Allocating Stochastic Public Expenditures under Revenue Volatility: The Role of Strict Budget Constraints*

(Preliminary version, do not cite)

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February 16, 2014

Abstract

This paper analyzes the impact of budgetary uncertainty on the allocation of public funds. In particular, the government is confronted in its allocation of public funds by uncertainty on the expenditure side of the budget in the form of cost shocks and by volatility on the revenue side, affecting public provision via the requirement for fiscal discipline. Comparison of the stochastic model to its deterministic counterpart reveals welfare losses as a consequence of both sources of uncertainty. The government's inability to adjust allocations in the short run, however, is found to alter the losses. Given the market pressure for fiscal discipline in the form of increased costs of public good provision in case of a public deficit, uncertainty is found to induce hedging via lower public goods provision and rainy-day funds, albeit possibly at the cost of larger welfare losses. Finally, while the buffers' success is dependent on the government's knowledge of the respective probability density functions, partial contemporaneous adjustment reduces welfare losses with certainty. Nevertheless, both the preferences for (i.e. demand elasticities) and the relative costs of the public goods are found to determine the strategy for an optimal budget process with partial adjustment under uncertainty.

Keywords: Public Good Provision, Fiscal Revenue Volatility, Price Shocks, Fiscal Discipline

JEL codes: D81, E62, H41, H50, H62

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*While accepting full responsibility for any mistakes, I gratefully acknowledge helpful comments by Stef Proost and Eric Schmitt. This paper is produced as part of track A4 of the "Steunpunt Fiscaliteit en Begroting" (http://www.steunpuntfb.be), a project funded by the Flemish Government. The views expressed in this paper are those of the author and do not necessarily represent those of the Flemish Government. The latter can also not be held accountable for the uses of this text.

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Extended Abstract

With the Great Recession came a surge in fiscal policy research concerning the impact of fiscal consolidations and austerity measures, the power of fiscal policy in a liquidity trap and its interlinkages with monetary policy. Yet, the concerns how to tackle the challenges posed by the financial crisis and the ensuing recession were highly intertwined with the uncertainty underlying economic decision making. After all expectations often had to be revised and further fiscal adjustments were required.

Given the importance of budgetary uncertainty, stochastic analysis of the fiscal balance and the resulting debt stock is now standard practice. The EC and IMF typically provide confidence intervals for their budgetary projections. Similarly, fan-charts and scenario analyses of future debt paths are provided The IMF's World Economic Outlook and the EC's Sustainability Reports are prime examples. More recently, policy advise is also pushed towards (Value-at-Risk) indicators of fiscal sustainability based on Monte Carlo simulations taking into account the uncertainty in economic growth rates, interest rates and the policy makers' reaction to increasing debt levels (see e.g. Celasun et al., 2007; Budina and van Wijnbergen, 2008; van Ewijk et al., 2013). In addition, dynamic stochastic general equilibrium models and vector autoregression analyses are continued to be applied to infer on the macroeconomic impact of fiscal policy shocks.

Looking at the components making up the deficit, fiscal revenue volatility, on the one hand, is well documented. For instance, it is well known that personal and corporate income tax revenues highly depend on economic activity. Moreover, an increase in the short run volatility of fiscal revenues, starting as early as the 1970s, was recently documented and mainly attributed to tax rate changes (Seegert, 2013). Expenditure volatility, on the other hand, has been left seemingly untouched. Except for social security transfers and expenditures on education, changes are thought to be political in nature. Still, from US data it follows that the volatility in fiscal revenues leads the volatility in public expenditures in time. There thus seems to be some interdependence via the budget constraint and its requirement for fiscal discipline. Yet, surprisingly little work has tried to explain the origin and dynamics of these shocks to public expenditures. Nor is there a theoretical framework to analyze the impact of such uncertainty on public welfare or the optimal budget process to take this into account.

The **purpose of this paper** is to explore the impact of budgetary uncertainty on the allocation of public funds. In particular, I consider a government whose expenditures are made up of the provision of multiple public goods with different (short run) marginal benefit curves, i.e. different demand curves. For succinctness, the marginal cost of provision of the first public good is standardized at one, with the others' marginal cost proportional to one. Nonetheless, to model the requirement for fiscal discipline an additional cost is added to the government's objective function if the allocation would result in a public deficit.

Hence, there is an asymmetry as the cost is higher in case of deficit spending. This cost can for instance be interpreted as the interest burden as a result of an increasing debt stock or the accompanying negative spillovers within a monetary union or federation. Solving the government's maximization problem (thus, deciding on the provision of public goods) results in the equilibrium allocation, i.e. **the deterministic benchmark**.

Given that preferences for public goods are rather stable, the most credible way to add **expenditure uncertainty** to the model is via their cost of provision. An uncertain cost might for instance result from shocks to wages, shocks to energy prices (e.g. the oil price) or a very harsh winter. To model the equilibrium general continuous probability density functions for the cost shocks to both public goods are presumed. It follows that the ex post first-best allocation can only be achieved if policy makers deviate from their ex ante committed public good provisions. In particular, under the condition of no adjustment of public good allocations once the shock has occurred, policy makers can only achieve a second-best allocation. Comparison with the optimal allocation in the deterministic benchmark thus reveals *a first channel of welfare losses*.

In addition to expenditure uncertainty, **uncertainty on the revenue side** of the budget is considered. If policy makers do not know the exact realization of fiscal revenues ex ante, *a second channel of welfare losses* will result from revenue volatility. Similar to the result for expenditure shocks, policy makers bound by initial budgets are restricted to a second-best allocation. For example, fiscal revenues might just as well turn out to be lower than expected and hence lower public provisions might have been preferable ex post.

When applying **a strict budget constraint**, with a penalty in case of a deficit, policy makers - basing their decisions on the expected instead of the realized expenditures and revenues - will be confronted with additional uncertainty about the cost of public provisions. Hence a new allocation is computed. Next, the new allocation is compared to the stochastic model without the penalty.

In case of revenue uncertainty policy makers' expectations of the cost of provision depend on the distribution of public revenues, as the disciplinary cost of a deficit depends on this. Specifically, it is found that policy makers under the restraint of a strict budget constraint anticipate a higher cost due to the asymmetric penalty structure. Thus, the resulting allocations of public goods are smaller than their deterministic counterparts as well as their stochastic counterparts without the penalty. Therefore, the disciplinary cost brings short run allocations closer to the optimum in case lower than benchmark revenues are collected. Nonetheless, if actual revenues are higher, discipline is still in force as public expenditures are shown to be capped at the benchmark due to the resulting cost structure.

Under expenditure uncertainty, two cases are distinguished. First, for reasons of clarity, homogeneous shocks are considered, i.e. all public goods are affected by one and the same cost shock. Doing so policy makers are also found to reduce their public provisions to anticipate the expected positive penalty as a result of the asymmetric cost structure. Consequently, as with revenue uncertainty, the markets' *retaliation for fiscal indiscipline* via the cost of profligacy spurs policy makers to hedge against disadvantageous shocks, while maintaining discipline in advantageous times. Next, differentiating shocks among public goods does not change this result.

Even though the penalty as a result of the (expected) shocks will be anticipated by ex ante allocations different from the deterministic benchmark, the first and second channel of welfare losses may persist. Moreover, in addition to the distortions caused by the predetermination of the ex ante quantities of public provision, a disciplinary market mechanism may create additional distortions. In particular, the anticipation may just as well prove to have been unnecessary.

Nevertheless, it is to be noted that extending the framework to a multiperiod setting, allowing for the anticipatory reduction in public expenditures to be employed as rainy-day funds, does not entirely dispel such possible distortions. After all, the buffers' success is dependent on the government's knowledge of the respective probability density functions.

A resulting, pressing question for practitioners in public administrations is which expenditure category is best adjusted or used as a buffer if possible. Indeed, it is not unimaginable that certain categories of public expenditures are more easily adjustable than others. For example, large public investment projects might be perceived to be more acceptable to postpone in times of tight budget constraints than current expenditures for existing services. Allowing for **partial contemporaneous adjustment** (i.e. adjustment of a number of public goods once the shocks have occurred) illustrates the ability of the government to counter welfare losses due to uncertainty. The effectiveness with which a governments might benefit from such an adjustment nonetheless depends on the slope of the demand curve of the adjustable public good as well as its costs relative to the other goods. Using these criteria a strategy for optimizing the budget process under uncertainty are set forth.

In sum, one does best not consider the factors underlying volatility on the expenditure side in separation, as often done for the different categories on the revenue side of the budget. Looking at the budget as it is, a (dynamic) system of communicating vessels, helps to explain that not only cost uncertainty on the expenditure side is crucial for public good provisions and social welfare. Given the pressure for fiscal discipline, revenue volatility unmistakably plays its role too. Both types of uncertainty are found to result in welfare losses. Yet, the impact of uncertainty is not only detrimental from a disciplinary point of view.

1 The Rise of Uncertainty

With the Great Recession came a surge in fiscal policy research concerning the impact of fiscal consolidations and austerity measures, the power of fiscal policy in a liquidity trap and its interlinkages with monetary policy. Yet, the concerns how to tackle the challenges posed by the financial crisis and the ensuing recession were highly intertwined with the uncertainty underlying economic decision making. After all expectations often had to be revised and further fiscal adjustments were required.

Given the importance of budgetary uncertainty, stochastic analysis of the fiscal balance and the resulting debt stock is now standard practice. The EC and IMF typically provide confidence intervals for their budgetary projections. Similarly, fan-charts and scenario analyses of future debt paths are provided The IMF's World Economic Outlook and the EC's Sustainability Reports are prime examples. More recently, policy advise is also pushed towards (Value-at-Risk) indicators of fiscal sustainability based on Monte Carlo simulations taking into account the uncertainty in economic growth rates, interest rates and the policy makers' reaction to increasing debt levels (see e.g. Celasun et al., 2007; Budina and van Wijnbergen, 2008; van Ewijk et al., 2013). In addition, dynamic stochastic general equilibrium models and vector autoregression analyses are continued to be applied to infer on the macroeconomic impact of fiscal policy shocks.

Looking at the components making up the deficit, fiscal revenue volatility, on the one hand, is well documented. For instance, it is well known that personal and corporate income tax revenues highly depend on economic activity. Moreover, an increase in the short run volatility of fiscal revenues, starting as early as the 1970s, was recently documented and mainly attributed to tax rate changes (Seegert, 2013). Expenditure volatility, on the other hand, has been left seemingly untouched. Except for social security transfers and expenditures on education, changes are mainly thought to be political in nature. Still, from US data it follows that the volatility in fiscal revenues leads the volatility in public expenditures in time. There thus seems to be interdependence via the budget constraint and its requirement for fiscal discipline. Yet, surprisingly little work has tried to explain the origin and dynamics of these shocks to public expenditures. Nor is there a theoretical framework to analyze the impact of such uncertainty on public welfare or the optimal budgeting process to take this into account.

The purpose of this paper is to explore the impact of budgetary uncertainty on the allocation of public funds and via this channel the impact on public welfare. In particular, a model is set up in which a government chooses to allocate public funds over multiple public goods. The government is confronted in its allocation of public funds by uncertainty on the expenditure side of the budget in the form of cost shocks. Moreover, uncertainty on the revenue side affects public provision via the requirement for fiscal discipline. Comparison of the stochastic model to its deterministic counterpart reveals welfare losses as a consequence of both sources of uncertainty in combination with the government's inability to adjust allocations in the short run. Given the market pressure for fiscal discipline in the form of increased costs of public good provision in case of a public deficit, fiscal revenue uncertainty, however, is also found to have a hedging impact, albeit possibly at the cost of larger welfare losses. Consequently, the model helps to improve the budget process by explaining the dynamics of expenditure shocks and revenue volatility. Finally, it aids policy makers in their choice which expenditures to adjust or use as buffers by suggesting a more optimal allocation in case of uncertainty.

