EUROPEAN CENTRAL BANK

EUROSYSTEM

# The Bayesian Estimation, Analysis and Regression (BEAR) Toolbox

**Version 2.3**

## 16 IWH-CIREQ

## Macroeconometric Workshop

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Romain Legrand, Alistair Dieppe and Björn van Roye
External Developments Division

*The views expressed in this presentation do not necessarily reflect those of the ECB.*

# Motivation

## Why Bayesian?

- Many countries have limited data availability
- Over-parameterization issue
- Bayesian VARs then preferable to traditional, maximum likelihood VARs

## Issues with existing applications

- Some software / codes for Bayesian VARs do already exist:
  - Eviews, RATS, Villani, Koop, IRIS, Dynare ...
- 3 main issues:
  - limited features: few priors available (RATS/Villani: normal-diffuse), limited number of applications (Eviews: only IRFs, no forecasts, FEVD, historical decomposition, conditional forecasts)
  - not user-friendly: requires advanced knowledge in mathematical software programming; no detailed user guide available
  - lack of flexibility: difficult to add new features (no coding guide), sometimes not even possible at all (Eviews)

# Objectives:

- Develop a toolbox satisfying 3 main objectives:
  - Easy to use for desk economists and user-friendly with a graphical interface and user’s guide.
  - Comprehensive: all applications (basic and advanced) gathered in one single application.
  - Easy to understand, augment and adapt: Support the code with a theoretical guide for transparency.

# 3 estimation techniques of VAR models:

- **OLS (maximum likelihood) VAR**

- **Standard Bayesian VAR:**

$$A(L)y_t = Cx_t + \varepsilon_t$$

- **Mean-adjusted Bayesian VAR (Villani, 2009)**

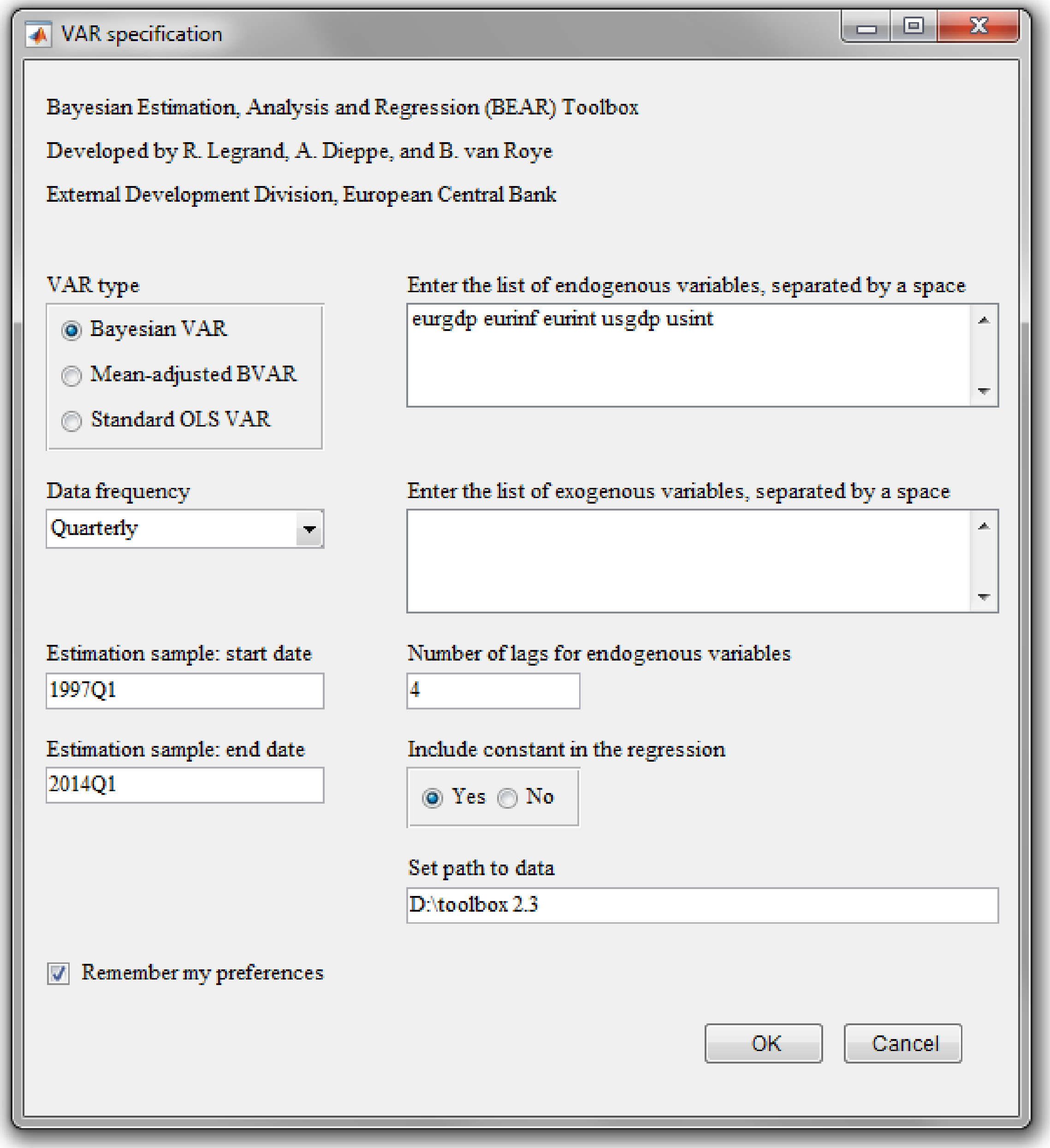
$$A(L)\left(y_t - Fx_t\right) = \varepsilon_t$$

  – Taking expectations on both sides and rearranging, one obtains:

$$E(y_t) = Fx_t$$

  – The long-run value of the VAR is simply the deterministic, or exogenous component of the model.
  – Prior on steady-state improves forecasting performance.

# Selection of estimation method, sample period and variables in BEAR



# Priors in BEAR

– Minnesota: only β is estimated; Σ assumed to be fixed and known; Prior for β $N \sim (\beta_0, \Omega_0)$, Σ: use OLS values.

– Normal-Wishart (natural conjugate): Minnesota too restrictive: want to estimate both β and Σ; Normal Wishart allows this, but at a cost : prior for β depends on Σ; Prior for β : $N \sim (\beta_0, \Sigma \otimes \Phi_0)$; Prior for Σ: IW ~ $(S_0, \alpha_0)$.

– Independent Normal-Wishart: Estimate both β and Σ, prior for β independent from Σ: Prior for β : $N \sim (\beta_0, \Omega_0)$ and prior for Σ: *IW* ~ $(S_0, \alpha_0)$. Possible but requires simulation methods.

– Normal-diffuse: Σ estimated, but agnostic about prior information; Prior for β : $N \sim (\beta_0, \Omega_0)$; Prior for Σ is diffuse (uninformative).

