Real exchange rates and economic fundamentals: An investigation based on a Markov-STAR model

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Outline

Motivation

- Rogoff (1996): PPP puzzle $\rightarrow$ high half-lifes for real exchange rates
- Taylor and Sarno (2003): Nonlinear adjustments in real exchange rates
- ESTAR approach: e.g. Michael et al. (1997), Taylor et al. (2001)
- Kilian and Taylor (2003), Rapach and Wohar (2006): nonlinear models improve forecast
- Engel (1994), Bergman and Hansson (2005): Markov switching model
- Failure of rejecting a unit root can be explained

Motivation

- Kaufmann et al. (2014) propose test for ESTAR against Markov-Switching in real exchange rates
- They found that real exchange rates of developing countries are rather Markov-Switching and of developed countries are rather ESTAR
- Markov-Switching describes abrupt changes possibly due to policy changes
- ESTAR describes smooth changes caused by trading behavior and for inflation targeting

Motivation

- Engel (2000) argues that equilibrium is time varying due to differences in productivity growth
- Engel and Kim (1999) suggest models with time varying equilibrium:

$$\begin{aligned} q_t &= y_t + x_t \\ y_t &= y_{t-1} + u_t \\ u_t &\sim N(0, \sigma_t^2) \end{aligned}$$

- q_t real exchange rate
- Their model has random walk as equilibrium with structural changes in volatility
- Transitory component x_t is $AR(2)$-process with time-varying volatility
- Alternatively, Hedgwood and Papell (1998) allow for exogenous structural breaks in equilibrium component

Motivation

- We propose equilibrium component to be Markov-Switching
- Transitory component is modeled as ESTAR

$\Rightarrow$ Our equilibrium is constant over periods with positive probability (other than Engel and Kim, 1999) and breaks are endogenous (other than Hedgwood and Papell, 1998)

- Our transitory component is nonlinear and depends on distance between actual real exchange rate and switching equilibrium values

Model

- ESTAR (p) model:

$$q_t = \left(\sum_{k=1}^{p} \psi_k q_{t-k}\right)(1 - G(q_{t-d}, \gamma)) + \left(\sum_{k=1}^{p} \theta_k q_{t-k}\right) G\left(q_{t-d}, \gamma\right) + \varepsilon_t,$$

$$\varepsilon_t \overset{iid}{\sim} \left(0, \sigma^2\right), \quad G\left(q_{t-d}, \gamma\right) = 1 - \exp\left(-\gamma q_{t-d}^2\right), \quad \gamma > 0$$

- Markov-Switching Model:

$$q_t = \mu_{s_t} + \phi_{1,s_t} q_{t-1} + \ldots + \phi_{p,s_t} q_{t-p} + \varepsilon_t, \quad \varepsilon_t \sim \left(0, \sigma_{s_t}^2\right)$$

$\Rightarrow$ The MSAR(1)-ESTAR(1) (MS-STAR) model:

$$q_t = \mu_{s_t} + \phi q_{t-1} + \psi G(q_{t-1}; \gamma, c) q_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim \left(0, \sigma_{s_t}^2\right)$$

Model

- MS-STAR model:

$$q_t = \mu_{s_t} + \phi q_{t-1} + \psi G(q_{t-1};\gamma, c) q_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim \left(0, \sigma^2_{s_t}\right)$$

- We set $\phi = 1$ and $\psi = -1$ resulting in

$$q_t = \mu_{s_t} + (1 - G(q_{t-1};\gamma, c)) q_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim \left(0, \sigma^2_{s_t}\right)$$

→ $\gamma \to \infty$: markov switching only model, $G(\cdot) \to 1 \Rightarrow q_t = \mu_{s_t} + \varepsilon_t$

→ $\gamma \to 0$: random walk with switching drift, $G(\cdot) \to 0 \Rightarrow q_t = q_{t-1} + \mu_{s_t} + \varepsilon_t$

