4878	0.561	0.6098	0.5
9	0.1174	0.1673	0.2591	0.1124		9	0.475	0.6	0.675	0.5
10	0.125	0.1869	0.2849	0.1171		10	0.4872	0.6154	0.641	0.5
11	0.1313	0.2051	0.3101	0.1212		11	0.5263	0.6053	0.6316	0.5
12	0.1375	0.2239	0.3372	0.125		12	0.4865	0.6216	0.6486	0.5

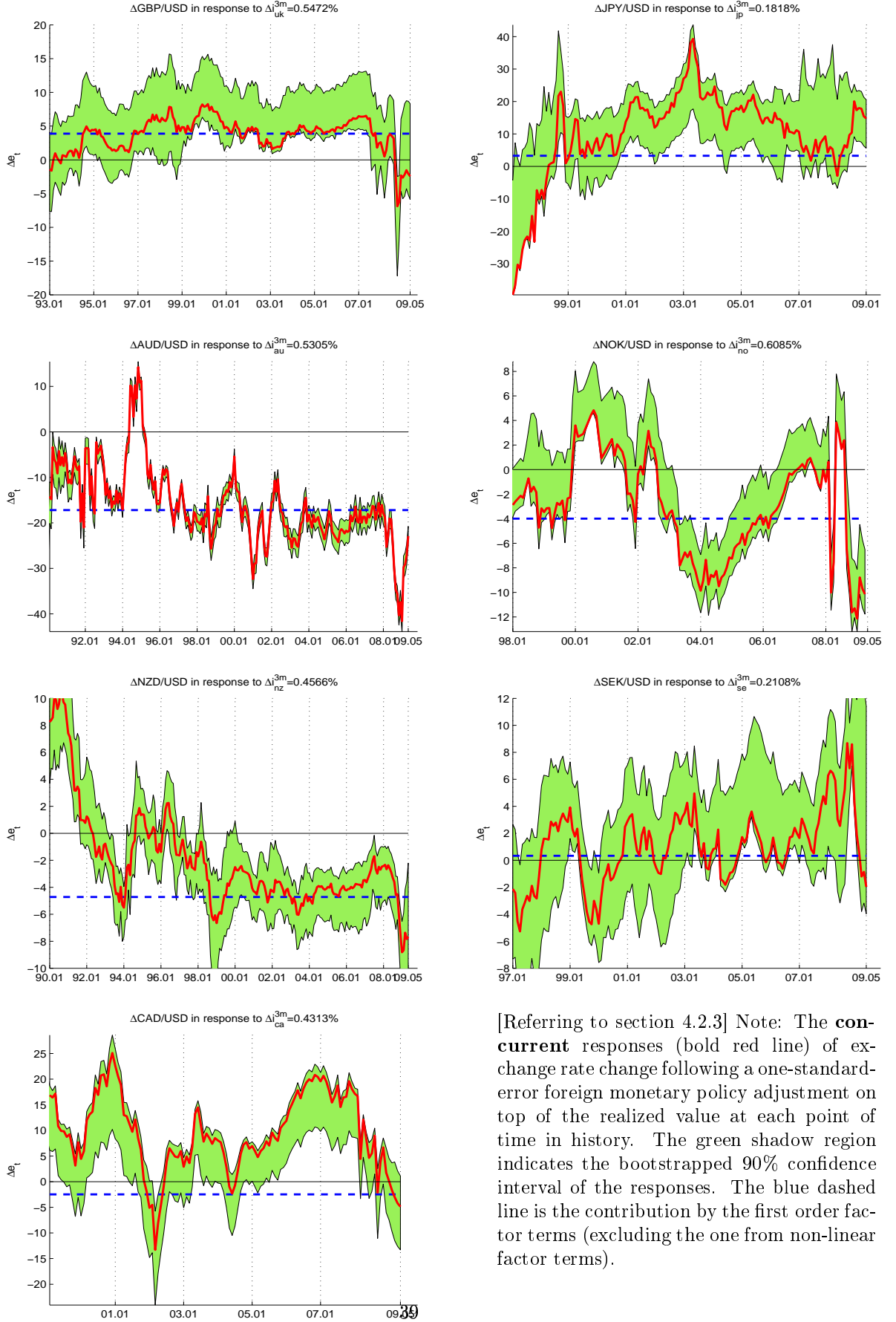
[Referring to section 4.1.2] Evaluation and comparison of models on how well they can forecast the exchange rate change/return, for the AUD during the sample period of 1990.01-2009.05, and for the CAD during 1999.01-2009.05. **Time- $(t+h)$ information is used for forecasting**, i.e. the contemporaneous values for the right-hand-side variables are used. Rolling estimation with window size 173 for the AUD. Forecast window 2004.06-2009.05 for the AUD and 2005.06-2009.05 for the CAD. The notations used here are the same as in the previous table.