– Dummy observation: Prior is uninformative for both β and Σ; Prior information transmitted by the way of artificial observations (dummy observations) added to data.

**Options:**

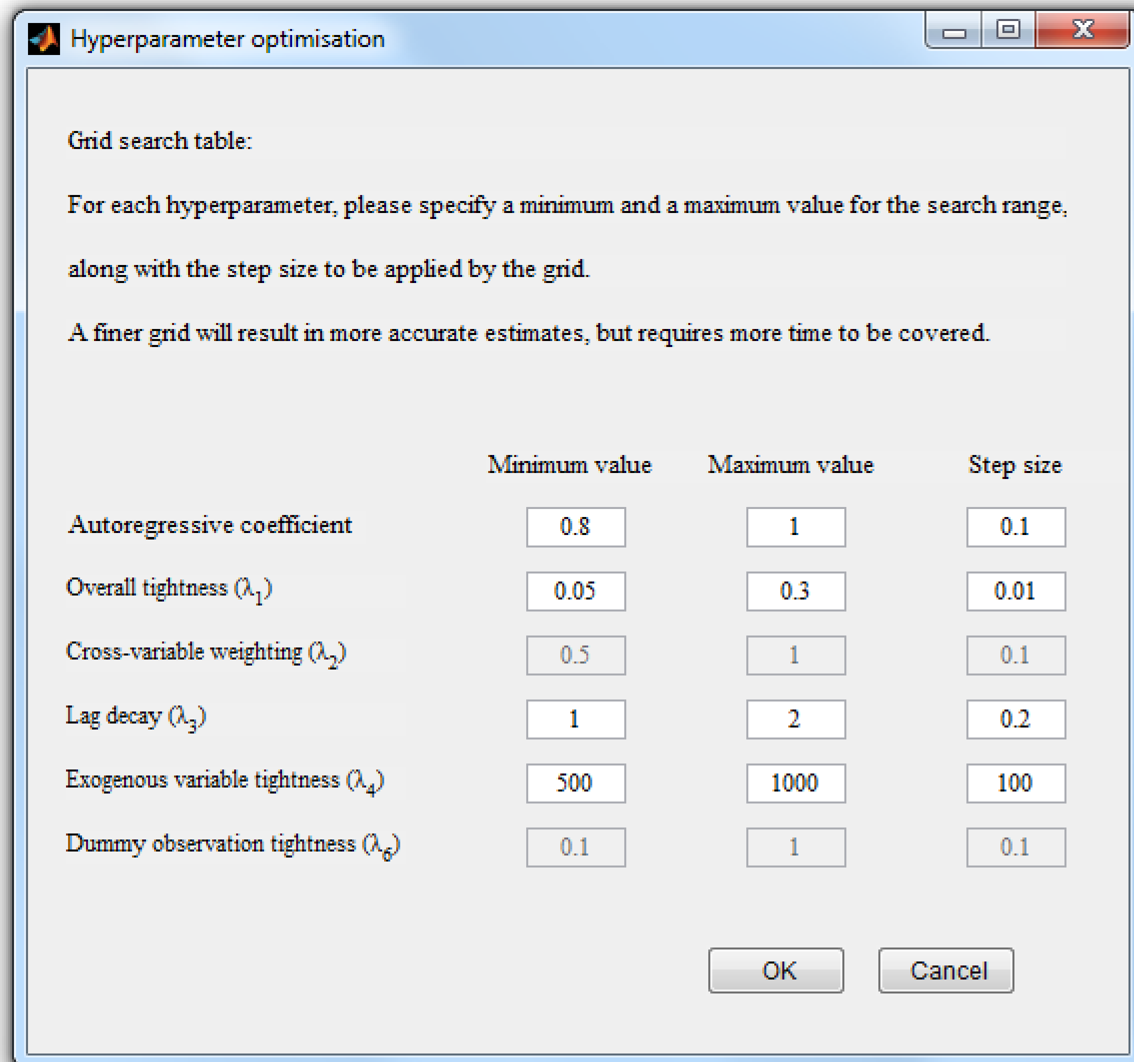
– Grid search for hyperparameters based on highest marginal likelihood
– Block exogeneity
– Sums-of-coefficients and dummy initial observation

# Hyperparameter optimisation by grid search

- Principle:
  - Hyperparameters determine the prior distribution
  - Prior distribution determines the posterior
  - Posterior determines the performance of the model
  - Hence: hyperparameters determine the performance of the model
  - Optimize model performance by choosing best hyperparameter values (criterion: marginal likelihood)

- Process:
  - Grid search: create a grid of values, estimate the marginal likelihood for any possible combination in the grid
  - Retain the set that maximizes the marginal likelihood.

# Hyperparameter optimisation by grid search



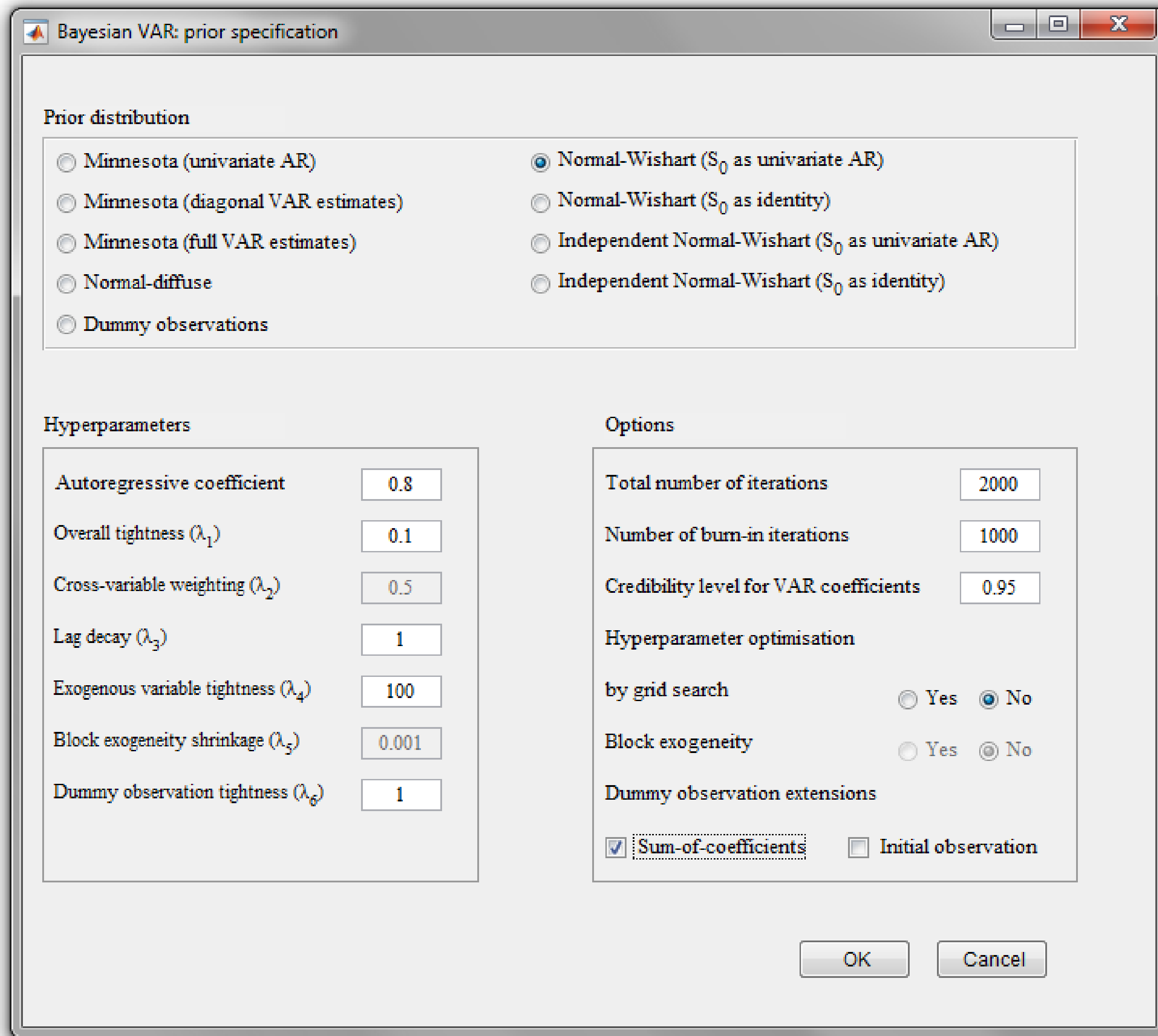
# Block exogeneity, Dummy observation application