The rest of this paper is structured as follows. Section 2 outlines the deterministic model used as a benchmark for analysis. Next, section 3 provides some introductory examples of the results under budgetary uncertainty, which will be generalized and studied in depth subsequently. Then, in section 4 a first channel of welfare losses is identified in a general setting by adding uncertainty to the allocation problem in the form of cost shocks on the expenditure side of the budget. Section 5 focuses on a second stochastic component, this time on the revenue side, affecting the allocation of public goods and welfare via the strict borrowing constraint. Section 6 brings the two sources of uncertainty together. Then, in section 7 the process of providing buffer provisions for future shocks is added. This has to be distinguished from the government's ability to adjust allocations in the short run. The assumption that a government is unable to divert from its ex ante commitments is relaxed by allowing partial contemporaneous adjustments in section 8. Finally, section 9 provides in some concluding remarks.

2 The Benchmark Case

2.1 Basic Setup

Consider a government whose expenditures are made up of the provision of multiple public goods. In the setup presented below the public goods are labeled i, with $i \in \{A, B\}$. Consequently, the quantities of both services provided by the government are denoted as q_A and q_B, respectively. Both goods have different short run marginal benefit or demand curves (MB_i), derived from the general benefit curve $B(q_A, q_B)$. Yet, for simplicity, assume that the marginal cost of provision for public good A is standardized at 1, i.e. $MC_A = 1$. Let the marginal cost of good B be proportional to this by factor ω: $MC_B = \omega$. Hence, total costs are the sum of public goods provided: $C(q_A, q_B) = q_A + \omega q_B$. This assumption will be relaxed later. Furthermore, if quantities are such that for a price of 1 and ω respectively the optimal supply is $\{q_A^0, q_B^0\}$, the public funds allocated to both public goods can be illustrated as in figure 1.

To enable comparison of this basic benchmark to later cases, assume the following general, linear marginal benefit functions:[1]

[1]One could see such general linear marginal functions as a second-order Taylor approximation of the actual total benefit and cost functions (see e.g. Weitzman, 1974). In case of uncertainty, as illustrated in sections 4 and 5, such (empirical) derivation is justifiable as long

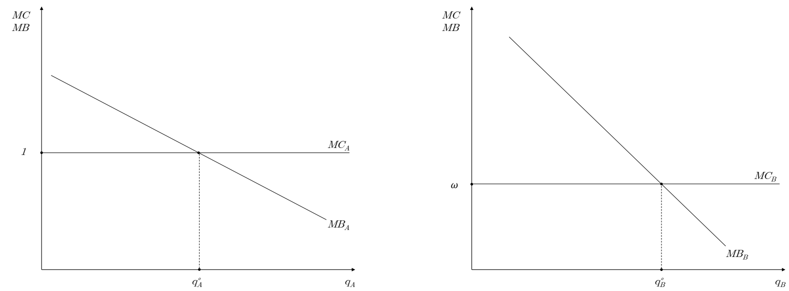

Figure 1: The Marginal Costs and Benefits of the Public Goods

$$MB_A(q_A) = a + bq_A \tag{1}$$

$$MB_B(q_B) = c + dq_B \tag{2}$$

with $a > 0$ and $c > 0$ to assure the existence of an equilibrium. Moreover, $b < 0$ and $d < 0$ are presumed to hold, i.e. the benefits of both public goods are concave downward. Then, if the government solves the following maximization problem

$$\underset{\{q_A, q_B\}}{\text{Max}} \Big[B(q_A, q_B) - C(q_A, q_B) \Big], \tag{3}$$

the optimal quantities of both public goods can be straightforwardly determined via the following first order conditions (FOCs):

$$q_A^0 = \frac{1-a}{b} \tag{4}$$

$$q_B^0 = \frac{\omega - c}{d}. \tag{5}$$

Finally, to refrain from any impact of the incurred public revenues on expenditure allocation, thus far public revenues are assumed to amount to $T^0 = q_A^0 + \omega q_B^0$. Hence, the public budget is balanced in equilibrium. This assumption is relaxed shortly.

2.2 Strict Budget Constraints

It is not unimaginable that an incurred public deficit would have disadvantageous effects on the economy. For instance, a deficit would imply an additional interest burden. This can be added to the problem by adding an additional cost or penalty of an additional unit of public deficit with the following structure:

$$p = \begin{cases} \bar{p}, & \text{if } \big(C(q_A, q_B) - T^0\big) > 0 \\ 0, & \text{if } \big(C(q_A, q_B) - T^0\big) \leq 0 \end{cases} \tag{6}$$

as the random term characterizing uncertainty is sufficiently small.

where $\bar{p} > 0$. As a result, the government will solve the following optimization problem (instead of (3)):

$$\underset{\{q_A, q_B\}}{\text{Max}} \Big[B(q_A, q_B) - C(q_A, q_B) - p\big(C(q_A, q_B) - T^0\big) \Big]. \tag{7}$$

Inserting the respective costs of public expenditures in (7) and simplifying yields $\underset{\{q_A, q_B\}}{\text{Max}} \Big[B(q_A, q_B) + pT^0 - (1+p)(q_A + \omega q_B) \Big]$.[2] This results in the following first order conditions:

$$q_A^{00} = \frac{(1+p) - a}{b} \tag{8}$$

$$q_B^{00} = \frac{(1+p)\omega - c}{d} \tag{9}$$

with p's structure defined as in (6).

As illustrated in figure 2, the optimization taking into account the stricter constraint does however not change the allocation of public goods as long as revenues are known ex ante to be T^0. As a result, $q_A^{00} = q_A^0$ and $q_B^{00} = q_B^0$ will still hold and the budget is balanced too.

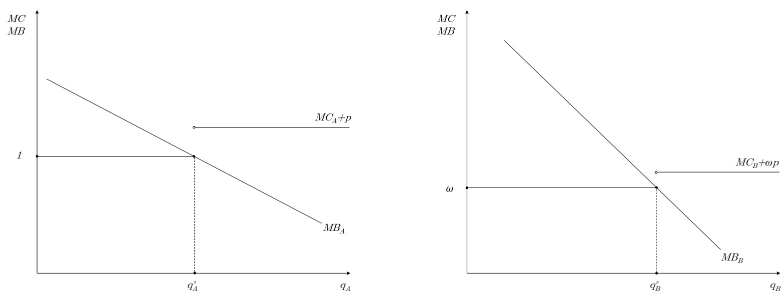

Figure 2: The Marginal Costs and Benefits of the Public Goods in case of Costly Deficits

In case $p = \bar{p}$ holds for $\big(C(q_A, q_B) - T^0\big) \leq 0$ as well, lower provisions of both public goods will be chosen by the government and a budget surplus results (as illustrated in appendix A.2). This is a logical result as the symmetry of p would, in addition to the economic cost of a deficit, imply a gain for the government via an additional fee in case of austere budgetary behavior.

3 Introductory Examples of Uncertainty

Given that preferences for public goods are rather stable, the most credible way to add expenditure uncertainty to the model is via their cost of provision. An

[2] Similarly, equations (7) and (6) can be combined to result in the following maximization problem $\underset{\{q_A, q_B\}}{\text{Max}} \Big[B(q_A, q_B) - (q_A + \omega q_B) - \bar{p}. \max\big(q_A + \omega q_B - T^0, 0\big) \Big]$ resulting in the same allocation, as shown in appendix A.1.

uncertain cost might for instance result from shocks to wages, shocks to energy prices (e.g. the oil price) or a very harsh winter.

In the model the uncertainty in costs is presumed to take the form of a disturbance term, Θ_i, to marginal costs, with $i \in \{A, B\}$. In other words, an unexpected shock is assumed to be restricted to shifts in the marginal curves, leaving their curvature unchanged. After all, the curvature is constant so far. Nonetheless, policy makers are often forced to commit themselves to certain public good allocations in the short run. As a result of such ex ante commitment contemporaneous adjustment, i.e. when the shock is realized, might not be possible even though it is not the optimal allocation ex post.

Example 1 First, consider the original benchmark case without any cost of a public deficit, but with stochastic shocks to the expenditures' costs. More specifically, assume the shocks have a simple discrete probability distribution:

$$\Pr[\Theta_i] = \begin{cases} \frac{1}{2}, & \text{if } \theta_i = \theta_i^L \\ \frac{1}{2}, & \text{if } \theta_i = \theta_i^H \end{cases} \tag{10}$$

where the superscripts indicate the possible scenarios for each expenditure category: the marginal costs of public good i might turn out either lower (L) or higher (H) than expected with an equal probability. For succinctness, $\mathbb{E}[\Theta_i] = 0$ is assumed to hold, i.e. policy makers do not expect the shock upfront.

In that case, expenditure uncertainty via shocks to the marginal costs will result in the following optimization problem for policy makers:

$$\underset{\{q_A, q_B\}}{\text{Max}} \Big[B(q_A, q_B) - \mathbb{E}\left[(1 + \Theta_A)q_A + (\omega + \Theta_B)q_B\right]\Big] \tag{11}$$

or more specifically, using the probabilities from (10),

$$\underset{\{q_A, q_B\}}{\text{Max}} \left[B(q_A, q_B) - \sum_{j \in \{L,H\}} \sum_{j \in \{L,H\}} \frac{1}{4}\left[(1 + \theta_A^j)q_A - (\omega + \theta_B^j)q_B\right]\right]. \tag{12}$$

Consequently, the ex ante allocation will consist of public provisions

$$q_A = \frac{\overbrace{\sum_j \frac{1}{2}(1 + \theta_A^j)}^{=1+\mathbb{E}[\Theta_A]} - a}{b} = \frac{1 - a}{b} = q_A^0 \tag{13}$$

$$q_B = \frac{\overbrace{\sum_j \frac{1}{2}(1 + \theta_B^j)}^{=1+\mathbb{E}[\Theta_B]} - c}{d} = \frac{\omega - c}{d} = q_B^0. \tag{14}$$

Policy makers will thus choose the provided quantities of A and B solely based on the expected marginal costs resulting in $\{q_A^0, q_B^0\}$, since the expected marginal costs from the deterministic benchmark will still hold.

Consequently, if the expenditure allocation of goods A and B is fixed ex ante and can only be adjusted in the long run, the first-best optimum (see section 4.1)

$$q_A^1 = \frac{(1+\theta_A) - a}{b} \tag{15}$$

$$q_B^1 = \frac{(\omega+\theta_B) - c}{d} \tag{16}$$

will not be attained. More specifically, ex post it would be optimal if the provided quantities of both public goods were taking into account the realized costs. After all, such an equilibrium is optimal since it equates the actual marginal benefits to the actual marginal costs. Nevertheless, so far it is assumed that policy makers are restraint to their initial allocation. Thus, only the second-best allocation in (13) and (14) can be attained, illustrating the consequences of such commitment in case of uncertainty.

The restriction to adjust expenditures once the shocks were realized is relaxed in section 8. There, the trade off among the different categories of public expenditures to adjust based on their elasticities and relative costs is especially insightful for practitioners.

Example 2 Next, introduce the penalty in case of a deficit. The government then solves

$$\underset{\{q_A,q_B\}}{\text{Max}}\Big[B(q_A,q_B) - \mathbb{E}\Big[(1+p)\big((1+\Theta_A)q_A + (\omega+\Theta_B)q_B\big) - pT^0\Big]\Big] \tag{17}$$

instead.