Figure 3: Exchange rate change in response to U.S. monetary policy change



[Referring to section 4.2.3] Note: The **current** responses (bold red line) of exchange rate change following a one-standard-error domestic (the U.S.) monetary policy adjustment on top of the realized value at each point of time in history. The green shadow region indicates the bootstrapped 90% confidence interval of the responses. The blue dashed line is the contribution by the first order factor terms (excluding the one from non-linear factor terms).

Figure 4: Exchange rate change in response to foreign monetary policy change



[Referring to section 4.2.3] Note: The **concurrent** responses (bold red line) of exchange rate change following a one-standard-error foreign monetary policy adjustment on top of the realized value at each point of time in history. The green shadow region indicates the bootstrapped 90% confidence interval of the responses. The blue dashed line is the contribution by the first order factor terms (excluding the one from non-linear factor terms).

Table 13: Robustness Check of Significance of the Risk Premium

	Reported		Full Sample		Early Days		Common		No-Crisis	
AUD	90.01-09.05	y	87.02-09.05	y	90.01-99.12	n	98.01-09.05	\sim y	98.01-08.01	n
NZD	90.01-09.05	y	90.01-09.05	y	90.01-99.12	y	98.01-09.05	\sim y	98.01-08.01	y
GBP	93.01-09.05	\sim y	79.01-09.05 ³⁰	y	79.01-95.12	y	98.01-09.05	y	98.01-08.01	y
SEK	93.01-09.05	y	93.01-09.05	y	93.01-00.12	y	98.01-09.05	y	98.01-08.01	y
CHF	96.01-09.05	y	88.01-09.05	\sim n	88.01-95.12	y	98.01-09.05	y	98.01-08.01	y
JPY	96.01-09.05	y	85.01-09.05	\sim n	85.01-95.12	\sim n	98.01-09.05	y	98.01-08.01	y
NOK	98.01-09.05 '	y	98.01-09.05	y	n.a.		98.01-09.05	y	98.01-08.01	y-
CAD	99.01-09.05	y	86.01-09.05	n	86.01-97.12	y	98.01-09.05	\sim y	98.01-08.01	y

[Referring to section 5.1] Robustness check of the *ARDL* model with risk premium for the dynamic exchange rate, *equation 1*, for all currencies at different time horizons. The definition of 'Full Sample', 'Early Days', 'Common' and 'No-Crisis' are shown in the text. In the table, an 'y' indicates the two criteria for robustness are met and there is strong evidence for the proposed $ARDL + F^{1st,2nd}$ model (risk premium is important and should include the second order of factors). An 'y-' means a relatively weaker 'y'. The ' \sim y' and ' \sim n' mean the second orders factors are significant, but the goodness of fit for the $ARDL + F^{1st,2nd}$ as measured by *AIC* is very close to but a bit worse than the $ARDL + F^{1st}$ model for ' \sim y' and than $ARDL$ model for ' \sim n'. This should be taken as evidence for the $ARDL + F^{1st}$ model. 'n' means the goodness of fit for the $ARDL + F^{1st,2nd}$ and $ARDL + F^{1st}$ model are not better than the $ARDL$ model, which means there is no strong evidence for the factor models even if the second order factors are significant.