- Block exogeneity:
  - Makes it possible to make certain variables exogenous w.r.t to other variables
  - Implements additional prior shrinkage to cut the transmission channel
  - Typical example: small open economy model

- Dummy observation prior:
  - Possible numerical issue if estimating large models (n=50+ variables)
  - Dummy observation prior: greatly reduces computation size (by $n^2$)
  - Uses artificial (dummy) observations to transmit prior information

- Dummy observation :
  - Create extra dummy observations to transmit additional information
  - “Sums-of-coefficients”: accounts for unit roots
  - “Dummy initial observation”: accounts for cointegration

# Prior distributions, hyperparameters, and options



# SVAR and applications in BEAR

**Structural VAR:**

- Choleski and triangular factorisation
- Sign restrictions (Arias, Rubio-Ramírez, Waggoner, 2014)
  - Latest methodology allowing for both sign and zero restrictions
  - Extended to allow also for magnitude restrictions.

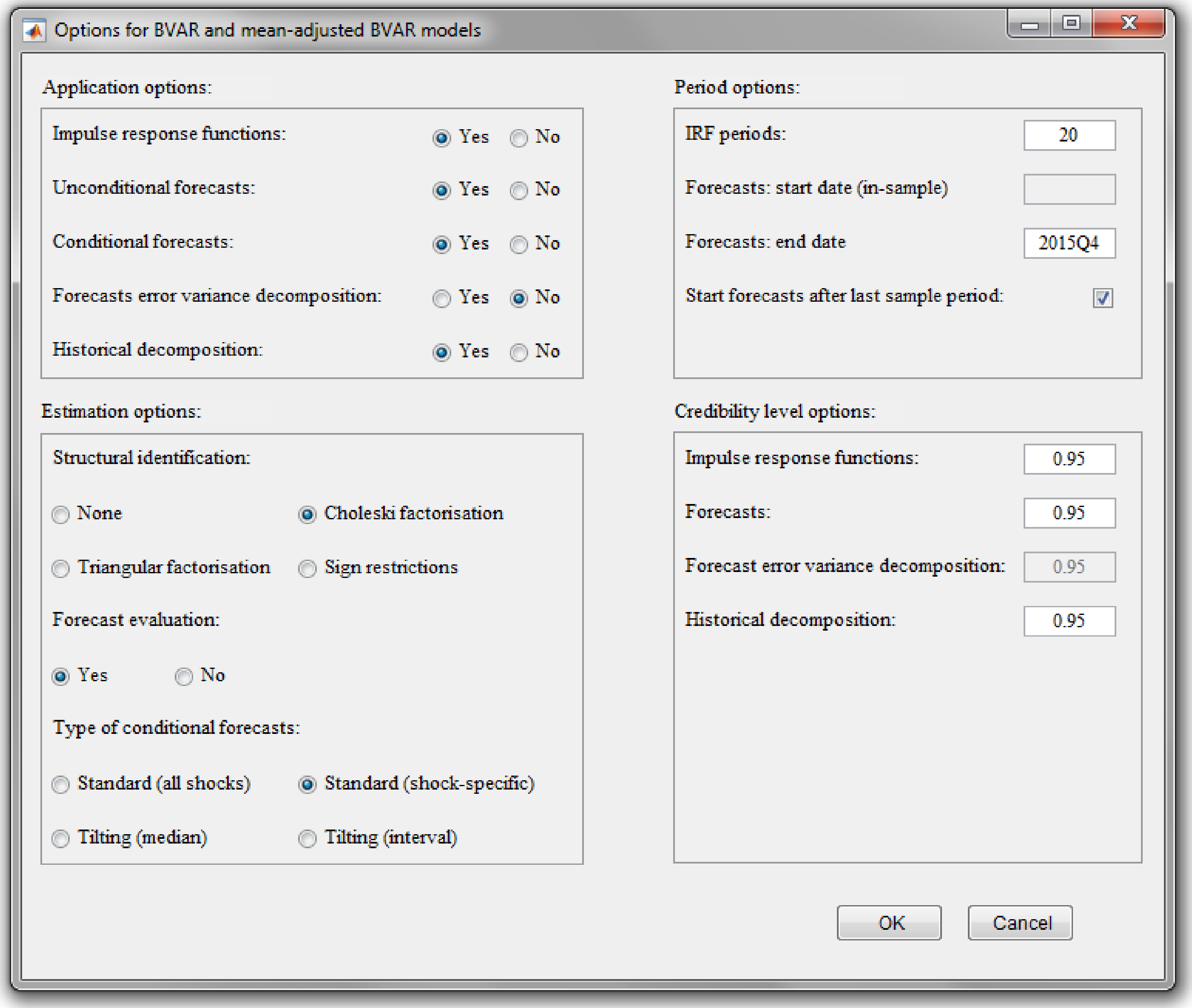
**Applications:**

- Impulse response functions
- Forecasts (unconditional)
- Forecast error variance decomposition
- Historical decomposition
- Conditional forecasts (Waggoner and Zha, 1999)
  - Adapted to allow forecasts to be generated by specific shocks.
  - Example condition on an interest rate path – driven by monetary policy shocks, or alternatively by inflation or GDP growth shocks or a combination of shocks
- Tilting of predictive distribution (relative entropy)
  - (Robertson, Tallman and Whiteman, 2005)
  - Alternative method for conditional forecasts