Model

- Estimation: Combination of grid search and likelihood estimation

$$L(\mu, P, \sigma; \gamma, c) = \sum_{i=1}^{N} \sum_{t=1}^{T} \log f_{it} \quad \text{with} \quad f_{it} = \mathbf{1}'(\xi_{it|t-1} \odot \eta_{it}),$$

$$\eta_{it} = \left(2\pi\sigma_i^2\right)^{-1/2} \exp\left(-0.5\varepsilon_{it}^2/\sigma_i^2\right),$$

$$\varepsilon_{it} = q_t - \mu_i - \phi q_{t-1} - \psi G(q_{t-1}; \gamma, c) q_{t-1},$$

$$\xi_{t|t} := Pr(s_t = i | q_t; \theta)$$

- Smoothed transition probabilities are calculated via the algorithm of Kim (1993)
- grid search for value of $\gamma_k \in [.01, 15] \Rightarrow L_k = L(\theta_k)$ with $\theta_k = (\mu_k, P_k, \sigma_k; \gamma_k)$ and $k^* = \arg\max_k L_k$

Model

- Macro Variables X:
  (i) Take first row of smoothed transition probabilities $\xi_{t|T}$, i.e. fix $i = 1$ in $\tilde{p}_i := Pr(s_t = i|q_T; \theta)$
  (ii) Recode the first row of $\xi_{t|T}$ into a binary variable Y such that

$$Y := \begin{cases} 0 & \text{for } \tilde{p}_1 \leq 0.5 \\ 1 & \text{for } \tilde{p}_1 > 0.5. \end{cases}$$

  (iii) Fit a Logit model to Y with macro variables X
- estimated coefficients $\hat{\beta}$:= marginal effect of X on $\ln(\tilde{p}_1/(1-\tilde{p}_1))$
- If e.g. $\hat{\beta} < 0$, $\tilde{p}_1 < 0.5 \Rightarrow$ increase of the macro variable increases probability of being in regime 2

Empirical Application

- Monthly data for 18 exchange rates to US Dollar from 01/60 to 02/14 $\Rightarrow T = 652$ (at most)
- IMF data, direct quotation: $q_t \uparrow \Rightarrow$ depreciation of domestic currency
- Macro variables:
  - $\rightarrow$ output gap: US output minus domestic output, Engel and West (2006)
  - $\rightarrow$ inflation differential: US inflation minus domestic inflation
  - $\rightarrow$ uncertainty: index by Nick Bloom (http://www.policyuncertainty.com)

Empirical Application

Country	$\hat{\mu}_1$	$\hat{\mu}_2$	$\hat{p}_1$	$\hat{p}_2$	$\hat{\sigma}_1$	$\hat{\sigma}_2$	γ^*	T
Brazil	0.0021	-0.0023	0.8801	0.9818	0.0618	0.0101	0.01	413
Canada	0.0011	-0.0006	0.8855	0.9908	0.0179	0.0031	0.01	652
Chile	0.0600	-0.0022	0.9990	0.9990	0.0001	0.0211	1.75	64
Denmark	0.0211	-0.0204	0.9499	0.9378	0.0148	0.0151	0.40	568
Finland	0.0001	-0.0014	0.9985	0.9916	0.0000	0.0280	1.15	468
France	0.0001	-0.0008	0.9990	0.9792	0.0000	0.0287	0.01	468
Germany	0.0208	-0.0256	0.9501	0.9438	0.0170	0.0153	15.00	96
Greece	0.0013	0.0001	0.9806	0.9990	0.0258	0.0001	0.55	492
Italy	0.0001	-0.0008	0.9990	0.9749	0.0001	0.0246	0.15	468
Japan	0.0102	-0.0376	0.9627	0.9102	0.0159	0.0196	0.03	652
Mexico	0.0031	-0.0100	0.9774	0.9990	0.0444	0.0001	0.03	652
Netherlands	0.0140	-0.0277	0.9604	0.9485	0.0149	0.0141	0.30	468
Norway	0.0002	-0.0002	0.9990	0.9602	0.0005	0.0241	0.15	652
Portugal	0.0108	-0.0317	0.9588	0.9389	0.0171	0.0142	0.40	468
Spain	0.0170	-0.0215	0.9536	0.9438	0.0169	0.0155	0.30	468
Sweden	0.0001	-0.0005	0.9990	0.9668	0.0003	0.0266	0.60	652
Turkey	0.0196	-0.0039	0.8583	0.9762	0.0869	0.0188	0.01	544
UK	0.0172	-0.0199	0.9156	0.9357	0.0161	0.0130	2.50	316

Tabelle: Estimation results of the Markov-STAR model
$q_t = \mu_{s_t} + \phi q_{t-1} + \psi G(q_{t-1}; \gamma, c) q_{t-1} + \varepsilon_t$ with $\phi = 1$ and $\psi = -1$.