Table 14: Robustness check in the presence of macro fundamentals. GBP and SEK

Δe_t	GBP (93.01-06.04)		SEK (97.01-09.05)	
	$\dots + F^{1^{st}, 2^{nd}}$	$\dots + F^{1^{st}, 2^{nd}} + Macro$	$\dots + F^{1^{st}, 2^{nd}}$	$\dots + F^{1^{st}, 2^{nd}} + Macro$
...
$\pi_t - \pi_t^*$		4.037740		-1.932342
$\Delta(m_t - m_t^*)$		-1.628168		-0.523363
$\Delta(y_t - y_t^*)$		-0.226431		0.108401
$\Delta(i_t - i_t^*)$		-16.56216		42.11352***
$y_{t-1}^{gap} - y_{t-1}^{gap*}$		-0.194011		-0.457220
$\pi_{t-1} - \pi_{t-1}^*$		-0.066324		-3.555682
<i>Adjusted - R</i> ²	0.256646	0.246244	0.344932	0.375426
<i>AIC</i>	9.243196	9.285574	9.550694	9.531860
Prob χ_{macro}^2	-	0.6890	-	0.0085 ***
Prob $\chi_{Factor12}^2$	0.0000***	0.0000 ***	0.0000***	0.0000 ***

Table 15: Robustness check in the presence of macro fundamentals. AUD and NZD

Δe_t	AUD (90.01-09.05)		NZD (90.01-09.05)	
	$\dots + F^{1^{st}, 2^{nd}}$	$\dots + F^{1^{st}, 2^{nd}} + Macro$	$\dots + F^{1^{st}, 2^{nd}}$	$\dots + F^{1^{st}, 2^{nd}} + Macro$
...
$\pi_t - \pi_t^*$		3.728521		8.584208
$\Delta(m_t - m_t^*)$		0.497165		-0.443546
$\Delta(y_t - y_t^*)$		-0.976477		-0.534847
$\Delta(i_t - i_t^*)$		-21.11592*		5.920638
$y_{t-1}^{gap} - y_{t-1}^{gap*}$		-0.563279		-1.009491*
$\pi_{t-1} - \pi_{t-1}^*$		-3.573037		-2.152234
<i>Adjusted - R</i> ²	0.214802	0.216439	0.177333	0.216439
<i>AIC</i>	9.943615	9.964044	9.635847	9.964044
Prob χ_{macro}^2	-	0.3973	-	0.2181
Prob $\chi_{Factor12}^2$	0.0000***	0.0000 ***	0.0000***	0.0000 ***

[Referring to section 5.1.1] Notes: Robustness check when including the conventional macro fundamentals in the working model of this paper. This can be accomplished by checking the joint significance of the coefficients of macro fundamentals and factors for the $ARDL + F^{1^{st}, 2^{nd}} + Macro$ model. The notations used in this table are: $\dots + F^{1^{st}, 2^{nd}} \equiv ARDL + F^{1^{st}, 2^{nd}}$: ARDL model augmented with factor in non-linear form (including the second order factors). $\dots + F^{1^{st}, 2^{nd}} + Macro \equiv ARDL + F^{1^{st}, 2^{nd}} + Macro$: ARDL model augmented with factors in non-linear form and traditional macro fundamentals.