**Evaluation criteria:**

- Classical (maximum likelihood) criteria :
  - model: sum of squared residuals, R-squared and adjusted R-squared
  - forecasts: RMSE, MAE, MAPE, Theil's U
- Bayesian-specific criteria:
  - model: marginal likelihood
  - forecasts: continuous ranked probability score, log score

# Selection of options in BEAR



# Structural shock identification by sign restrictions

- Methodology
  - developed by Arias et al. (2014)
  - Generalisation of the kind of restrictions provided e.g. by Choleski
  - generalise to 3 types of restrictions: sign restrictions, zero restrictions and magnitude restrictions
  - Can apply to any variable, at any period(s)

- Possible identification pitfalls: technical VS. theoretical
  - Technical: any type of restriction, and any number of restrictions works!
  - Theoretical: number of restrictions must be sufficient to identify shocks unambiguously (2 shocks don’t trigger the same responses for all the variables)

# Structural shock identification by sign restrictions

- Example
  - VAR model with 2 variables (GDP, unemployment), and 2 shocks identified (demand shock, monetary shock).
  - Restriction 1:

| | dem. Shock | mon. shock |
|---|---|---|
| Y | + | + |
| U | – | – |

Identified? No: the two shocks have similar effects on the two variables.

Impossible to determine which shock is demand, which one is monetary.

  - Restriction 2:

| | dem. Shock | mon. shock |
|---|---|---|
| Y | + | + |
| U | – | |

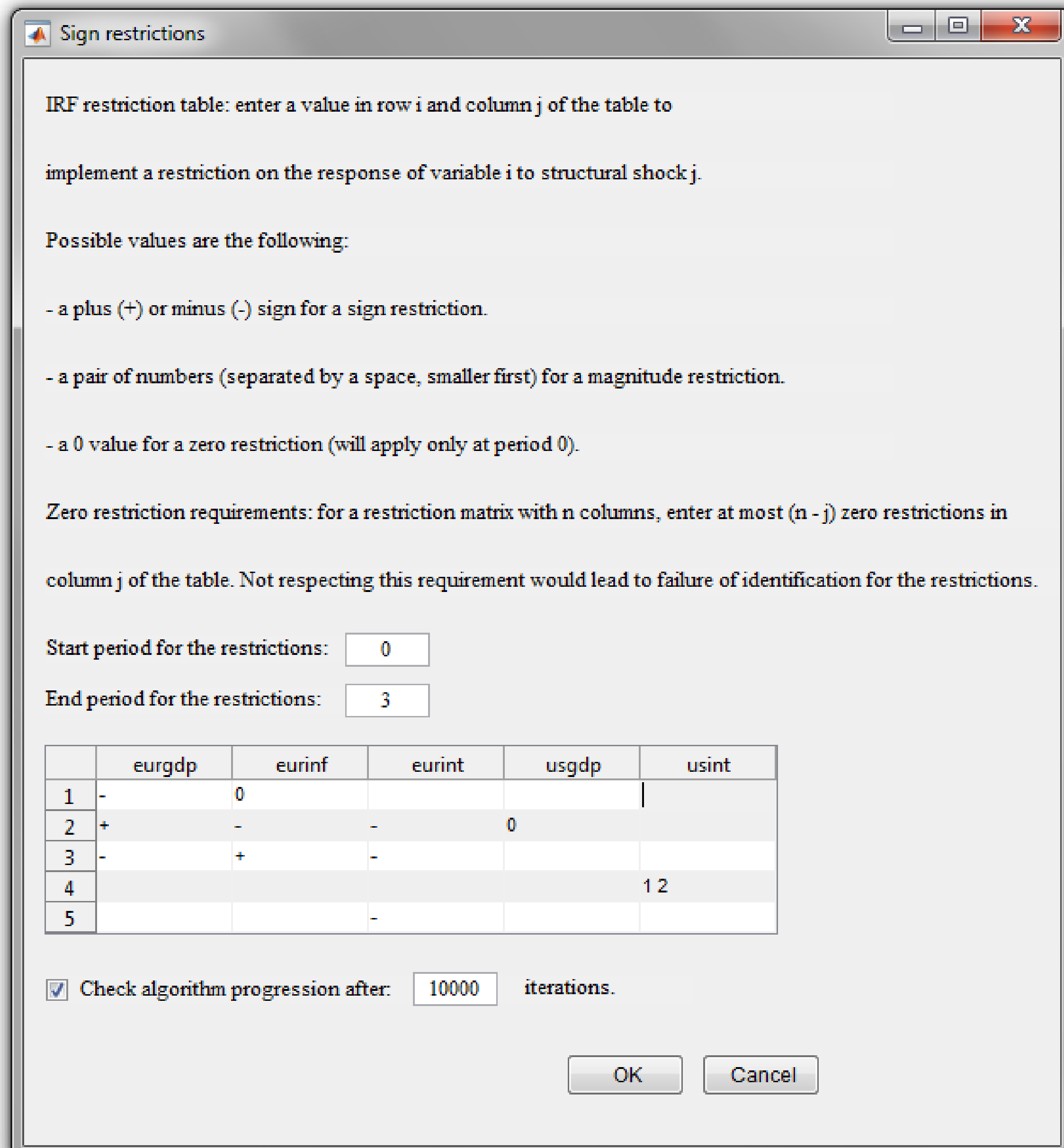
Identified? Still not: no constraint implies that shocks *may* have similar effects on the two variables

  - Restriction 3:

| | dem. Shock | mon. shock |
|---|---|---|
| Y | + | + |
| U | – | + |

Identified? Yes!  Economically relevant? Perhaps not.