Empirical Application

Country	Cons	p-value	Outp gap	p-value	Inflation	p-value	Bloom	p-value	McF R^2	T
Brazil	**-1.2619**	0.0240	2.1166	0.1675	0.2104	0.1265	**0.0083**	0.0178	0.3498	277
Canada	**-2.5149**	0.0000	-0.8764	0.5863	**22.3886**	0.0007	**0.0269**	0.0000	0.2127	638
Chile	-1.4458	0.8621	-5.8024	0.8216	95.0608	0.1064	-0.0180	0.6647	0.3274	50
Denmark	**1.1618**	0.0033	0.1359	0.9115	6.1457	0.2108	**-0.0090**	0.0009	0.1700	482
Finland	**-1.2216**	0.0415	**6.9882**	0.0022	**-11.2645**	0.0435	**-0.0158**	0.0045	0.1860	456
France	**1.4334**	0.0059	**5.2411**	0.0026	-8.7965	0.1933	**-0.0217**	0.0000	0.2611	456
Germany	**5.7656**	0.0042	**30.4195**	0.0001	**-160.3895**	0.0002	**-0.0516**	0.0011	0.3935	84
Greece	3.2912	0.2530	-3.0250	0.8110	-19.7693	0.4640	-0.0011	0.9510	0.9720	71
Italy	**-0.9686**	0.0834	**8.4582**	0.0000	1.7153	0.6583	-0.0025	0.6483	0.1774	456
Japan	**3.1656**	0.0000	**2.7421**	0.0000	3.0448	0.3900	**-0.0115**	0.0000	0.0883	638
Mexico	0.1086	0.8485	0.8032	0.7558	**-2.9040**	0.0016	0.0060	0.1086	0.4300	410
Netherlands	**3.7095**	0.0000	**6.9630**	0.0000	-2.3859	0.6104	**-0.0222**	0.0000	0.2008	456
Norway	**-3.0725**	0.0000	-1.4693	0.1025	**-14.1791**	0.0091	**0.0054**	0.0678	0.0349	638
Portugal	**4.1083**	0.0000	1.6396	0.1463	-3.0523	0.1354	**-0.0264**	0.0000	0.1743	456
Spain	**2.2098**	0.0000	**3.1035**	0.0003	**5.7628**	0.0564	**-0.0147**	0.0004	0.1923	456
Sweden	**-2.3789**	0.0231	**-5.7314**	0.0588	28.2894	0.1957	-0.0015	0.8031	0.7718	206
Turkey	**-2.6346**	0.0007	**7.8153**	0.0000	**1.6519**	0.0714	**0.0078**	0.0573	0.4395	350
UK	0.7189	0.1648	0.4888	0.8291	**20.8430**	0.0176	-0.0026	0.4259	0.0689	302

Tabelle: Logit regression results. Significant coefficients for $\alpha = 0.1$ in bold.

(i) Interpretation of Bloom Index?

(ii) Is the mean switch or the variance switch the main driver?

(iii) Contribution of smooth transition part?

Empirical Application

- Bloom Index: hard to interpret
- disentangle switch in mean and switch in volatility effects

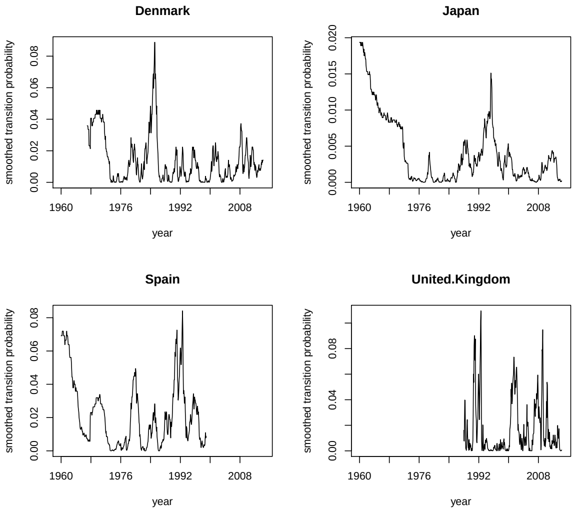
$\Rightarrow$ fix μ in MS-STAR model, estimate model and fit Logit