Suppose that only in the case of the combinations of shocks $\{\theta_A^H, \theta_B^H\}$ the state of the world is such that a deficit is insurmountable. In the three other scenarios of shocks, the realized costs are low enough to prevent any deficit. Therefore, the expectation in the case of a stricter budget constraint equals: $\sum_j \sum_j \frac{1}{4}\Big[(1+\theta_A^j)q_A - (\omega+\theta_B^j)q_B\Big]$ as before, plus $\frac{1}{4}\Big[\bar{p}\big((1+\theta_A^H)q_A + (\omega+\theta_B^H)q_B - T^0\big)\Big]$ as a result of the possible deficit. Hence, the ex ante allocation will be

$$q_A = \frac{\overbrace{\sum_j \frac{1}{2}(1+\theta_A^j)}^{=1+\mathbb{E}[\theta_A]} + \overbrace{\frac{1}{4}\bar{p}(1+\theta_A^H)}^{>0} - a}{b} \tag{18}$$

$$q_B = \frac{\overbrace{\sum_j \frac{1}{2}(1+\theta_B^j)}^{=1+\mathbb{E}[\theta_B]} + \overbrace{\frac{1}{4}\bar{p}(\omega+\theta_B^H)}^{>0} - c}{d}. \tag{19}$$

Thus, policymakers now anticipate the positive probability of an additional cost by lowering public good provisions before the shocks might arise. Nonetheless, again the optimum allocation will not be achieved. Yet, the anticipation hedges the cost shocks in case $\{\theta_A^H, \theta_B^H\}$. It might, however, just as well bring the allocation farther away from the ex post optimum. For example, if both realized marginal costs are lower than expected, $\{\theta_A^L, \theta_B^L\}$.

In addition to expenditure uncertainty, uncertainty on the revenue side of the budget is now illustrated. In reality budgetary policy is characterized with a considerable amount of revenue uncertainty. For example, public budgets are drawn up based on revenue projections. Yet, these projections are not necessarily realized ex post. Output fluctuations for instance are well known to affect revenue collections via corporate and personal income taxation. As a result, public expenditure allocations might be affected.

Example 3 For illustrative purposes expenditure shocks are left aside for a moment. If policy makers do not know the exact realization of fiscal revenues ex ante, they are uncertain about the realized fiscal balance too. In case there is no strict budget constraint such uncertainty does not result in an allocation different from the benchmark allocation, $\{q_A^0, q_B^0\}$. If a penalty is in force, however, the picture is different. As T is no longer fixed at $T^0 = q_A^0 + \omega q_B^0$, p is now also conditional on the realized value of T, characterized by disturbance term Λ.

Consider a distribution of the realized T over q according to the simple discrete probability density function

$$\Pr[T(\Lambda)] = \begin{cases} \frac{1}{2}, & \text{if } T(\Lambda) = T^0 + \lambda^L \quad << q_A^0 + \omega q_B^0 \\ \frac{1}{2}, & \text{if } T(\Lambda) = T^0 + \lambda^H \quad >> q_A^0 + \omega q_B^0 \end{cases} \tag{20}$$

The government will thus solve the following maximization problem:

$$\begin{aligned} &\underset{\{q_A, q_B\}}{\text{Max}} \Big[B(q_A, q_B) - (q_A + \omega q_B) - \mathbb{E}\big[p(q_A + \omega q_B - T^0 - \Lambda)\big]\Big] \\ &= \underset{\{q_A, q_B\}}{\text{Max}} \Big[B(q_A, q_B) - (q_A + \omega q_B) - \frac{1}{2}\bar{p}(q_A + \omega q_B - T^0 - \lambda^L)\Big], \end{aligned} \tag{21}$$

by choosing the following allocation

$$q_A = \frac{1 + \frac{1}{2}\bar{p} - a}{b} \tag{22}$$

$$q_B = \frac{\omega + \frac{1}{2}\bar{p}\omega - c}{d} \tag{23}$$

Basing their decisions on the expected revenues, $\mathbb{E}[T(\Lambda)]$, instead of the realized revenues will thus create additional uncertainty about the cost of public provisions. Specifically, policy makers' expectations of the cost of provision depends on the distribution of public revenues, as the disciplinary cost of a deficit (p) depends on it.

In sum, as with shocks to the costs, revenue volatility urges governments subject to a strict budget constraint to anticipate the possible penalty in case of lower than expected fiscal revenues or higher than expected costs. Nonetheless, the anticipation does not ensure ex post optimal allocations. Given its importance in everyday policy, in what follows, light is shed on the allocations under uncertainty and the conditions of their subsequent welfare losses using more general probability distributions, introducing a possible buffer and deriving the corresponding optimal budget process.

4 Expenditure Shocks

4.1 No Costly Deficits, No Difference

Consider the original benchmark case without any cost of a public deficit. In that case, expenditure uncertainty via shocks to the marginal costs will result in the following revised optimization problem for policy makers:

$$\underset{\{q_A,q_B\}}{\text{Max}}\ \mathbb{E}\Big[B(q_A,q_B) - [C(q_A,q_B,\Theta_A,\Theta_B)]\Big] \tag{24}$$

or more specifically: $\underset{\{q_A,q_B\}}{\text{Max}}\Big[B(q_A,q_B) - \mathbb{E}\left[(1+\Theta_A)q_A + (\omega+\Theta_B)q_B\right]\Big]$. Thus, the uncertainty in costs is presumed to take the form of a disturbance term, Θ_i, to marginal costs. In other words, an unexpected shock is assumed to be restricted to shifts in the marginal curves, leaving their curvature unchanged. After all, the curvature is constant so far. For succinctness, $\mathbb{E}[\Theta_i] = 0$ is assumed to hold for $i \in \{A, B\}$, i.e. policy makers do not expect the shock upfront.[3]

If the expenditure allocation of goods A and B is fixed ex ante and can only be adjusted in the long run, the optimum will not be attained. This can be shown as follows. Presume $f_{\Theta_i}(\theta_i)$ is a univariate continuous probability density function of the shock to the expenditure costs of good i. In other words, there exists a function $f_{\Theta_i}(\theta_i)$ such that the cumulative density function $F_{\Theta_i}(\bar{\theta}_i) = \int_{-\infty}^{\bar{\theta}_i} f_{\Theta_i}(\theta_i)\mathrm{d}\theta_i$ exists for every real value $\bar{\theta}_i$.[4] Moreover, the joint density function of (θ_A, θ_B), denoted by $f_{\Theta_A,\Theta_B}(\cdot,\cdot)$, is defined to be $f_{\Theta_A,\Theta_B}(\theta_A,\theta_B) = \Pr[\Theta_A = \theta_A, \Theta_B = \theta_B]$. Then, the aforementioned maximization problem can be rewritten as

$$\underset{\{q_A,q_B\}}{\text{Max}}\left[B(q_A,q_B) - \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{\Theta_A,\Theta_B}(\theta_A,\theta_B)\left[(1+\theta_A)q_A - (\omega+\theta_B)q_B\right]\mathrm{d}\theta_A\mathrm{d}\theta_B\right]. \tag{25}$$

[3] The latter assumption is mainly for reasons of clarity as $\mathbb{E}[\Theta_i] \neq 0$ would bring little new to the fore and obscure derivations. A non-zero expectation would cause allocations diverging from the benchmark, even without costly deficits. Yet, it would not add anything to the points made further on in the text as they can be straightforwardly accommodated for such anticipation.

[4] Given the continuity of the marginal probability density functions, the cumulative density function is as well.

Consequently, the ex ante allocation will consist of public provisions

$$q_A = \frac{\overbrace{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{\Theta_A,\Theta_B}(\theta_A,\theta_B)(1+\theta_A)\mathrm{d}\theta_A\mathrm{d}\theta_B}^{=1+\int_{-\infty}^{\infty} f_{\Theta_A}(\theta_A)\theta_A\mathrm{d}\theta_A=1+\mathbb{E}[\theta_A]} - a}{b} = \frac{1-a}{b} = q_A^0 \tag{26}$$

$$q_B = \frac{\overbrace{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{\Theta_A,\Theta_B}(\theta_A,\theta_B)(\omega+\theta_B)\mathrm{d}\theta_A\mathrm{d}\theta_B}^{=\omega+\int_{-\infty}^{\infty} f_{\Theta_B}(\theta_B)\theta_B\mathrm{d}\theta_B=\omega+\mathbb{E}[\theta_B]} - c}{d} = \frac{\omega-c}{d} = q_B^0. \tag{27}$$

Policy makers will thus choose the provided quantities of A and B solely based on the expected marginal costs resulting in $\{q_A^0, q_B^0\}$, since $\mathbb{E}[MC_A(q_A,\Theta_A)] = 1$ and $\mathbb{E}[MC_B(q_B,\Theta_B)] = \omega$ are the only costs taken into account when considering the provision of an additional unit of each public good. Hence, the following welfare losses are incurred, as illustrated by the shaded triangles in figure 3:

$$\begin{aligned} L_A^g &= \frac{1}{2}\Big(MC_A(q_A^0,\theta_A) - \mathbb{E}[MC_A(q_A^0,\Theta_A)]\Big)\big(q_A^0 - q_A^1\big) \\ &= -\frac{\theta_A^2}{2b} \end{aligned} \tag{28}$$

$$\begin{aligned} L_B^g &= \frac{1}{2}\Big(MC_B(q_B^0,\theta_B) - \mathbb{E}[MC_B(q_B^0,\Theta_B)]\Big)\big(q_B^0 - q_B^1\big) \\ &= -\frac{\theta_B^2}{2d}, \end{aligned} \tag{29}$$

where the ex post optimum quantities, q_i^1, are comprised by the following allocation:

$$q_A^1 = \frac{(1+\theta_A)-a}{b} \tag{30}$$

$$q_B^1 = \frac{(\omega+\theta_B)-c}{d}. \tag{31}$$

Whether these provisions are smaller or larger than $\{q_A^0, q_B^0\}$ depends on whether the shocks to marginal costs θ_i turn out to be positive or negative, respectively. More specifically, $\frac{\partial q_i^1}{\partial \theta_i} < 0$ holds for both expenditure categories. Thus, if actual marginal costs are higher than expected ($\theta_i > 0$), the optimum allocation would entail lower provision of public goods than those chosen based on the expected costs, and vice versa.

Despite the expected shocks being zero, comparison with the optimal allocation in the deterministic benchmark reveals *a first channel of welfare losses* under the condition of no contemporaneous adjustment of public good allocations once the shocks have occurred. This can be summarized by proposition 1, from which it is clear that not only the size of the shocks but also the slopes of the marginal benefit curves (i.e. the elasticity of the public goods) is an important determinant of the losses.