# Implementation of sign restrictions in BEAR



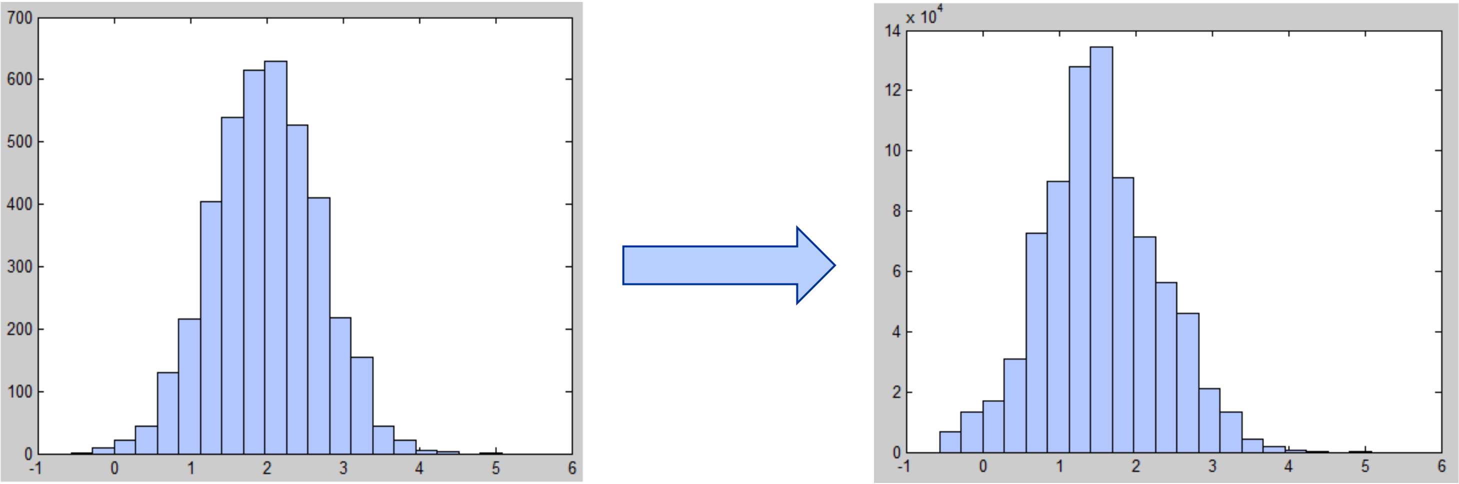
# New feature: conditional forecasts (tilting)

- Tilting: alternative to Waggoner-Zha approach:
  - Soft forecasts (variability allowed around condition values)
  - Agnostic about shocks

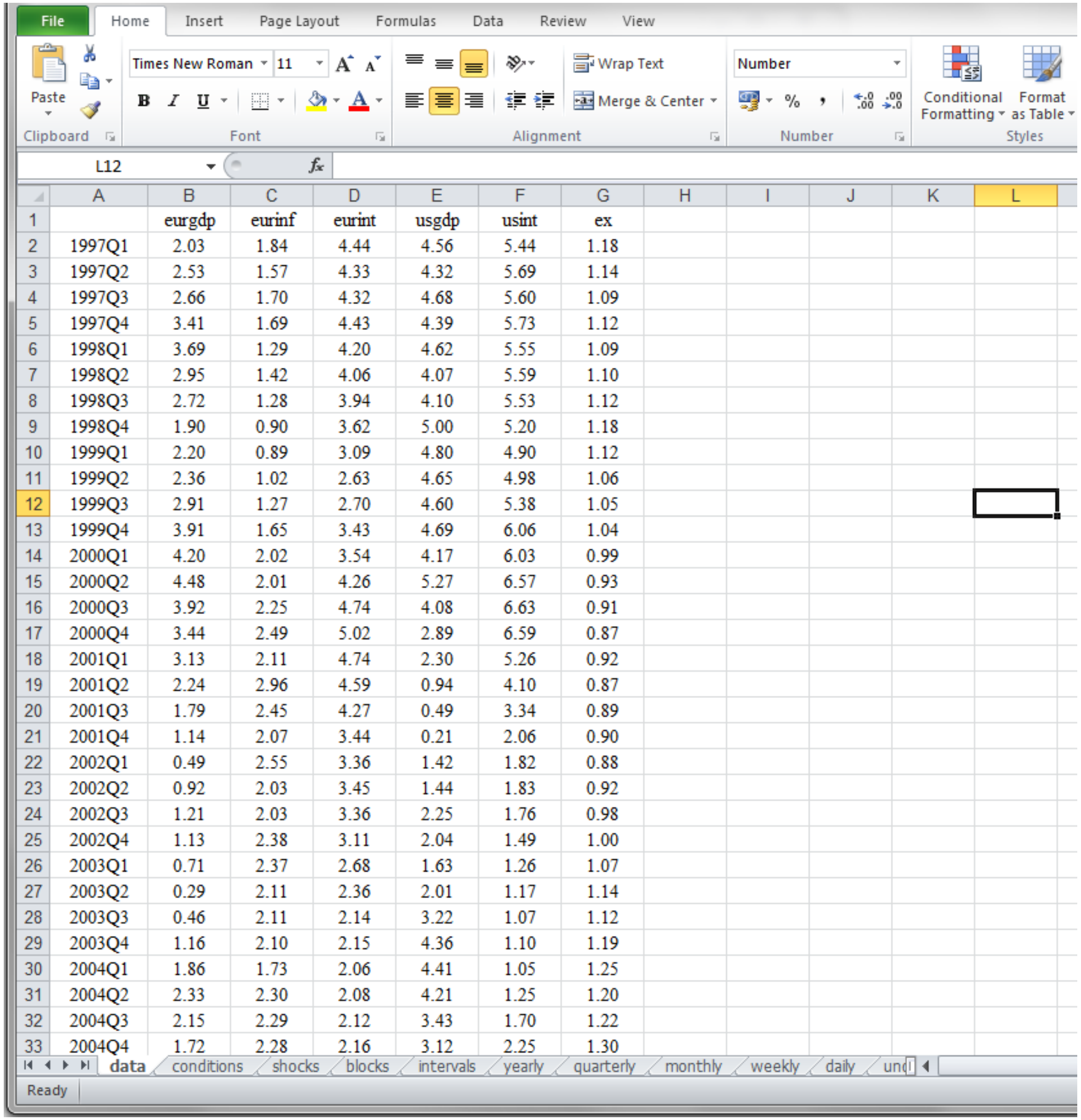
- Idea:

  – Modify (tilt) the normal forecast distribution to obtain a new distribution that satisfies the conditional forecasts

- The tilted distribution:



# Toolbox input: Excel data file



| | eurgdp | eurinf | eurint | usgdp | usint | ex |
|---|---|---|---|---|---|---|
| 1997Q1 | 2.03 | 1.84 | 4.44 | 4.56 | 5.44 | 1.18 |
| 1997Q2 | 2.53 | 1.57 | 4.33 | 4.32 | 5.69 | 1.14 |
| 1997Q3 | 2.66 | 1.70 | 4.32 | 4.68 | 5.60 | 1.09 |
| 1997Q4 | 3.41 | 1.69 | 4.43 | 4.39 | 5.73 | 1.12 |
| 1998Q1 | 3.69 | 1.29 | 4.20 | 4.62 | 5.55 | 1.09 |
| 1998Q2 | 2.95 | 1.42 | 4.06 | 4.07 | 5.59 | 1.10 |
| 1998Q3 | 2.72 | 1.28 | 3.94 | 4.10 | 5.53 | 1.12 |
| 1998Q4 | 1.90 | 0.90 | 3.62 | 5.00 | 5.20 | 1.18 |
| 1999Q1 | 2.20 | 0.89 | 3.09 | 4.80 | 4.90 | 1.12 |
| 1999Q2 | 2.36 | 1.02 | 2.63 | 4.65 | 4.98 | 1.06 |
| 1999Q3 | 2.91 | 1.27 | 2.70 | 4.60 | 5.38 | 1.05 |
| 1999Q4 | 3.91 | 1.65 | 3.43 | 4.69 | 6.06 | 1.04 |
| 2000Q1 | 4.20 | 2.02 | 3.54 | 4.17 | 6.03 | 0.99 |
| 2000Q2 | 4.48 | 2.01 | 4.26 | 5.27 | 6.57 | 0.93 |
| 2000Q3 | 3.92 | 2.25 | 4.74 | 4.08 | 6.63 | 0.91 |
| 2000Q4 | 3.44 | 2.49 | 5.02 | 2.89 | 6.59 | 0.87 |
| 2001Q1 | 3.13 | 2.11 | 4.74 | 2.30 | 5.26 | 0.92 |
| 2001Q2 | 2.24 | 2.96 | 4.59 | 0.94 | 4.10 | 0.87 |
| 2001Q3 | 1.79 | 2.45 | 4.27 | 0.49 | 3.34 | 0.89 |
| 2001Q4 | 1.14 | 2.07 | 3.44 | 0.21 | 2.06 | 0.90 |
| 2002Q1 | 0.49 | 2.55 | 3.36 | 1.42 | 1.82 | 0.88 |
| 2002Q2 | 0.92 | 2.03 | 3.45 | 1.44 | 1.83 | 0.92 |
| 2002Q3 | 1.21 | 2.03 | 3.36 | 2.25 | 1.76 | 0.98 |
| 2002Q4 | 1.13 | 2.38 | 3.11 | 2.04 | 1.49 | 1.00 |
| 2003Q1 | 0.71 | 2.37 | 2.68 | 1.63 | 1.26 | 1.07 |
| 2003Q2 | 0.29 | 2.11 | 2.36 | 2.01 | 1.17 | 1.14 |
| 2003Q3 | 0.46 | 2.11 | 2.14 | 3.22 | 1.07 | 1.12 |
| 2003Q4 | 1.16 | 2.10 | 2.15 | 4.36 | 1.10 | 1.19 |
| 2004Q1 | 1.86 | 1.73 | 2.06 | 4.41 | 1.05 | 1.25 |
| 2004Q2 | 2.33 | 2.30 | 2.08 | 4.21 | 1.25 | 1.20 |
| 2004Q3 | 2.15 | 2.29 | 2.12 | 3.43 | 1.70 | 1.22 |
| 2004Q4 | 1.72 | 2.28 | 2.16 | 3.12 | 2.25 | 1.30 |

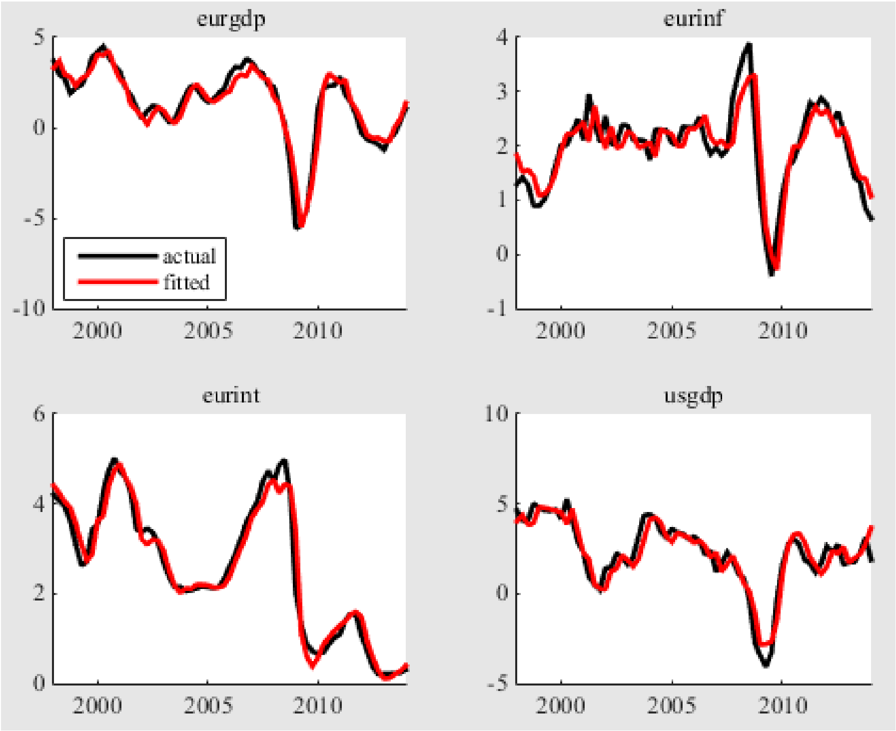
# Toolbox output: estimation results

```
BEAR toolbox estimates
Date: 15-Sep-2015   Time: 14:30


Bayesian VAR
structural decomposition: choleski factorisation
endogenous variables:  eurgdp  eurinf  eurint  usgdp
exogenous variables:  constant
estimation sample: 1997q1-2014q1
sample size (omitting initial conditions): 65
number of lags included in regression: 4
prior: normal-wishart (sigma as identity)
hyperparameters:
autoregressive coefficient (ar):          0.8
overall tightness (lambda1):              0.1
lag decay (lambda3):                      1
exogenous variable tightness (lambda4):   100



VAR coefficients (beta): posterior estimates

Endogenous: eurgdp
                  Median    St.dev   Low.bound   Upp.bound
   eurgdp(-1)      0.930     0.062      0.808      1.052
   eurgdp(-2)     -0.029     0.044     -0.115      0.057
   eurgdp(-3)     -0.023     0.030     -0.082      0.037
   eurgdp(-4)     -0.011     0.023     -0.056      0.034
   eurinf(-1)     -0.284     0.107     -0.495     -0.074
   eurinf(-2)     -0.165     0.076     -0.314     -0.015
   eurinf(-3)     -0.052     0.053     -0.157      0.053
   eurinf(-4)     -0.010     0.041     -0.091      0.070
   eurint(-1)     -0.028     0.108     -0.240      0.184
   eurint(-2)     -0.069     0.082     -0.230      0.092
   eurint(-3)     -0.018     0.057     -0.130      0.095
   eurint(-4)      0.009     0.044     -0.077      0.095
    usgdp(-1)      0.113     0.055      0.005      0.220
    usgdp(-2)     -0.019     0.038     -0.093      0.055
    usgdp(-3)     -0.013     0.026     -0.064      0.038
    usgdp(-4)     -0.004     0.020     -0.043      0.034
     Constant      1.287     0.300      0.697      1.877

Sum of squared residuals: 20.11
R-squared: 0.929
adj. R-squared: 0.905
```