- Results: increasing policy uncertainty leads to high volatilities in real exchange rates (100% of significant coefficients)

- Mean vs. volatility switch: LR–test
- $H_0 : \sigma_2 = 0$ vs. $H_1 : \sigma_2 \neq 0$, $LR = -2\ln(L_k^{**}/L_k^{*})$, L_k^{**} denotes maximized likelihood of restricted model
- Results: H_0 can never be rejected ($\alpha = .1$) $\rightarrow$ switch in mean seems to be main driver of the real exchange rate dynamics

Empirical Application

- Contribution of the smooth transition part?



Empirical Application

- Contribution of the smooth transition part?
- Calculation of half-lifes: generalized nonlinear impulse-response function, Koop et al. (1996)
- One standard deviation shock at time t, after how many years has half of the shock faded out?

Country	$\gamma = \gamma^*$	$\gamma = .01$	$\gamma = 15$	St. Dev.
Denmark	2.2074	11.0146	0.3067	0.1871
Japan	4.4847	6.3560	0.0833	0.3746
Spain	2.4880	10.5711	0.0833	0.2727
UK	1.2280	13.6058	0.6049	0.0835

Conclusion

- Propose exchange rate model with time-varying (MS) equilibrium and nonlinear ($ESTAR$) transitory regime
- Explain equilibrium changes by macro variables, mean switch: output gap and inflation, variance changes: uncertainty
- Economic theory/expectations met by MS-STAR model, varying equilibrium exchange rate closely connected with macro variables
- Smooth transition part plays crucial role
- Possible extensions: relax coefficient restrictions, let coefficients switch, more than 2 states, multivariate model, direct implementation of macro variables into MS-STAR, etc.

Tables

Country	$\hat{\mu}$	$\hat{p}_1$	$\hat{p}_2$	$\hat{\sigma}_1$	$\hat{\sigma}_2$	γ^*
Brazil	-0.0020	0.9820	0.8801	0.0101	0.0618	0.0100
Canada	-0.0005	0.9910	0.8854	0.0032	0.0180	0.0100
Chile	-0.0100	0.9990	0.9917	0.0000	0.0279	0.0100
Denmark	0.0004	0.9890	0.8978	0.0072	0.0313	0.0300
Finland	-0.0008	0.9900	0.8905	0.0062	0.0367	0.1000
France	0.0001	0.9990	0.9672	0.0001	0.0288	0.0300
Germany	-0.0064	0.9990	0.9356	0.0013	0.0295	6.5000
Greece	-0.0001	0.9990	0.9792	0.0000	0.0265	0.0300
Italy	0.0001	0.9990	0.9773	0.0000	0.0261	0.0100
Japan	0.0004	0.9832	0.8703	0.0079	0.0372	0.0100
Mexico	-0.0034	0.9782	0.8794	0.0079	0.0832	0.0100
Netherlands	0.0008	0.9822	0.9213	0.0025	0.0292	0.0300
Norway	0.0001	0.9990	0.9553	0.0002	0.0265	0.0100
Portugal	0.0001	0.9990	0.9565	0.0007	0.0265	0.1000
Spain	0.0002	0.9990	0.9377	0.0004	0.0278	0.0100
Sweden	0.0001	0.9988	0.9880	0.0000	0.0256	0.0300
Turkey	-0.0028	0.9751	0.8592	0.0183	0.0869	0.0300
UK	0.0020	0.9733	0.8720	0.0075	0.0323	1.8500

Tabelle: Estimation results of the Markov-STAR model $q_t = \mu_{s_t} + \phi q_{t-1} + \psi G(q_{t-1};\gamma, c)q_{t-1} + \varepsilon_t$ with μ fixed, $\phi = 1$ and $\psi = -1$.