Table 16: Robustness check in the presence of macro fundamentals. JPY and CHF

	JPY (96.01-09.05)			CHF (96.01-09.05)	
Δe_t	$\dots + F^{1^{st}, 2^{nd}}$	$\dots + F^{1^{st}, 2^{nd}} + Macro$		$\dots + F^{1^{st}, 2^{nd}}$	$\dots + F^{1^{st}, 2^{nd}} + Macro$
...
$\pi_t - \pi_t^*$		-2.167285			-0.252127
$\Delta(m_t - m_t^*)$		2.505537**			-1.646169
$\Delta(y_t - y_t^*)$		-0.537623			-0.941849
$\Delta(i_t - i_t^*)$		-3.826421			33.64608**
$y_{t-1}^{gap} - y_{t-1}^{gap*}$		-1.304823*			0.618132
$\pi_{t-1} - \pi_{t-1}^*$		-7.741083**			-8.552289
$Adjusted - R^2$	0.208859	0.257215		0.205917	0.244046
AIC	9.995506	9.960087		10.02602	10.00446
Prob χ_{macro}^2	-	0.0027 ***		-	0.0159 **
Prob $\chi_{Factor12}^2$	0.0000***	0.0000 ***		0.0000***	0.0000 ***

Table 17: Robustness check in the presence of macro fundamentals. NOK and CAD

	NOK (98.01-09.05)			CAD (99.01-09.05)	
Δe_t	$\dots + F^{1^{st}, 2^{nd}}$	$\dots + F^{1^{st}, 2^{nd}} + Macro$		$\dots + F^{1^{st}, 2^{nd}}$	$\dots + F^{1^{st}, 2^{nd}} + Macro$
...
$\pi_t - \pi_t^*$		0.556385			0.500650
$\Delta(m_t - m_t^*)$		-0.345970			-2.122777***
$\Delta(y_t - y_t^*)$		-0.238424			0.223045
$\Delta(i_t - i_t^*)$		-1.422903			-8.393160
$y_{t-1}^{gap} - y_{t-1}^{gap*}$		-0.434642			-0.080269
$\pi_{t-1} - \pi_{t-1}^*$		-6.341001*			-7.942008**
$Adjusted - R^2$	0.143231	0.129621		0.353113	0.379620
AIC	10.08251	10.13276		9.064817	9.053737
Prob χ_{macro}^2	-	0.4904		-	0.0556 *
Prob $\chi_{Factor12}^2$	0.000***	0.0000 ***		0.0004 ***	0.0011 ***

[Referring to section 5.1.1] Notes: Robustness check when including the conventional macro fundamentals in the working model of this paper. This can be accomplished by checking the joint significance of the coefficients of macro fundamentals and factors for the $ARDL + F^{1^{st}, 2^{nd}} + Macro$ model. The notations used in this table are: $\dots + F^{1^{st}, 2^{nd}} \equiv ARDL + F^{1^{st}, 2^{nd}}$: ARDL model augmented with factor in non-linear form (including the second order factors). $\dots + F^{1^{st}, 2^{nd}} + Macro \equiv ARDL + F^{1^{st}, 2^{nd}} + Macro$: ARDL model augmented with factors in non-linear form and traditional macro fundamentals.

Table 18: Regression results of exchange rate models with expectation variables. GBP and SEK