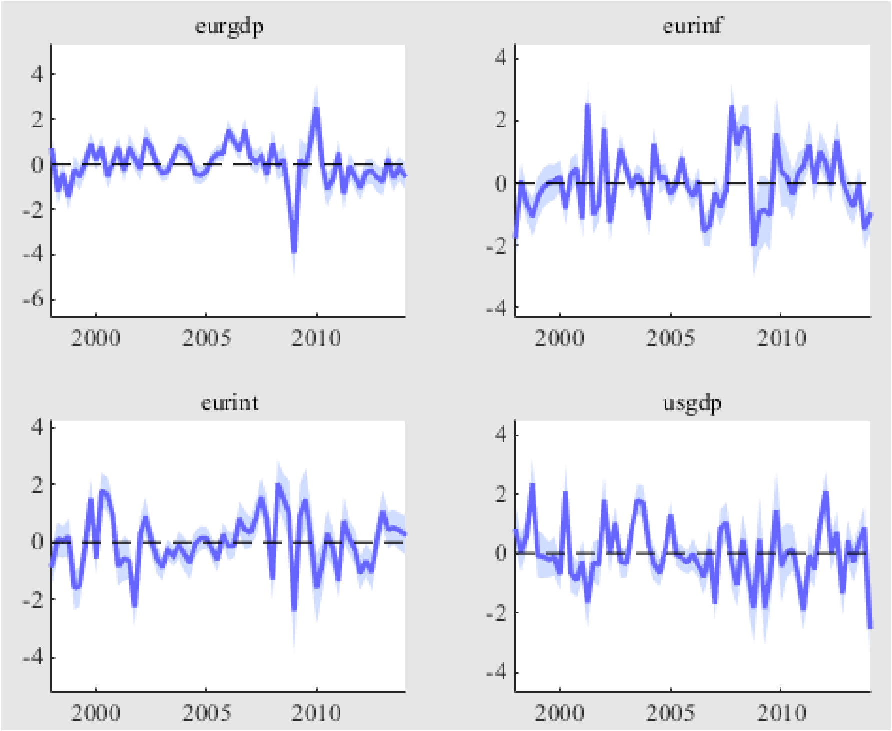
```
Log 10 marginal likelihood: -90.49


Roots of the characteristic polynomial (modulus):
0.887  0.407  0.236  0.215
0.886  0.281  0.231  0.195
0.886  0.281  0.231  0.173
0.788  0.236  0.215  0.173
No root lies outside the unit circle.
The estimated VAR model satisfies the stability condition


sigma (residual covariance matrix): posterior estimates
 0.402   0.073   0.117   0.158
 0.073   0.158   0.048   0.048
 0.117   0.048   0.118   0.037
 0.158   0.048   0.037   0.507


D (structural decomposition matrix): posterior estimates
 0.621   0.000   0.000   0.000
 0.119   0.339   0.000   0.000
 0.224   0.053   0.305   0.000
 0.187  -0.013  -0.118   0.695


gamma (structural disturbances covariance matrix): posterior estimates
 1.000   0.000   0.000   0.000
 0.000   1.000   0.000   0.000
 0.000   0.000   1.000   0.000
 0.000   0.000   0.000   1.000



Forecast evaluation:

Evaluation conducted over 3 periods (from 2014q2 to 2014q4).

Endogenous: eurgdp
                2014q2     2014q3     2014q4
RMSE:            1.115      1.615      1.974
MAE:             1.115      1.554      1.884
MAPE:          139.171    195.874    226.679
Theil's U:       0.410      0.507      0.548
CRPS:            0.145      0.223      0.268
Log score 1:    -3.217     -3.457     -3.389
Log score 2:    -2.015     -3.319     -4.287

Endogenous: eurinf
                2014q2     2014q3     2014q4
RMSE:            0.396      0.792      1.073
MAE:             0.396      0.722      0.976
MAPE:           70.662    221.892    412.643
Theil's U:       0.261      0.495      0.621
CRPS:            0.094      0.135      0.166
Log score 1:    -1.231     -3.510     -4.153
Log score 2:    -0.510     -1.740     -2.452
```