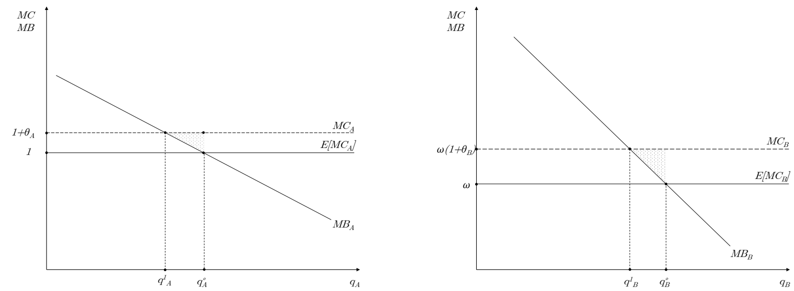

Figure 3: The Marginal Costs and Benefits of the Public Goods in case of Cost Uncertainty

Proposition 1. (Expenditure Uncertainty I) *Let $MB_i(q_i)$ be a downward sloping marginal benefit function and let $MC_i(\Theta_i)$ be a constant stochastic marginal cost function, with $i \in \{A, B\}$ and shocks $\theta_i \sim \mathcal{F}_{\Theta_i}(0, \sigma_i^2)$. Then, the government's inability to adjust the allocation $\{q_A^0, q_B^0\}$ to expenditure shocks in the short run, yields deadweight losses amounting to $L_A^g = -\frac{\theta_A^2}{2b}$ and $L_B^g = -\frac{\theta_B^2}{2d}$, respectively.*

4.2 Costly Deficits, Anticipated Uncertainty

Next, consider the case of a shock on the expenditure side taking into account the cost of a possible deficit. The government then solves

$$\underset{\{q_A,q_B\}}{\text{Max}}\ \mathbb{E}\Big[B(q_A,q_B) - C(q_A,q_B,\Theta_A,\Theta_B) - p(C(q_A,q_B,\Theta_A,\Theta_B) - T^0)\Big]$$

Or similarly $$\underset{\{q_A,q_B\}}{\text{Max}}\left[B(q_A,q_B) - \mathbb{E}\Big[(1+p)\big((1+\Theta_A)q_A + (\omega+\Theta_B)q_B\big) - pT^0\Big]\right] \tag{32}$$

with p taking taking on the conditional values as specified in (6) adjusted for cost uncertainty:

$$p = \begin{cases} \bar{p}, & \text{if } \big(C(q_A,q_B,\theta_A,\theta_B) - T^0\big) > 0 \\ 0, & \text{if } \big(C(q_A,q_B,\theta_A,\theta_B) - T^0\big) \leq 0 \end{cases} \tag{33}$$

As in section 4.1, solving this problem for the ex ante public good allocation requires the specification of a distribution for the shocks. Nevertheless, the asymmetric structure of p will have a distinct impact on ex ante behavior as policy makers will choose allocations conditional on the probability that shocks may cause a deficit and thus a penalty.

4.2.1 Homogeneous Shocks

As a means of illustration, first consider the case where both shocks to the marginal costs of both public goods are the same ($\theta_A = \theta_B = \theta$), with a distribution $f_\Theta(\theta)$ for such a shock. As illustrated below, the presence of an asymmetric factor comprising the adverse economic effects of public deficit results in an ex ante commitment different from the benchmark case without the economic cost of a deficit.

In particular, even with T^0 fixed policy makers do not know ex ante at which expenditure allocations an additional cost will be incurred due to a deficit. After all, they do not know the exact marginal cost due to the possible disturbance. Yet, this disturbance will affect the economic penalty incurred due to fiscal profligacy and the ex post optimality of their choice. Specifically, solving the expectation $\mathbb{E}\Big[(1+p)\big((1+\Theta_A)q_A + (\omega+\Theta_B)q_B - pT^0\big)\Big]$ in (32) gives the integral over Θ of all occurrences of the shock, $\Pr[\theta]\cdot\big[(1+\theta)q_A + (\omega+\theta)q_B\big]$, plus the integral over Θ of the occurrences of the shock for which there is a deficit, $\Pr[\theta]\cdot\big[\bar{p}\big((1+\theta)q_A + (\omega+\theta)q_B - T^0\big)\big]$.

In particular, by means of probability density function $f_\Theta(\theta)$, policy makers solve:[5]

$$\begin{aligned}
&\underset{\{q_A,q_B\}}{\text{Max}}\Bigg[B(q_A,q_B) - \int_{-\infty}^{\infty} f_\Theta(\theta)\,[(1+\theta)q_A + (\omega+\theta)q_B]\,\mathrm{d}\theta \\
&\qquad - \int_{-\infty}^{\infty} f_\Theta(\theta)\big[\bar{p}\cdot\max\big((1+\theta)q_A + (\omega+\theta)q_B - T^0, 0\big)\big]\mathrm{d}\theta\Bigg] \qquad (34)\\
= &\underset{\{q_A,q_B\}}{\text{Max}}\Bigg[B(q_A,q_B) - \int_{-\infty}^{\infty} f_\Theta(\theta)\,[(1+\theta)q_A + (\omega+\theta)q_B]\,\mathrm{d}\theta \\
&\qquad - \int_{-\infty}^{\infty} f_\Theta(\theta)\frac{1}{2}\bar{p}\underbrace{\big((1+\theta)q_A + (\omega+\theta)q_B - T^0 + \big|(1+\theta)q_A + (\omega+\theta)q_B - T^0\big|\big)}_{\substack{=0,\,\text{if }\left((1+\theta)q_A + (\omega+\theta)q_B - T^0\right)\le 0\\ >0,\,\text{if }\left((1+\theta)q_A + (\omega+\theta)q_B - T^0\right)> 0}}\mathrm{d}\theta\Bigg].
\end{aligned}$$

Consequently, the ex ante allocation is

$$q_A = \frac{1}{b}\left(1 + \underbrace{\int_{-\infty}^{\infty} f_\Theta(\theta)\theta\mathrm{d}\theta}_{=\mathbb{E}[\Theta]=0} + \int_{-\infty}^{\infty} f_\Theta(\theta)\frac{1}{2}\bar{p}\underbrace{(1+\theta)\left(1+\frac{(1+\theta)q_A + (\omega+\theta)q_B - T^0}{|(1+\theta)q_A + (\omega+\theta)q_B - T^0|}\right)}_{\substack{=0,\,\text{if }\left((1+\theta)q_A + (\omega+\theta)q_B - T^0\right)\le 0\\ >0,\,\text{if }\left((1+\theta)q_A + (\omega+\theta)q_B - T^0\right)> 0\text{ and }\theta>-1}}\mathrm{d}\theta - a\right)$$

[5]Distinguishing the realizations of the shock for which there will be a deficit is the result of an iterative process by policy makers. In particular, the positive expectation of a penalty $\int_0^\infty f_\Theta(\theta)\big[\bar{p}\big((1+\theta)q_A + (\omega+\theta)q_B - T^0\big)\big]\mathrm{d}\theta$ will cause the government to anticipate by lowering public provisions. Thereby, creating a lower probability of running a deficit and thus incurring a penalty. This will, in its turn, lower the anticipation and thus increase the probability of penalty. The process continues until an equilibrium allocation has been reached.

$$= \frac{1 + \int_{-\infty}^{\infty} f_\Theta(\theta)\frac{1}{2}\bar{p}(1+\theta)\left(1 + \frac{(1+\theta)q_A + (\omega+\theta)q_B - T^0}{|(1+\theta)q_A + (\omega+\theta)q_B - T^0|}\right)\mathrm{d}\theta - a}{b} \tag{35}$$

$$q_B = \frac{1}{d}\left(\omega + \underbrace{\int_{-\infty}^{\infty} f_\Theta(\theta)\theta \mathrm{d}\theta}_{=\mathbb{E}[\Theta]=0} + \int_{-\infty}^{\infty} f_\Theta(\theta)\frac{1}{2}\bar{p}\underbrace{(\omega+\theta)\left(1 + \frac{(1+\theta)q_A + (\omega+\theta)q_B - T^0}{|(1+\theta)q_A + (\omega+\theta)q_B - T^0|}\right)}_{\substack{= 0, \text{ if } \left((1+\theta)q_A + (\omega+\theta)q_B - T^0\right) \leq 0 \\ > 0, \text{ if } \left((1+\theta)q_A + (\omega+\theta)q_B - T^0\right) > 0 \text{ and } \theta > -1}}\mathrm{d}\theta - c\right)$$

$$= \frac{\omega + \int_{-\infty}^{\infty} f_\Theta(\theta)\frac{1}{2}\bar{p}(\omega+\theta)\left(1 + \frac{(1+\theta)q_A + (\omega+\theta)q_B - T^0}{|(1+\theta)q_A + (\omega+\theta)q_B - T^0|}\right)\mathrm{d}\theta - c}{d}. \tag{36}$$

Therefore, as long as the probabilities of unreasonably large downward shocks to public goods' prices are small, there will be anticipation via lower provisions of public goods.[6] [7]

Proposition 2. (Expenditure Uncertainty II) *Let $MB_i(q_i)$ be a downward sloping marginal benefit function and let $MC_i(\Theta)$ be a constant stochastic marginal cost function, with $i \in \{A, B\}$. Then, an asymmetric cost structure for the public deficit $\bar{p} > 0$ and shocks $\theta \sim \mathcal{F}_\Theta(0, \sigma^2)$ with small enough probability $f_\Theta(\theta)$ for unreasonably large downward shocks (i.e. $\Theta < -1$), yield short run public provisions $q_A < q_A^{00}$ and $q_B < q_B^{00}$ under expenditure uncertainty.*

In addition to the cost shocks, the incurred welfare losses are now also depending on the cost $\bar{p}$ of too high expenditures for given revenues (i.e. fiscal indiscipline). In particular, the ex post optimal allocation would have been:

$$q_A^{11} = \frac{(1+p)(1+\theta) - \alpha}{\beta} \tag{37}$$

$$q_B^{11} = \frac{(1+p)(\omega+\theta) - \chi}{\delta} \tag{38}$$

Therefore, the incurred deadweight losses are

$$L_A^g = \frac{1}{2}\Bigg(MC_A(q_A, \theta) + p(1+\theta) - \mathbb{E}[MC_A(q_A, \Theta)] \tag{39}$$
$$+ \int_{-\infty}^{\infty} f_\Theta(\theta)\frac{1}{2}\bar{p}(1+\theta)\left(1 + \frac{(1+\theta)q_A + (\omega+\theta)q_B - T^0}{|(1+\theta)q_A + (\omega+\theta)q_B - T^0|}\right)\mathrm{d}\theta\Bigg)\left(q_A - q_A^{11}\right)$$

[6] An alternative derivation of this result is presented in appendix A.4. Nonetheless, the method used in the main text is less burdensome once the assumption of homogeneous shocks is dropped.

[7] The public administration's stance with respect to risk nonetheless matters for the degree of anticipation (see e.g. Adar and Griffin (1976)). In particular, a risk neutral government will base its decisions on its expected marginal costs (incl. possible penalty), while a risk averse policy maker will pass judgment in a more behavioral manner (e.g. based on a utility function quantifying its valuation of risk). Nevertheless, anticipatory behavior is observed in case of risk neutrality as well.

$$= \begin{cases} -\frac{\left(\theta+\bar{p}(1+\theta)-\int_{-\infty}^{\infty} f_\Theta(\theta)\frac{1}{2}\bar{p}(1+\theta)\left(1+\frac{(1+\theta)q_A+(\omega+\theta)q_B-T^0}{|(1+\theta)q_A+(\omega+\theta)q_B-T^0|}\right)\mathrm{d}\theta\right)^2}{2\beta}, & \text{if } \left(C(q_A,q_B,\theta)-T^0\right)>0 \\ -\frac{\left(\theta-\int_{-\infty}^{\infty} f_\Theta(\theta)\frac{1}{2}\bar{p}(1+\theta)\left(1+\frac{(1+\theta)q_A+(\omega+\theta)q_B-T^0}{|(1+\theta)q_A+(\omega+\theta)q_B-T^0|}\right)\mathrm{d}\theta\right)^2}{2\beta}, & \text{if } \left(C(q_A,q_B,\theta)-T^0\right)\leq 0 \end{cases}$$

$$\begin{aligned} L_B^g = \frac{1}{2}\Bigg(&MC_B(q_B,\theta)+p(\omega+\theta)-\mathbb{E}[MC_B(q_B,\Theta)] \\ &\int_{-\infty}^{\infty} f_\Theta(\theta)\frac{1}{2}\bar{p}(\omega+\theta)\left(1+\frac{(1+\theta)q_A+(\omega+\theta)q_B-T^0}{|(1+\theta)q_A+(\omega+\theta)q_B-T^0|}\right)\mathrm{d}\theta\Bigg)(q_B-q_B^{11}) \end{aligned} \tag{40}$$

$$= \begin{cases} -\frac{\left(\theta+\bar{p}(\omega+\theta)-\int_{-\infty}^{\infty} f_\Theta(\theta)\frac{1}{2}\bar{p}(\omega+\theta)\left(1+\frac{(1+\theta)q_A+(\omega+\theta)q_B-T^0}{|(1+\theta)q_A+(\omega+\theta)q_B-T^0|}\right)\mathrm{d}\theta\right)^2}{2\delta}, & \text{if } \left(C(q_A,q_B,\theta)-T^0\right)>0 \\ -\frac{\left(\theta-\int_{-\infty}^{\infty} f_\Theta(\theta)\frac{1}{2}\bar{p}(\omega+\theta)\left(1+\frac{(1+\theta)q_A+(\omega+\theta)q_B-T^0}{|(1+\theta)q_A+(\omega+\theta)q_B-T^0|}\right)\mathrm{d}\theta\right)^2}{2\delta}, & \text{if } \left(C(q_A,q_B,\theta)-T^0\right)\leq 0 \end{cases}$$

Even though the penalty fines - thus, not the losses - as a result of the (expected) shock on the expenditure side will be anticipated by ex ante allocations different from the deterministic benchmark as a result of the asymmetrical cost structure, the first channel of welfare losses from section 4.1 persists. Moreover, in addition to the distortions caused by the predetermination of the ex ante quantities of public provision, a disciplinary market mechanism may create additional distortions.[8] In particular, the anticipation may just as well prove to have been unnecessary.