Δe_t	GBP (93.01-09.05)				SEK (93.01-09.05)		
	$ARDL$	$ARDL + \tilde{R}$	$ARDL + \tilde{R}, \tilde{\Sigma}$		$ARDL$	$ARDL + \tilde{R}$	$ARDL + \tilde{R}, \tilde{\Sigma}$
$constant$	5.051606	53.47841***	38.09224*		-0.212793	52.58799***	80.20335**
Δe_{t-1}	0.072882	-0.073139	-0.131747**		0.450109	0.387825***	0.370036***
Δe_{t-2}					-0.170434	-0.189277***	-0.190543***
Δe_{t-3}							
Δe_{t-4}							
Δe_{t-5}							
Δe_{t-6}							
Δe_{t-7}							
Δe_{t-8}							
$i_t - i_t^*$					7.361294	23.77210**	29.10528**
$i_{t-1} - i_{t-1}^*$	3.318426	18.11328***	13.60867***		-20.22149	-24.16103**	-28.97631**
$i_{t-2} - i_{t-2}^*$					12.65682	10.01266	12.11554
$i_{t-3} - i_{t-3}^*$							
$i_{t-4} - i_{t-4}^*$							
$i_{t-5} - i_{t-5}^*$							
$\bar{r}_{t,...,t+m t}^*$		16.54400***	13.59611***			22.89243***	24.87222**
$\bar{r}_{t,...,t+m t}$		-27.00200***	-21.41308***			-24.08511***	-32.19437***
$\bar{t}p_{t,...,t+m t}^*$		-0.066681	14.36035**			-21.59628	-20.51940
$\bar{t}p_{t,...,t+m t}$		-7.555211	-8.098500			-14.50292**	-17.98192**
σ_r^*			46.51901**				3.058705
σ_r			-12.19932				-156.2141*
σ_{tp}^*			-62.84368***				3.303434
σ_{tp}			4.751782				23.50929
$Adj. - R^2$	0.014807	0.154524	0.188009		0.161825	0.225891	0.228619
AIC	9.577392	9.453952	9.469410		9.604529	9.544460	9.559916
$Prob \chi_{\tilde{R}}^2$	-	0.0000***	0.0017 ***		-	0.0002 ***	0.0153 **
$Prob \chi_{\tilde{\Sigma}}^2$	-	-	0.0000***		-	-	0.1090
$Prob \chi_{\tilde{R}, \tilde{\Sigma}}^2$	-	-	0.0000***		-	-	0.0023 ***

[Referring to section 5.2.2] Comparison of models that with expectation variables, on how well they can fit the data for the GBP and SEK during the period of 1993.01-2009.05. The notations used in the tables are $ARDL$: Autoregressive distributed lags model discussed in section 2.3; $ARDL + \tilde{R}$: ARDL model augmented with means of expectations as defined in text; $ARDL + \tilde{R}, \tilde{\Sigma}$: ARDL model augmented with means and variances of expectations. The $Adj. - R^2$ and AIC are reported for each model and each currency. Especially, the nulls on the $\tilde{C} = 0_{6 \times 1}$, $\tilde{D} = 0_{6 \times 6}$, \tilde{C} and \tilde{D} are jointly zero are tested respectively, probabilities of the χ^2 statistics are reported. The '***', '**', '*' denote significance at 1%, 5% and 10% level respectively. All the inferences are adjusted with the Newey-West autocorrelation and heteroskedasticity consistent standard errors. The lag orders of the $ARDL$ model are selected by the AIC .

Table 19: Regression results of exchange rate models with expectation variables. AUD and NZD

Δe_t	AUD (90.01-09.05)				NZD (90.01-09.05)		
	$ARDL$	$ARDL + \tilde{R}$	$ARDL + \tilde{R}, \tilde{\Sigma}$		$ARDL$	$ARDL + \tilde{R}$	$ARDL + \tilde{R}, \tilde{\Sigma}$
$constant$	-0.987931	18.80270	74.91103*		0.456397	-3.212269	46.18975
Δe_{t-1}	0.148618	0.135738	0.119893		0.339334***	0.306045***	0.277770***
Δe_{t-2}	-0.093554	-0.108402	-0.107434				
Δe_{t-3}	0.137384	0.117418*	0.123672*				
Δe_{t-4}							
Δe_{t-5}							
Δe_{t-6}							
Δe_{t-7}							
Δe_{t-8}							
$i_t - i_t^*$	-23.07489***	-3.533960	-2.121386				
$i_{t-1} - i_{t-1}^*$	22.31848***	11.73386	13.02233		0.113527	4.559638	4.379514
$i_{t-2} - i_{t-2}^*$							
$i_{t-3} - i_{t-3}^*$							
$i_{t-4} - i_{t-4}^*$							
$i_{t-5} - i_{t-5}^*$							
$\bar{r}_{t,...,t+m t}^*$		17.96603*	16.14432*			10.16583**	4.808765
$\bar{r}_{t,...,t+m t}$		-22.61981**	-33.14268**			-10.32528**	-14.61201**
$\bar{t}p_{t,...,t+m t}^*$		15.64105*	29.22874**			5.224399	8.144440
$\bar{t}p_{t,...,t+m t}$		-19.74823*	-19.00202*			-5.122769	-4.007528
σ_r^*			36.29833*				18.98951*
σ_r			-119.9203				-126.9446
σ_{tp}^*			-25.04547				-20.13576
σ_{tp}			-7.169475				6.151540
$Adj. - R^2$	0.031807	0.077604	0.090584		0.105866	0.113188	0.110665
AIC	10.08084	10.05313	10.05519		9.646137	9.654705	9.674024
$\text{Prob} \chi_{\tilde{R}}^2$	-	0.0415 ***	0.0916 *		-	0.1490	0.2411
$\text{Prob} \chi_{\tilde{\Sigma}}^2$	-	-	0.1839		-	-	0.4195
$\text{Prob} \chi_{\tilde{R}, \tilde{\Sigma}}^2$	-	-	0.1032		-	-	0.1095