Tables

Country	Cons	p-value	Outp gap	p-value	Inflation	p-value	Bloom	p-value
Brazil	**1.2199**	0.0290	-2.0712	0.1764	-0.2153	0.1174	**-0.0081**	0.0201
Canada	**2.3750**	0.0000	0.9886	0.5351	**-21.2466**	0.0011	**-0.0254**	0.0000
Chile	-3.2782	0.1074	4.5629	0.6382	**82.5774**	0.0310	0.0129	0.1286
Denmark	**-2.4563**	0.0000	0.9396	0.5976	**17.9266**	0.0268	0.0040	0.2367
Finland	**4.3886**	0.0000	1.0099	0.4947	-4.7237	0.2255	**-0.0428**	0.0000
France	**1.4334**	0.0059	**5.2411**	0.0026	-8.7965	0.1933	**-0.0217**	0.0000
Germany	-0.0354	0.9893	6.7519	0.3806	-34.3305	0.4656	-0.0202	0.3228
Greece	-3.7843	0.1542	9.0071	0.4445	46.8058	0.1180	0.0128	0.4166
Italy	0.3635	0.4857	**6.5754**	0.0003	-0.8291	0.8102	**-0.0118**	0.0225
Japan	**1.5532**	0.0000	**2.8777**	0.0000	**-12.8065**	0.0001	**-0.0142**	0.0000
Mexico	**2.2328**	0.0000	-3.0232	0.1631	**1.0405**	0.0559	**-0.0077**	0.0119
Netherlands	**2.3628**	0.0000	1.7446	0.2276	9.7440	0.1002	**-0.0395**	0.0000
Norway	0.1599	0.5716	**1.7546**	0.0101	**-8.1598**	0.0420	**-0.0116**	0.0000
Portugal	**-2.3218**	0.0003	1.5063	0.2886	-3.3301	0.2133	0.0005	0.9389
Spain	0.7946	0.1314	0.5160	0.5240	4.1769	0.1658	**-0.0168**	0.0005
Sweden	**-2.4018**	0.0206	**-5.9916**	0.0460	34.5575	0.1104	-0.0016	0.7884
Turkey	**2.5982**	0.0004	**-7.4661**	0.0000	**-1.4762**	0.0814	**-0.0087**	0.0250
UK	**-1.0304**	0.0557	-2.6436	0.2638	**32.8191**	0.0008	0.0031	0.3610

Tabelle: Logit regression results with μ fixed. Significant coefficients for $\alpha = 0.1$ in bold.

Tables

Country	$\hat{\mu}_1$	$\hat{\mu}_2$	$\hat{p}_1$	$\hat{p}_2$	$\hat{\sigma}$	γ^*	LR
Brazil	-0.0067	0.1069	0.9905	0.9322	0.0300	0.1000	0.23
Canada	-0.0055	0.0151	0.9469	0.9369	0.0091	0.5000	0.12
Chile	-0.0140	0.0252	0.9603	0.9467	0.0120	7.0000	0.15
Denmark	-0.0205	0.0210	0.9382	0.9492	0.0150	0.4000	0.00
Finland	-0.0063	0.0406	0.9753	0.9653	0.0194	0.0100	0.36
France	-0.0078	0.0367	0.9710	0.9173	0.0161	0.4000	1.09
Germany	-0.0246	0.0215	0.9387	0.9488	0.0163	15.0000	0.01
Greece	-0.0094	0.0307	0.9522	0.9483	0.0156	0.3500	0.38
Italy	-0.0081	0.0316	0.9726	0.9550	0.0151	0.3000	0.74
Japan	-0.0372	0.0115	0.9283	0.9612	0.0167	0.0500	0.01
Mexico	-0.0018	0.3635	0.9990	0.9990	0.0263	0.2000	0.74
Netherlands	-0.0281	0.0144	0.9428	0.9632	0.0146	0.3500	0.01
Norway	-0.0244	0.0140	0.9292	0.9580	0.0138	0.4000	0.04
Portugal	-0.0298	0.0107	0.9287	0.9567	0.0163	0.3000	0.23
Spain	-0.0084	0.0354	0.9746	0.9360	0.0163	0.0100	0.00
Sweden	-0.0109	0.0279	0.9571	0.9593	0.0154	0.3000	0.26
Turkey	-0.0045	0.1650	0.9970	0.9935	0.0300	0.0500	0.10
UK	-0.0182	0.0187	0.9257	0.9243	0.0148	3.0000	0.01

Tabelle: Estimation results of the Markov-STAR model
$q_t = \mu_{s_t} + \phi q_{t-1} + \psi G(q_{t-1}; \gamma, c) q_{t-1} + \varepsilon_t$ with σ fixed, $\phi = 1$ and $\psi = -1$.