The realized deficit and cost of the ex ante allocation depend on the realized shocks. As summarized by proposition 2, policy makers are found to reduce their provisions to anticipate the expected positive penalty as a result of the asymmetric cost structure. Therefore, the disciplinary cost brings short run allocations closer to the optimum in case higher than benchmark marginal costs are realized. Nonetheless, if actual marginal costs are lower, discipline is still in force as public expenditures are capped at the benchmark due to the resulting cost structure.

Lemma 1. (Retaliation for Fiscal Indiscipline via Cost Shocks) *With a cost for fiscal indiscipline as specified in equation* (6) *revenue volatility creates one-sided anticipatory behavior by reducing public goods' provisions to* $\{q_A, q_B\}$*. Hence, retaliation for fiscal indiscipline both hedges against the impact of higher public expenditure costs and disciplines in case marginal costs fall below expectations.*

Consequently, the markets' *retaliation for fiscal indiscipline* via the cost of a public deficit spurs policy makers to hedge against disadvantageous shocks on the expenditure side, while maintaining discipline in advantageous times. The conditions under which such discipline is harmful for welfare are explored further in section 4.2.2.

[8]The result that the additional cost due to a public deficit matters for the welfare losses still holds in case of 'perfect' anticipation of the penalty due to a symmetric p. After all, the anticipation codetermines the deviation of the ex ante allocation from the optimum determined by the shocks, as illustrated in appendix A.3.

4.2.2 Heterogeneous Shocks

Now, again consider the objective function (32) with heterogeneous shocks and the general continuous probability density function $f_{\Theta_i}(\theta_i)$ for the shocks to expenditure costs. Policy makers thus choose quantities q_A and q_B by balancing benefits against expected costs, i.e. the integral over vector (Θ_A, Θ_B) of $Pr[\theta_A \cap \theta_B] \cdot \big[(1+\theta_A)q_A + (\omega+\theta_B)q_B\big]$ plus the integral over $Pr[\theta_A \cap \theta_B] \cdot \big[\bar{p}\big((1+\theta_A)q_A + (\omega+\theta_B)q_B - T^0\big)\big]$ in case of a deficit.

The corresponding maximization problem equals:

$$\underset{\{q_A,q_B\}}{\text{Max}} \Bigg[B(q_A,q_B) - \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{\Theta_A,\Theta_B}(\theta_A,\theta_B)\left[(1+\theta_A)q_A - (\omega+\theta_B)q_B\right] \mathrm{d}\theta_A \mathrm{d}\theta_B \\ - \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{\Theta_A,\Theta_B}(\theta_A,\theta_B)\left[\bar{p}\cdot \max\big((1+\theta_A)q_A + (\omega+\theta_B)q_B - T^0, 0\big)\right] \mathrm{d}\theta_A \mathrm{d}\theta_B \Bigg].$$

with, as in 4.2.1, the incurred penalty being a function of the realizations of both shocks and the allocation itself. Therefore, the ex ante allocation of public goods chosen by the government is:

$$\begin{aligned} q_A = \frac{1}{b}\Bigg[1 + &\underbrace{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{\Theta_A,\Theta_B}(\theta_A,\theta_B)\theta_A \mathrm{d}\theta_A \mathrm{d}\theta_B}_{=\int_{-\infty}^{\infty} f_{\Theta_A}(\theta_A)\theta_A \mathrm{d}\theta_A = \mathbb{E}[\Theta_A]=0} \\ &+ \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{\Theta_A,\Theta_B}(\theta_A,\theta_B)\frac{1}{2}\bar{p}\underbrace{(1+\theta_A)\left(1+\frac{(1+\theta_A)q_A + (\omega+\theta_B)q_B - T^0}{|(1+\theta_A)q_A + (\omega+\theta_B)q_B - T^0|}\right)}_{\substack{=0,\text{ if } \left((1+\theta_A)q_A + (\omega+\theta_B)q_B - T^0\right)\leq 0 \\ >0,\text{ if } \left((1+\theta_A)q_A + (\omega+\theta_B)q_B - T^0\right)> 0 \text{ and } \theta_A > -1}} \mathrm{d}\theta_A \mathrm{d}\theta_B - a\Bigg] \\ = \frac{1 + \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{\Theta_A,\Theta_B}(\theta_A,\theta_B)\frac{1}{2}\bar{p}(1+\theta_A)\left(1+\frac{(1+\theta_A)q_A + (\omega+\theta_B)q_B - T^0}{|(1+\theta_A)q_A + (\omega+\theta_B)q_B - T^0|}\right)\mathrm{d}\theta_A \mathrm{d}\theta_B - a}{b} \end{aligned} \tag{41}$$

$$\begin{aligned} q_B = \frac{1}{d}\Bigg[\omega + &\underbrace{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{\Theta_A,\Theta_B}(\theta_A,\theta_B)\theta_B \mathrm{d}\theta_A \mathrm{d}\theta_B}_{=\int_{-\infty}^{\infty} f_{\Theta_B}(\theta_B)\theta_B \mathrm{d}\theta_B = \mathbb{E}[\Theta_B]=0} \\ &+ \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{\Theta_A,\Theta_B}(\theta_A,\theta_B)\frac{1}{2}\bar{p}\underbrace{(\omega+\theta_B)\left(1+\frac{(1+\theta_A)q_A + (\omega+\theta_B)q_B - T^0}{|(1+\theta_A)q_A + (\omega+\theta_B)q_B - T^0|}\right)}_{\substack{=0,\text{ if } \left((1+\theta_A)q_A + (\omega+\theta_B)q_B - T^0\right)\leq 0 \\ >0,\text{ if } \left((1+\theta_A)q_A + (\omega+\theta_B)q_B - T^0\right)> 0 \text{ and } \theta_B > -\omega}} \mathrm{d}\theta_A \mathrm{d}\theta_B - c\Bigg] \\ = \frac{\omega + \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{\Theta_A,\Theta_B}(\theta_A,\theta_B)\frac{1}{2}\bar{p}(\omega+\theta_B)\left(1+\frac{(1+\theta_A)q_A + (\omega+\theta_B)q_B - T^0}{|(1+\theta_A)q_A + (\omega+\theta_B)q_B - T^0|}\right)\mathrm{d}\theta_A \mathrm{d}\theta_B - c}{d}. \end{aligned} \tag{42}$$

Proposition 2 can thus be generalized to the for heterogeneous shocks.

Proposition 3. (Expenditure Uncertainty IIbis) *Let $MB_i(q_i)$ be a downward sloping marginal benefit function and let $MC_i(\Theta_i)$ be a constant stochastic marginal cost function, with $i \in \{A, B\}$. Then, an asymmetric cost structure for the public deficit $\bar{p} > 0$ and shocks $\theta_i \sim \mathcal{F}_{\Theta_i}(0, \sigma_i^2)$ with small enough probability $f_{\Theta_i}(\theta_i)$ for unreasonably large downward shocks (i.e. $\Theta_A < -1$ and $\Theta_B < -\omega$), yield short run public provisions $q_A < q_A^{00}$ and $q_B < q_B^{00}$ under expenditure uncertainty.*

As shown above, the optimality of the government's chosen allocation, however, depends on the cost shocks (see section 4.1) and the extent to which such shocks are correctly anticipated (see section 4.2.1). Since, the uncertainty-driven welfare losses are:

$$L_A^g = \begin{cases} -\dfrac{\left(\theta_A + \bar{p}(1+\theta_A) - \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{\Theta_A,\Theta_B}(\theta_A,\theta_B)\frac{1}{2}\bar{p}(1+\theta_A)\left(1+\frac{(1+\theta_A)q_A+(\omega+\theta_B)q_B-T^0}{|(1+\theta_A)q_A+(\omega+\theta_B)q_B-T^0|}\right)\mathrm{d}\theta_A\mathrm{d}\theta_B\right)^2}{2b}, \\ \qquad\qquad \text{if } \left(C(q_A,q_B,\theta_A,\theta_B) - T^0\right) > 0 \\ -\dfrac{\left(\theta_A - \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{\Theta_A,\Theta_B}(\theta_A,\theta_B)\frac{1}{2}\bar{p}(1+\theta_A)\left(1+\frac{(1+\theta_A)q_A+(\omega+\theta_B)q_B-T^0}{|(1+\theta_A)q_A+(\omega+\theta_B)q_B-T^0|}\right)\mathrm{d}\theta_A\mathrm{d}\theta_B\right)^2}{2b}, \\ \qquad\qquad \text{if } \left(C(q_A,q_B,\theta_A,\theta_B) - T^0\right) \leq 0 \end{cases} \tag{43}$$

$$L_B^g = \begin{cases} -\dfrac{\left(\theta_B + \bar{p}(\omega+\theta_B) - \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{\Theta_A,\Theta_B}(\theta_A,\theta_B)\frac{1}{2}\bar{p}(\omega+\theta_B)\left(1+\frac{(1+\theta_A)q_A+(\omega+\theta_B)q_B-T^0}{|(1+\theta_A)q_A+(\omega+\theta_B)q_B-T^0|}\right)\mathrm{d}\theta_A\mathrm{d}\theta_B\right)^2}{2d}, \\ \qquad\qquad \text{if } \left(C(q_A,q_B,\theta_A,\theta_B) - T^0\right) > 0 \\ -\dfrac{\left(\theta_B - \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{\Theta_A,\Theta_B}(\theta_A,\theta_B)\frac{1}{2}\bar{p}(\omega+\theta_B)\left(1+\frac{(1+\theta_A)q_A+(\omega+\theta_B)q_B-T^0}{|(1+\theta_A)q_A+(\omega+\theta_B)q_B-T^0|}\right)\mathrm{d}\theta_A\mathrm{d}\theta_B\right)^2}{2d}, \\ \qquad\qquad \text{if } \left(C(q_A,q_B,\theta_A,\theta_B) - T^0\right) \leq 0 \end{cases} \tag{44}$$

whether the anticipation following the deficit penalty actually succeeds at reducing distortions depends on the actual costs of provision. The aforementioned anticipation is counterproductive if the cost shocks are negative (i.e. the costs of provision are lower) or if they are positive and the shocks' values does not exceed half of the anticipated reduction in public expenditures, nor compensate each other. In the end, the latter is an important qualification because in the case of heterogeneous shocks opposite shocks may cancel each other out over the entire budget.