[Referring to section 5.2.2] Comparison of models that with expectation variables, on how well they can fit the data for the AUD and NZD during the period of 1990.01-2009.05. The notations used here are the same as in the previous table.

Table 20: Regression results of exchange rate models with expectation variables. JPY and CHF

Δe_t	JPY (96.01-09.05)				CHF (96.01-09.05)		
	$ARDL$	$ARDL + \tilde{R}$	$ARDL + \tilde{R}, \tilde{\Sigma}$		$ARDL$	$ARDL + \tilde{R}$	$ARDL + \tilde{R}, \tilde{\Sigma}$
$constant$	13.85854**	17.82083	44.59713*		7.828034	51.47989***	118.5425***
Δe_{t-1}	-0.029807	-0.076892	-0.085555		-0.025616	-0.156617	-0.214426*
Δe_{t-2}	0.052889	0.034277	0.016529		-0.031518	-0.158794**	-0.214310***
Δe_{t-3}	0.051886	0.044973	0.030635		-0.000585	-0.134005*	-0.202867***
Δe_{t-4}	-0.126041*	-0.096892	-0.105065		-0.156540	-0.270170***	-0.326389***
Δe_{t-5}	-0.141809*	-0.139827	-0.157501*		0.024588	-0.055232	-0.105350
Δe_{t-6}	-0.064520	-0.062969	-0.082306		-0.060323	-0.118015	-0.155489
Δe_{t-7}					-0.087961	-0.093332	-0.111345
Δe_{t-8}					-0.058058	-0.057113	-0.076118
$i_t - i_t^*$					4.263529	35.48326***	25.74151**
$i_{t-1} - i_{t-1}^*$	-9.630588	51.67464**	49.25467**		-1.056968	-0.859017	-3.139582
$i_{t-2} - i_{t-2}^*$	23.60070	-0.395568	4.635691		-36.80506	-38.18719	-33.03191
$i_{t-3} - i_{t-3}^*$	-25.51819	-26.30274	-26.81562		29.96227	37.43218**	33.17237**
$i_{t-4} - i_{t-4}^*$	25.93667	30.85263	32.59334				
$i_{t-5} - i_{t-5}^*$	-17.95750	-32.02627	-34.78036				
$\bar{r}_{t,...,t+m t}^*$		62.69724***	50.75497**			45.39369***	38.03451***
$\bar{r}_{t,...,t+m t}$		-36.77281***	-43.54631***			-58.72329***	-59.89601***
$\bar{t}p_{t,...,t+m t}^*$		7.572569	11.52264			43.99960***	51.34943***
$\bar{t}p_{t,...,t+m t}$		15.58203*	15.22046			-18.40315	-38.60343***
σ_r^*			-3474.719**				-155.8028
σ_r			-35.99504				-297.7682**
σ_{tp}^*			30.71830				-4.972996
σ_{tp}			-6.579628				40.16002
$Adj. - R^2$	0.069476	0.116267	0.118840		0.013888	0.142123	0.159350
AIC	10.13415	10.03381	10.05261		10.15344	10.03642	10.03765
$\text{Prob } \chi_{\tilde{R}}^2$	-	0.0000 ***	0.0002 ***			0.0000 ***	0.0000 ***
$\text{Prob } \chi_{\tilde{\Sigma}}^2$	-	-	0.1378				0.0851 **
$\text{Prob } \chi_{\tilde{R}, \tilde{\Sigma}}^2$	-	-	0.0000 ***				0.0001 ***

[Referring to section 5.2.2] Comparison of models that with expectation variables, on how well they can fit the data for the JPY and CHF during the period of 1996.01-2009.05. The notations used here are the same as in the previous table.

Table 21: Regression results of exchange rate models with expectation variables. NOK and CAD

Δe_t	NOK (98.01-09.05)				CAD (99.01-09.05)		
	$ARDL$	$ARDL + \tilde{R}$	$ARDL + \tilde{R}, \tilde{\Sigma}$		$ARDL$	$ARDL + \tilde{R}$	$ARDL + \tilde{R}, \tilde{\Sigma}$
$constant$	1.632000	-27.26461	-45.67158		2.285305	24.82102*	57.18412**
Δe_{t-1}	0.165972	0.139623	0.044861		0.307091***	0.289726***	0.242445***
Δe_{t-2}					0.057781	0.031145	0.026617
Δe_{t-3}					-0.119025	-0.135985	-0.162611
Δe_{t-4}					0.239679**	0.209022**	0.216185**
Δe_{t-5}					-0.014012	-0.010851	0.046324
Δe_{t-6}					-0.249555**	-0.252537**	-0.227411**
Δe_{t-7}							
Δe_{t-8}							
$i_{t-1} - i_{t-1}^*$	0.272399	2.268678	-7.562408		1.161302	9.260196	5.507445
$i_{t-2} - i_{t-2}^*$					-17.99161	-16.79831	-16.63913
$i_{t-3} - i_{t-3}^*$					23.36692*	24.72350**	24.80259**
$i_{t-4} - i_{t-4}^*$					21.97614**	20.20368**	18.47577*
$i_{t-5} - i_{t-5}^*$					-29.29930***	-33.94504***	-37.20897***
$\bar{r}_{t,...,t+m t}^*$		13.99852	9.824741			15.49376*	12.89360
$\bar{r}_{t,...,t+m t}$		-9.026967**	-4.066482			-17.73629*	-35.55993**
$\bar{t}p_{t,...,t+m t}^*$		36.07518	137.1314*			10.35198	56.40581*
$\bar{t}p_{t,...,t+m t}$		-12.72682*	-26.67426***			-20.41900	-18.31073
σ_r^*			16.96689*				316.8675**
σ_r			-290.1149**				-207.5318*
σ_{tp}^*			-242.0556*				-48.34425*
σ_{tp}			61.12793*				-33.68473
$Adj. - R^2$	0.013185	0.013034	0.065297		0.179898	0.198723	0.269768
AIC	10.10530	10.13355	10.10628		9.187586	9.192323	9.126091
$\text{Prob } \chi_{\tilde{R}}^2$	-	0.1576	0.0467 **		-	0.2289	0.1198
$\text{Prob } \chi_{\tilde{\Sigma}}^2$	-	-	0.0527 *		-	-	0.1605
$\text{Prob } \chi_{\tilde{R}, \tilde{\Sigma}}^2$	-	-	0.0105 **		-	-	0.1292

[Referring to section 5.2.2] Comparison of models that with expectation variables, on how well they can fit the data for the NOK and CAD during the period of 1998.01-2009.05 and 1999.01-2009.05, respectively. The notations used here are the same as in the previous table.