Proposition 4. (Expenditure Uncertainty III) *Let $MB_i(q_i)$ be a downward sloping marginal benefit function and let $MC_i(\Theta_i)$ be a constant stochastic marginal cost function, with $i \in \{A, B\}$. Then, the anticipatorily lowered public provisions as a result of an asymmetric cost structure for the public deficit $\bar{p} > 0$ and shocks $\theta_i \sim \mathcal{F}_{\Theta_i}(0, \sigma_i^2)$, are counterproductive in hedging against expenditure uncertainty if the shocks are negative or if they are positive and do not exceed half of the anticipated reduction in public expenditures.*

[TBC]

5 Revenue Uncertainty

If policy makers do not know the exact realization of fiscal revenues ex ante, they are uncertain about the realized fiscal balance too. In case there is no strict budget constraint such uncertainty does not result in an allocation different from $\{q_A^0, q_B^0\}$ from section 4.1. If a penalty is in force, however, the picture is different. As T is no longer fixed at $T^0 = q_A^0 + \omega q_B^0$, p is now also conditional on the realized value of T, characterized by disturbance term Λ:

$$p = \begin{cases} \bar{p}, & \text{if } \big(C(q_A, q_B) - T(\lambda)\big) > 0 \\ 0, & \text{if } \big(C(q_A, q_B) - T(\lambda)\big) \leq 0 \end{cases} \tag{45}$$

In particular, basing their decisions on the expected revenues, $\mathbb{E}[T(\Lambda)]$, instead of the realized revenues will create additional uncertainty about the cost of public provisions. Specifically, policy makers' expectations of the cost of provision depends on the distribution of public revenues, as the disciplinary cost of a deficit (p) depends on it.

Consider a distribution of Λ according to $\Lambda \sim \mathcal{F}_\Lambda(0, \sigma_t^2)$ and a corresponding continuous probability density function $f_\Lambda(\lambda)$. Hence, the distribution of the realized $T(\lambda)$, i.e. $T^0 + \lambda$, over q according to probability density function $f_T\big(T(\lambda)\big)$, with $T(\Lambda) \sim \mathcal{F}_T(q_A^0 + \omega q_B^0, \sigma_t^2)$. Hence, the corresponding government's maximization,

$$\begin{aligned} &\underset{\{q_A, q_B\}}{\text{Max}} \Big[B(q_A, q_B) - (q_A + \omega q_B) - \mathbb{E}\big[p(q_A + \omega q_B - T^0 - \lambda)\big]\Big] \\ &= \underset{\{q_A, q_B\}}{\text{Max}} \Big[B(q_A, q_B) - (q_A + \omega q_B) - \Pr[p = \bar{p} | T(\lambda)] \cdot \bar{p}\big(q_A + \omega q_B - T(\lambda)\big)\Big] \\ &= \underset{\{q_A, q_B\}}{\text{Max}} \Big[B(q_A, q_B) - (q_A + \omega q_B) - \int_{-\infty}^{C(q_A, q_B)} \bar{p}\big(q_A + \omega q_B - T(\lambda)\big) f_T\big(T(\lambda)\big) \mathrm{d}T\Big], \end{aligned} \tag{46}$$

will result in the following quantities:

$$q_A^2 = \frac{1 + \overbrace{\int_{-\infty}^{q_A + \omega q_B} \bar{p} f_T\big(T(\lambda)\big) \mathrm{d}T}^{>0} - a}{b} \tag{47}$$

$$q_B^2 = \frac{\omega + \overbrace{\int_{-\infty}^{q_A + \omega q_B} \bar{p} \omega f_T\big(T(\lambda)\big) \mathrm{d}T}^{>0} - c}{d}. \tag{48}$$

Since the continuous probability density function is positive over its domain (i.e. $f_T\big(T(\lambda)\big) > 0$), allocation $\{q_A^2, q_B^2\}$ results in proposition 5.[9]

Proposition 5. (Revenue Uncertainty I) *Let $MB_i(q_i)$ be a downward sloping marginal benefit function and let MC_i be a constant marginal cost function, with $i \in \{A, B\}$. Then, an asymmetric cost structure for the public deficit $\bar{p} > 0$ and public revenues $T(\Lambda) \sim \mathcal{F}_T(q_A^0 + \omega q_B^0, \sigma_t^2)$, yields short run public provisions $q_A^2 < q_A^{00}$ and $q_B^2 < q_B^{00}$ under revenue uncertainty.*

Policy makers will thus take the strict budget constraint into consideration when allocating public expenditures ex ante. Consequently, given the asymmetric structure of the penalty, the allocation of public goods will be lower than in the benchmark case without revenue uncertainty.

Furthermore, it can be straightforwardly shown that a higher perceived economic cost of the public deficit will urge policy makers to pursue a more austere expenditure policy and cut in the provision of both public services. In particular, $\frac{\partial q_i^2}{\partial \bar{p}} < 0$ will hold for both goods $i \in \{A, B\}$ as a result of the downward sloping demand curves.

The ex post optimal allocation is the allocation $\{q_A^{00}, q_B^{00}\}$ tailored to the fact that p is now a function of the disturbance term on the revenue side. Hence, it is the initial benchmark with p now given by the conditionals of (45). Call this allocation $\{q_A^{01}, q_B^{01}\}$, then proposition 6 will hold.

Proposition 6. (Revenue Uncertainty II) *Let $MB_i(q_i)$ be a downward sloping marginal benefit function and let MC_i be a constant marginal cost function, with $i \in \{A, B\}$. Then, an asymmetric cost structure for the public deficit $\bar{p} > 0$ and public revenues $T(\Lambda) \sim \mathcal{F}_T(q_A^0 + \omega q_B^0, \sigma_t^2)$, yields welfare losses L_A^t and L_B^t in case the government is unable to adjust the allocation $\{q_A^2, q_B^2\}$ to revenue volatility.*

With the deadweight losses specified as follows:[10]

$$L_A^t = \frac{1}{2}\left(MC_A(q_A^2) + p - \mathbb{E}[MC_A(q_A^2)] - \int_{-\infty}^{q_A + \omega q_B} \bar{p} f_T\big(T(\lambda)\big)\mathrm{d}T\right)\big(q_A^2 - q_A^{01}\big)$$
$$= \begin{cases} -\frac{\left(\bar{p} - \int_{-\infty}^{q_A+\omega q_B} \bar{p} f_T\left(T(\lambda)\right)\mathrm{d}T\right)^2}{2b}, & \text{if } \big(C(q_A, q_B) - T(\lambda)\big) > 0 \\ -\frac{\left(-\int_{-\infty}^{q_A+\omega q_B} \bar{p} f_T\left(T(\lambda)\right)\mathrm{d}T\right)^2}{2b}, & \text{if } \big(C(q_A, q_B) - T(\lambda)\big) \leq 0 \end{cases} \tag{49}$$

[9] The allocation is obtained using differentiation under the integral:

$$\frac{\partial}{\partial x}\left(\int_{a(x)}^{b(x)} f(x,t)\mathrm{d}t\right) = f\big(x, b(x)\big)b'(x) - f\big(x, a(x)\big)a'(x) + \int_{a(x)}^{b(x)} f'_x(x,t)\mathrm{d}t,$$

with primes indicating first order derivatives. Furthermore, to accommodate for the improper integral $\lim_{a \to -\infty} f_\Lambda(a) = 0$ is presumed.

[10] Due to the assumption that the marginal curves are linear, there is no difference in losses whether realized revenues are higher or lower than expected. A difference in the deadweight losses would occur in the face of non-linearities. Yet, it would not change anything to the points made here.

$$L_B^t = \frac{1}{2}\bigg(MC_B(q_B^2) + p - \mathbb{E}[MC_B(q_B^2)] - \int_{-\infty}^{q_A+\omega q_B} \bar{p}\omega f_T\big(T(\lambda)\big)\mathrm{d}T\bigg)\big(q_B^2 - q_B^{01}\big)$$
$$= \begin{cases} -\frac{\left(\bar{p}-\int_{-\infty}^{q_A+\omega q_B} \bar{p}\omega f_T\left(T(\lambda)\right)\mathrm{d}T\right)^2}{2d}, & \text{if } \big(C(q_A,q_B) - T(\lambda)\big) > 0 \\ -\frac{\left(-\int_{-\infty}^{q_A+\omega q_B} \bar{p}\omega f_T\left(T(\lambda)\right)\mathrm{d}T\right)^2}{2d}, & \text{if } \big(C(q_A,q_B) - T(\lambda)\big) \le 0 \end{cases} \tag{50}$$

Obviously, the realized deficit and cost of the ex ante allocation depend on the realized revenues. Policy makers are found to reduce their provisions to anticipate the expected positive penalty as a result of the asymmetric cost structure. Therefore, the disciplinary cost brings short run allocations closer to the optimum in case lower than benchmark revenues are collected. Nonetheless, if actual revenues are higher, discipline is still in force as public expenditures are capped at the benchmark due to the resulting cost structure. Consequently, the markets' *retaliation for fiscal indiscipline* via the cost of public expenditures spurs policy makers to hedge against disadvantageous revenue shocks, while maintaining discipline in advantageous times albeit possibly at the cost of lower welfare.

Lemma 2. (Retaliation for Fiscal Indiscipline via Revenue Volatility) *With a cost for fiscal indiscipline as specified in equation* (45) *revenue volatility creates one-sided anticipatory behavior by reducing public goods' provisions to* $\{q_A^2, q_B^2\}$*. Hence, retaliation for fiscal indiscipline both hedges against the impact of lower fiscal revenues and disciplines in case fiscal revenues exceed expectations.*

In brief, although an anticipatory reduction of the supply of public goods might hedge against part of the revenue uncertainty, *a second channel of welfare losses* due to revenue volatility will persist. For example, fiscal revenues might just as well turn out to be lower than expected and an even lower provision might have been preferable ex post. Hence, conclusions similar to those for expenditure shocks apply.

6 Compounded Uncertainty

To further generalize the model and its findings, expenditure cost shocks and revenue volatility are now considered in unison. Firstly, section 6.1 does this for two public good categories, as before. Secondly, section 6.2 extends the model from two to N different types of expenditure categories.

6.1 Full-fledged Model

To further generalize the model and its findings, expenditure cost shocks and revenue volatility are now considered in unison. Combining the revenue volatility from section 5 with the heterogeneous cost shocks from section 4.2.2 results in a model with three stochastic random variables. Take the joint probability density function of those three variables $f_{\Theta_A,\Theta_B,\Lambda}(\theta_A,\theta_B,\lambda)$ as given. Moreover, the budget is balanced or: $\lambda = (1+\theta_A)q_A + (\omega+\theta_B)q_B - T^0$. Then, defining

$$\chi(q_A, q_B, \theta_A, \theta_B) \equiv (1+\theta_A)q_A + (\omega+\theta_B)q_B - T^0, \tag{51}$$

the government solves the following maximization problem:

$$\begin{aligned}\underset{\{q_A,q_B\}}{\text{Max}} \Bigg[& B(q_A,q_B) - \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{\Theta_A,\Theta_B}(\theta_A,\theta_B)\left[(1+\theta_A)q_A - (\omega+\theta_B)q_B\right]\mathrm{d}\theta_A\mathrm{d}\theta_B \\ & - \int_{\chi(q_A,q_B,\theta_A,\theta_B)}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{\Theta_A,\Theta_B,\Lambda}(\theta_A,\theta_B,\lambda)\left[\bar{p}\cdot\big((1+\theta_A)q_A + (\omega+\theta_B)q_B - (T^0+\lambda)\big)\right]\mathrm{d}\theta_A\mathrm{d}\theta_B\mathrm{d}\lambda\Bigg].\end{aligned}$$

Consequently, the ex ante allocation of public goods chosen by the government again portray anticipation via lower public provisions:

$$\begin{aligned} q_A = & \frac{1}{b}\Bigg(1 + \underbrace{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{\Theta_A,\Theta_B}(\theta_A,\theta_B)\theta_A\mathrm{d}\theta_A\mathrm{d}\theta_B}_{=\int_{-\infty}^{\infty} f_{\Theta_A}(\theta_A)\theta_A\mathrm{d}\theta_A = \mathbb{E}[\Theta_A]=0} \\ & + \int_{\chi(q_A,q_B,\theta_A,\theta_B)}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{\Theta_A,\Theta_B,\Lambda}(\theta_A,\theta_B,\lambda)\bar{p}\overbrace{(1+\theta_A)}^{>0,\text{if } \theta_A>-1}\mathrm{d}\theta_A\mathrm{d}\theta_B\mathrm{d}\lambda - a\Bigg) \\ = & \frac{1+\int_{\chi(q_A,q_B,\theta_A,\theta_B)}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{\Theta_A,\Theta_B,\Lambda}(\theta_A,\theta_B,\lambda)\bar{p}(1+\theta_A)\mathrm{d}\theta_A\mathrm{d}\theta_B\mathrm{d}\lambda - a}{b}\end{aligned} \tag{52}$$

$$\begin{aligned} q_B = & \frac{1}{d}\Bigg(\omega + \underbrace{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{\Theta_A,\Theta_B}(\theta_A,\theta_B)\theta_B\mathrm{d}\theta_A\mathrm{d}\theta_B}_{=\int_{-\infty}^{\infty} f_{\Theta_B}(\theta_B)\theta_B\mathrm{d}\theta_B = \mathbb{E}[\Theta_B]=0} \\ & + \int_{\chi(q_A,q_B,\theta_A,\theta_B)}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{\Theta_A,\Theta_B,\Lambda}(\theta_A,\theta_B,\lambda)\bar{p}\overbrace{(\omega+\theta_B)}^{>0,\text{if } \theta_B>-\omega}\mathrm{d}\theta_A\mathrm{d}\theta_B\mathrm{d}\lambda - a\Bigg) \\ = & \frac{\omega+\int_{\chi(q_A,q_B,\theta_A,\theta_B)}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{\Theta_A,\Theta_B,\Lambda}(\theta_A,\theta_B,\lambda)\bar{p}(\omega+\theta_B)\mathrm{d}\theta_A\mathrm{d}\theta_B\mathrm{d}\lambda - c}{d}.\end{aligned} \tag{53}$$

Note that the conditionality on $\big((1+\theta_A)q_A + (\omega+\theta_B)q_B - T^0\big) > 0$ from section 4.2 is still present, bus is now imposed via $\chi(q_A, q_B, \theta_A, \theta_B)$.

Proposition 7. (Compounded Uncertainty I) *Let $MB_i(q_i)$ be a downward sloping marginal benefit function and let $MC_i(\Theta_i)$ be a constant stochastic marginal cost function, with $i \in \{A, B\}$. Then, an asymmetric cost structure for the public deficit $\bar{p} > 0$, public revenues $T(\Lambda) \sim \mathcal{F}_T(q_A^0 + \omega q_B^0, \sigma_t^2)$, and expenditure shocks $\theta_i \sim \mathcal{F}_{\Theta_i}(0, \sigma_i^2)$ with small enough probability $f_{\Theta_i}(\theta_i)$ for unreasonably large downward shocks (i.e. $\Theta_A < -1$ and $\Theta_B < -\omega$), yield short run public provisions $q_A < q_A^{00}$ and $q_B < q_B^{00}$ under expenditure uncertainty.*

Despite the clear similarities to the results in sections 4 and 5, the welfare losses as a result of the compounded uncertainty-driven distortions are not simply the sum of L_i^g and L_i^t. Nonetheless, parallel to the aforementioned cases with both types of uncertainty in unison, the anticipation of the possibly disadvantageous outcomes is not necessarily preferable over the case without any anticipation. The realizations for which this is the case with compounded uncertainty, will depend on the realization of Λ as well. Since the uncertainty-driven distortions to economic efficiency are:

$$L_A = \begin{cases} -\frac{\left(\theta_A+\bar{p}(1+\theta_A)-\int_{\chi(q_A,q_B,\theta_A,\theta_B)}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{\Theta_A,\Theta_B,\Lambda}(\theta_A,\theta_B,\lambda)\bar{p}(1+\theta_A)\mathrm{d}\theta_A\mathrm{d}\theta_B\mathrm{d}\lambda\right)^2}{2b}, \\ \qquad \text{if } \left(C(q_A,q_B,\theta_A,\theta_B)-T(\lambda)\right)>0 \\ -\frac{\left(\theta_A-\int_{\chi(q_A,q_B,\theta_A,\theta_B)}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{\Theta_A,\Theta_B,\Lambda}(\theta_A,\theta_B,\lambda)\bar{p}(1+\theta_A)\mathrm{d}\theta_A\mathrm{d}\theta_B\mathrm{d}\lambda\right)^2}{2b}, \\ \qquad \text{if } \left(C(q_A,q_B,\theta_A,\theta_B)-T(\lambda)\right)\leq 0 \end{cases} \tag{54}$$

$$L_B = \begin{cases} -\frac{\left(\theta_B+\bar{p}(\omega+\theta_B)-\int_{\chi(q_A,q_B,\theta_A,\theta_B)}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{\Theta_A,\Theta_B,\Lambda}(\theta_A,\theta_B,\lambda)\bar{p}(\omega+\theta_B)\mathrm{d}\theta_A\mathrm{d}\theta_B\mathrm{d}\lambda\right)^2}{2d}, \\ \qquad \text{if } \left(C(q_A,q_B,\theta_A,\theta_B)-T(\lambda)\right)>0 \\ -\frac{\left(\theta_B-\int_{\chi(q_A,q_B,\theta_A,\theta_B)}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{\Theta_A,\Theta_B,\Lambda}(\theta_A,\theta_B,\lambda)\bar{p}(\omega+\theta_B)\mathrm{d}\theta_A\mathrm{d}\theta_B\mathrm{d}\lambda\right)^2}{2d}, \\ \qquad \text{if } \left(C(q_A,q_B,\theta_A,\theta_B)-T(\lambda)\right)\leq 0 \end{cases} \tag{55}$$

shocks $(\theta_A, \theta_B, \lambda)$ for which λ is greater or equal to $(q_i - q_i^{00})$ (i.e. the anticipation) and the cost shock is (a) negative or (b) positive with the shock's value not exceeding half of the anticipated reduction in marginal benefits, will actually imply a range of realizations of expenditure costs and fiscal revenues for which anticipation is counterproductive. On the other hand, if λ is smaller than $(q_i - q_i^{00})$ this does not hold. Then, anticipation has an adverse effect in case the expenditure shock is smaller than λ minus half of the anticipated reduction in marginal benefits. The remark with respect to compensation between different expenditure categories within the overall budget as a result of heterogeneous shocks, however, still holds (see section 4.2.2).

Proposition 8. (Compounded Uncertainty II) *Let $MB_i(q_i)$ be a downward sloping marginal benefit function and let $MC_i(\Theta_i)$ be a constant stochastic marginal cost function, with $i \in \{A, B\}$. Then, the anticipatorily lowered public provisions as a result of an asymmetric cost structure for the public deficit $\bar{p} > 0$, public revenues $T(\Lambda) \sim \mathcal{F}_T(q_A^0 + \omega q_B^0, \sigma_t^2)$, and shocks $\theta_i \sim \mathcal{F}_{\Theta_i}(0, \sigma_i^2)$, are counterproductive in hedging against expenditure uncertainty if:*

a. $\lambda \geq (q_i - q_i^{00})$ and the shock θ_i is either negative or positive but does not exceed half of the anticipated reduction in marginal benefits or;

b. $\lambda < (q_i - q_i^{00})$ and the shock θ_i is smaller than λ minus half of the anticipated reduction in marginal benefits.

[TBC]

6.2 N-Vector Model

[TBC]

6.3 Discussion

[TBC]

7 Buffer Provisions for the Future

So far, the government's maximization problem, however, inclined policy makers to spend as much as possible to maximize the (social) benefits under the cost and revenue conditions. In particular, there was no other uses for the fiscal funds than for expenditure based benefits. Nonetheless, governments might be interested to shift expenditures through time via buffers (ϵ). Moreover, they may be restricted in their actions by European restrictions, e.g. a debt target.

Nevertheless, extending the framework to a multiperiod setting, allowing for the anticipatory reduction in public expenditures to be employed as rainy-day funds, does not entirely dispel the possible distortions due to anticipation. After all, the buffers' success is dependent on the government's knowledge of the respective probability density functions.

[TBC]

8 The Budgeting Process Under Uncertainty

A detrimental factor of the uncertainty-driven welfare losses was the assumption of a government's inability to adjust the allocation ex post. It is, however, not unimaginable that certain categories of public expenditures give leeway to adjustment in the short run without incurring too much political damage. Draft budgetary plans and fiscal rules might actually foresee a limited room for maneuver and allow for adjustment of these expenditure categories. Ex post adjustments might also be the result of time inconsistent behavior by policy makers. Consequently, in this section the assumption that the government is unable to divert from its ex ante commitments is relaxed to accommodate for such scenarios.

Additionally, certain categories of public expenditures are probably more interesting to adjust than others. For example, large public investment projects might be perceived to be more acceptable to postpone in times of tight budget constraints than current expenditures for existing services. A resulting and pressing question for practitioners in public administrations therefore is which expenditure category is best adjusted if there are multiple options.

Allowing for partial contemporaneous adjustment (i.e. of a fraction of the public goods provided) illustrates the ability of the government to counter possible welfare losses due to uncertainty. For example, suppose that good A can be adapted more easily in the short run than B. More specifically, assume that good A can be adjusted contemporaneously with the divergences of both expenditure costs and revenues from their expectations, while B is only adjustable in the long term. Then, instead of providing in the allocation of public goods $\{q_A, q_B\}$, the government will provide in $\{q_A^{11}, q_B\}$. This allocation can be either higher or lower than the original allocation from section 6. As a result of the adjustment, however, L_A will be zero.

8.1 Preference-based Adjustment

The effectiveness with which a governments might benefit from such an adjustment nonetheless depends on the slope of the demand curve of the adjustable public good, i.e. the elasticity of the respective public good. For instance, for a homogeneous shock to the costs of provision $\omega = 1$, the losses due to uncertainty increase as the slope of the marginal benefit curve becomes flatter. Consequently, the budgeting process would be better of with a public good with a flatter curve being more flexible to adjust.

In the figures above, for instance, public good A would be the preferred good to adjust. The steeper slope of good B's marginal benefit curve vis-á-vis good A's curve reflects that citizens prefer a sharper decrease in the provision of good A over one in good B in case of a comparable increase in both their cost of provision.

[**TBC**]

8.2 Cost-based Adjustment

In addition to the relative shocks, the shocks' distributions and the relative slopes of both expenditure categories, the relative difference in costs codetermines the suitability for adjustment of the various categories of expenditures. As presented in the deterministic benchmark model, both public goods have proportional costs: 1 and ω, respectively. As shown in equations (54) and (55), the losses by uncertainty-driven distortions are depending on this proportionality as well. In fact, $\omega > 1$ would plea in favor of adjusting good B. The optimal strategy for adjustment thus entails a combination of the preference-based and the cost-based arguments for adjustment.

[**TBC**]

9 Concluding Remarks

In sum, one does best not consider the factors underlying volatility on the expenditure side in separation, as often done for the different categories on the revenue side of the budget. Looking at the budget as it is, a dynamic system of communicating vessels, helps to explain that not only cost uncertainty on the expenditure side is crucial for public good provisions and welfare. Given the pressure for fiscal discipline, revenue volatility unmistakable plays its role too.

The model presented here results in four clear conclusions. Firstly, both expenditures' cost uncertainty and revenue volatility are straightforwardly found to result in welfare losses, irrespective of whether a strict budget constraint is in force or not.

Secondly, adding a penalty in case of a deficit to incorporate the market's requirement for fiscal discipline, however, urges policy makers to anticipate for the cases in which shocks are disadvantageous and a penalty would be incurred. In particular, both types of uncertainty are found to make governments provide lower levels of public goods when an asymmetric penalty structure favoring fiscal discipline is in force. After all, that anticipatory behavior hedges against the disadvantageous impact of uncertainty if costs turn out to be higher and/or public revenues turn out to be lower than expected.

Thirdly, while an explicit cost for fiscal profligacy hedges against disadvantageous shocks, it also warrants fiscal discipline in case ex post realized costs and revenues are lower and higher than expected, respectively. The anticipatory behavior can nonetheless be counterproductive as well. In case the public goods' costs are lower than expected and/or fiscal revenues were underestimated, higher public expenditures would have been optimal ex post and welfare losses are thus higher than without anticipation.

Finally, relaxing the assumption of a government's inability to adjust the expenditures ex post, raises the pressing question for practitioners in public administrations which expenditure category is best adjusted if there are multiple options. The model shows that, in addition to the realized shocks to the expenditures' costs of provision, two inherent characteristics of the expenditures are to be considered. Specifically, relatively more expensive and more elastic goods are to be adjusted.

Withal, in contrast with the disciplinary pressure of the market cost in case of a deficit, partial adjustment reduces the uncertainty-driven distortions with certainty. The latter result therefore pleads in favor of draft budgetary plans and fiscal rules foreseeing a limited room for maneuver and allowing for the adaptation of those expenditure categories that are relatively the most effectively adjustable.

A Mathematical Appendix

A.1 Benchmark with Strict Budget Constraint

The government will solve the following optimization problem

$$\underset{\{q_A,q_B\}}{\text{Max}} \Big[B(q_A,q_B) - (q_A+\omega q_B) - \bar{p}.\max\big(q_A+\omega q_B - T^0, 0\big)\Big]. \tag{56}$$

Working out the maximum of the fiscal deficit and a balanced budget using $\max(x,y) = \frac{1}{2}(x+y+|x-y|)$, results in the policy makers solving the following problem:

$$\underset{\{q_A,q_B\}}{\text{Max}} \Big[B(q_A,q_B) - (q_A+\omega q_B) - \frac{1}{2}\bar{p}\Big(q_A+\omega q_B - T^0 + \big|q_A+\omega q_B - T^0\big|\Big)\Big]. \tag{57}$$

Next, given that the first order derivative of a function $f(x) = |u(x)|$, comprising an absolute value function of x, equals $\frac{df(x)}{dx} = \frac{u}{|u|}u'$ results in the allocations mentioned in section 2.2:

$$\begin{aligned} q_A &= \frac{1}{b}\left(1 + \frac{1}{2}\bar{p} + \frac{1}{2}\bar{p}\cdot\frac{q_A+\omega q_B - T^0}{|q_A+\omega q_B - T^0|} - a\right) \\ &= \begin{cases} \frac{1+\bar{p}-a}{b}, & \text{if } (q_A+\omega q_B - T^0) > 0 \\ \frac{1-a}{b}, & \text{if } (q_A+\omega q_B - T^0) \leq 0 \end{cases} \end{aligned} \tag{58}$$

$$\begin{aligned} q_A &= \frac{1}{d}\left(1 + \frac{1}{2}\bar{p} + \frac{1}{2}\bar{p}\cdot\frac{q_A+\omega q_B - T^0}{|q_A+\omega q_B - T^0|} - c\right) \\ &= \begin{cases} \frac{(1+\bar{p})\omega-c}{d}, & \text{if } (q_A+\omega q_B - T^0) > 0 \\ \frac{\omega-c}{d}, & \text{if } (q_A+\omega q_B - T^0) \leq 0 \end{cases} \end{aligned} \tag{59}$$

A.2 Benchmark with a Symmetric Penalty Structure

In case of a symmetric penalty the government solves

$$\underset{\{q_A,q_B\}}{\text{Max}} \Big[B(q_A,q_B) - C(q_A,q_B) - p\big(C(q_A,q_B) - T^0\big)\Big], \tag{60}$$

with $p = \bar{p}$ instead of the structure from (6). Given that $\bar{p} > 0$ always holds in case of a symmetric penalty, it can be easily seen that $q_A^{00} < q_A^0$ and $q_B^{00} < q_B^0$. Moreover, it follows that $q_A^{00} < q_B^{00}$ if and only if $b > d$.

Consequently, the fiscal balance will be higher than in the benchmark case (i.e. $D < 0$). Inserting the quantities specified by the first order conditions results in:

$$\begin{aligned} D &= (1+\bar{p})\big(C(q_A^{00}, q_B^{00}) - T^0\big) \\ &= (1+\bar{p})(q_A^{00} + \omega q_B^{00} - q_A^0 - \omega q_B^0) \\ &= \bar{p}(1+\bar{p})\left(\frac{1}{b} + \frac{\omega^2}{d}\right). \end{aligned} \tag{61}$$

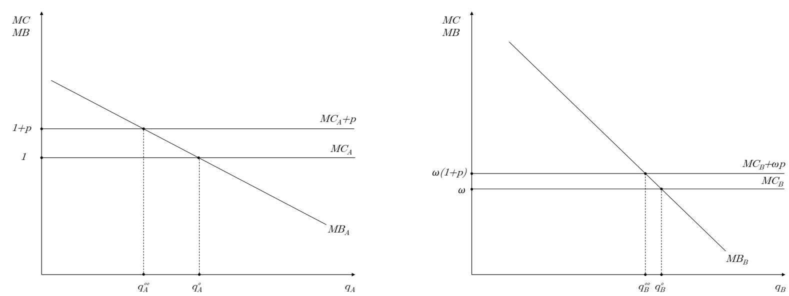

Figure 4: The Marginal Costs and Benefits of the Public Goods in case of a Symmetric p

A.3 Expenditure Shocks with a Symmetric Penalty Structure

In case of a symmetric penalty and possible cost shocks on the expenditure side the government solves

$$\underset{\{q_A,q_B\}}{\text{Max}}\ \mathbb{E}\Big[B(q_A,q_B)-C(q_A,q_B,\Theta_A,\Theta_B)-p(C(q_A,q_B,\Theta_A,\Theta_B)-T^0)\Big], \quad (62)$$

with $p=\bar{p}$ for all pairs $\{q_A,q_B\}$. This again results in the following optimum allocations:

$$q_A^{11} = \frac{(1+p)(1+\theta_A)-a}{b} \quad (63)$$

$$q_B^{11} = \frac{(1+p)(\omega+\theta_B)-c}{d} \quad (64)$$

Whether these provisions are smaller or larger than $\{q_A^{00}, q_B^{00}\}$ depends on whether the shocks turn out to be positive or negative, respectively. In particular, given the downward sloping demand curves of the public goods the following holds:

$$\frac{\partial q_A^{11}}{\partial\theta_A}<0 \qquad \text{and} \qquad \frac{\partial q_B^{11}}{\partial\theta_B}<0. \quad (65)$$

The fiscal balance in the optimum too will depend on the shock. Inserting the quantities specified by the first order conditions results in:

$$\begin{aligned} D =&(1+\bar{p})(q_A^{11}+\omega q_B^{11}-q_A^0-\omega q_B^0)\\ &(1+\bar{p})\left(\frac{\theta_A+\bar{p}(1+\theta_A)}{b}+\frac{\omega\theta_B+\omega\bar{p}(\omega+\theta_B)}{d}\right). \end{aligned} \quad (66)$$

Thus, in case of a positive shock to marginal costs of public good provision the government would have been better of running a larger budget surplus. If the shock on the other hand would have been negative, the government could have done better (with hindsight) by running a deficit instead of a surplus.

Governments nevertheless anticipate the penalty as it applies to any allocation of public goods they make. As illustrated by the following ex ante allocation:

$$q_A = \frac{(1+\bar{p}) - a}{b} = q_A^{00} \tag{67}$$

$$q_B = \frac{(1+\bar{p})\omega - c}{d} = q_B^{00}, \tag{68}$$

where the equality with $\{q_A^{00}, q_B^{00}\}$ holds for $p = \bar{p}$.

In spite of that, the governments do not accommodate for the cost shocks themselves and this channel of losses thus remains. Furthermore, as a result of this channel the anticipation of $\bar{p}$ may overshoot its cause. Hence, both θ_i and $\bar{p}$ are present in the welfare losses incurred if the expenditure allocations of A and B are fixed ex ante:

$$\begin{aligned} L_A^g &= \frac{1}{2}\Big(MC_A(q_A^{00}, \theta_A) + \bar{p}(1+\theta_A) - \mathbb{E}[MC_A(q_A^{00}, \Theta_A)] - \bar{p} \Big)\big(q_A^{00} - q_A^{11}\big) \\ &= -\frac{(\theta_A + \theta_A\bar{p})^2}{2b} \end{aligned} \tag{69}$$

$$\begin{aligned} L_B^g &= \frac{1}{2}\Big(MC_B(q_B^{00}, \theta_B) + \bar{p}(\omega+\theta_B) - \mathbb{E}[MC_B(q_B^{00}, \Theta_B)] - \omega\bar{p} \Big)\big(q_B^{00} - q_B^{11}\big) \\ &= -\frac{(\theta_B + \theta_B\bar{p})^2}{2d}. \end{aligned} \tag{70}$$

A.4 Homogeneous Expenditure Shocks: Alternative Derivation

Instead of using absolute value functions to work out the $\max(\cdot,\cdot)$, the first endpoint of the integral in (34) can be replaced. In particular, in case of homogeneous shocks the budget will be balanced if:

$$\theta(q_a, q_B) = \frac{T^0 - q_a - \omega q_B}{q_A + q_B}. \tag{71}$$

As a consequence, the integral in (34) can be rewritten as follows:

$$\begin{aligned} \underset{\{q_A, q_B\}}{\text{Max}} \Bigg[& B(q_A, q_B) - \int_{-\infty}^{\infty} f_\Theta(\theta)\,[(1+\theta)q_A + (\omega+\theta)q_B]\,\mathrm{d}\theta \\ & - \int_{\theta(q_A,q_B)}^{\infty} f_\Theta(\theta)\big[\bar{p}\cdot\big((1+\theta)q_A + (\omega+\theta)q_B - T^0\big)\big]\,\mathrm{d}\theta \Bigg] \end{aligned} \tag{72}$$

List of Symbols

q_i	The amount of public good i provided, with $i \in \{A, B\}$
$B(q_A, q_B, \theta_A, \theta_B)$	The overall (stochastic) benefit function
$C(q_A, q_B, \theta_A, \theta_B)$	The overall (stochastic) cost function
$MC_i(q_i, \theta_i)$	The (stochastic) cost of one additional unit of expenditure on good i
$MB_i(q_i)$	The benefit of one additional unit of expenditure on good i, i.e. the respective demand function
a	The intercept of $MB_A(q_A)$
b	The slope of $MB_A(q_A)$
c	The intercept of $MB_B(q_B)$
d	The slope of $MB_B(q_B)$
ω	The proportional cost of public good B
θ_i	The shock to the marginal cost of public good i
T	Fiscal revenues
D	The budget deficit, i.e. $(q_A + q_B - T)$
p	The economic cost of fiscal indiscipline
$\bar{p}$	The economic cost of fiscal indiscipline specified per unit of deficit incurred
L_i^g	Welfare losses for good i in case of expenditure uncertainty
L_i^t	Welfare losses for good i in case of revenue uncertainty

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