# Toolbox output: graphical diagnostics

## In-sample fit

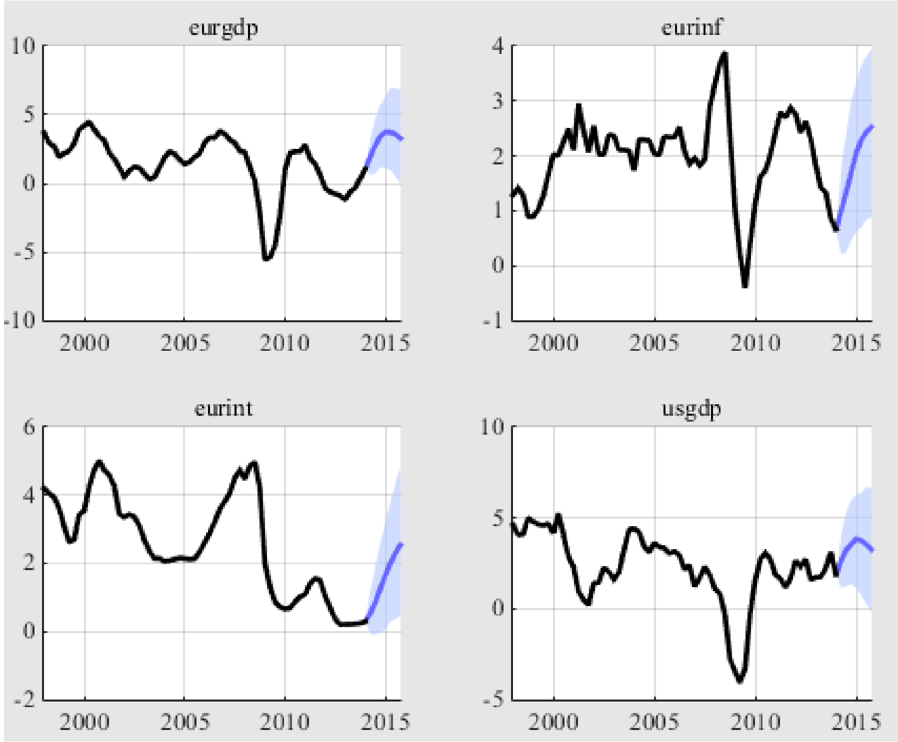



## Structural shocks

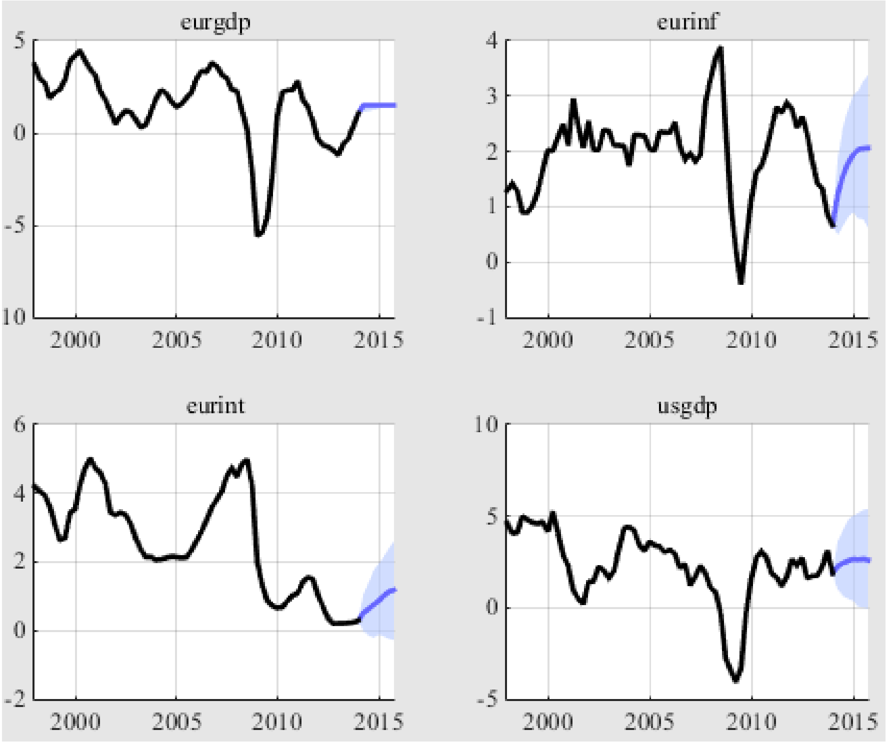


# Toolbox output : graphs

## Unconditional forecast




## Conditional forecast



# Toolbox output: EXCEL results file

| | response of eurgdp to eurgdp shocks | | | | response of eurgdp to eurinf shocks | | | | response of eurgdp to eurint shocks | | | | response of eurgdp to usgdp shocks | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | lw. bound | median | up. bound | | lw. bound | median | up. bound | | lw. bound | median | up. bound | | lw. bound | median | up. bound |
| 1 | 0.533175 | 0.629773 | 0.75139 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 2 | 0.46607 | 0.573736 | 0.711956 | 2 | -0.18912 | -0.10339 | -0.01045 | 2 | -0.07674 | -0.01275 | 0.051099 | 2 | 0.002413 | 0.07278 | 0.144349 |
| 3 | 0.320644 | 0.450997 | 0.616943 | 3 | -0.40798 | -0.25965 | -0.10864 | 3 | -0.14136 | -0.04409 | 0.052583 | 3 | -0.00091 | 0.116624 | 0.230119 |
| 4 | 0.119026 | 0.289647 | 0.482075 | 4 | -0.59339 | -0.39823 | -0.21183 | 4 | -0.19152 | -0.07223 | 0.033806 | 4 | -0.01878 | 0.136192 | 0.288767 |
| 5 | -0.08745 | 0.12144 | 0.324864 | 5 | -0.72178 | -0.48663 | -0.28628 | 5 | -0.22549 | -0.09334 | 0.02998 | 5 | -0.03604 | 0.140645 | 0.328429 |
| 6 | -0.26916 | -0.02569 | 0.201438 | 6 | -0.78159 | -0.51384 | -0.30601 | 6 | -0.24901 | -0.10201 | 0.034456 | 6 | -0.04867 | 0.142446 | 0.344274 |
| 7 | -0.41067 | -0.14233 | 0.088483 | 7 | -0.78866 | -0.49047 | -0.2698 | 7 | -0.26183 | -0.09996 | 0.056164 | 7 | -0.0735 | 0.137903 | 0.3486 |
| 8 | -0.50059 | -0.22415 | 0.000585 | 8 | -0.74216 | -0.42598 | -0.20798 | 8 | -0.26259 | -0.08763 | 0.070144 | 8 | -0.08071 | 0.128333 | 0.35344 |
| 9 | -0.5554 | -0.26929 | -0.05844 | 9 | -0.64879 | -0.3336 | -0.09823 | 9 | -0.25005 | -0.07283 | 0.092197 | 9 | -0.10503 | 0.110725 | 0.356359 |
| 10 | -0.58312 | -0.27682 | -0.07236 | 10 | -0.57327 | -0.22909 | 0.01658 | 10 | -0.23834 | -0.05315 | 0.11478 | 10 | -0.13119 | 0.095178 | 0.352321 |
| 11 | -0.55716 | -0.2596 | -0.05095 | 11 | -0.49202 | -0.13218 | 0.142813 | 11 | -0.21586 | -0.03441 | 0.134321 | 11 | -0.14651 | 0.077357 | 0.339469 |
| 12 | -0.50523 | -0.2214 | -0.00576 | 12 | -0.3837 | -0.03971 | 0.25812 | 12 | -0.18224 | -0.01555 | 0.148512 | 12 | -0.15148 | 0.058691 | 0.326038 |
| 13 | -0.45898 | -0.16735 | 0.047437 | 13 | -0.2973 | 0.032021 | 0.35451 | 13 | -0.15891 | 0.002052 | 0.156356 | 13 | -0.15445 | 0.039926 | 0.304853 |
| 14 | -0.38824 | -0.11359 | 0.102406 | 14 | -0.20844 | 0.088648 | 0.428456 | 14 | -0.13432 | 0.014353 | 0.160927 | 14 | -0.16067 | 0.026939 | 0.266898 |
| 15 | -0.33582 | -0.05963 | 0.178849 | 15 | -0.15252 | 0.119296 | 0.472584 | 15 | -0.11935 | 0.022979 | 0.160951 | 15 | -0.15892 | 0.01456 | 0.244606 |
| 16 | -0.27291 | -0.01627 | 0.240233 | 16 | -0.12149 | 0.126952 | 0.483908 | 16 | -0.10064 | 0.024396 | 0.161997 | 16 | -0.16039 | 0.007992 | 0.214939 |
| 17 | -0.22901 | 0.020083 | 0.284288 | 17 | -0.11202 | 0.121557 | 0.486311 | 17 | -0.08369 | 0.027554 | 0.15309 | 17 | -0.15642 | 0.001061 | 0.18862 |
| 18 | -0.17641 | 0.043593 | 0.322954 | 18 | -0.12653 | 0.105881 | 0.4665 | 18 | -0.07156 | 0.024503 | 0.153796 | 18 | -0.1592 | -0.0013 | 0.155541 |
| 19 | -0.13859 | 0.056834 | 0.335787 | 19 | -0.15678 | 0.080132 | 0.424689 | 19 | -0.05977 | 0.02105 | 0.139971 | 19 | -0.14365 | -0.00154 | 0.148199 |
| 20 | -0.11698 | 0.061475 | 0.341911 | 20 | -0.18612 | 0.052267 | 0.37475 | 20 | -0.05594 | 0.015894 | 0.14049 | 20 | -0.1306 | -0.00122 | 0.128572 |

| | response of eurinf to eurgdp shocks | | | | response of eurinf to eurinf shocks | | | | response of eurinf to eurint shocks | | | | response of eurinf to usgdp shocks | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | lw. bound | median | up. bound | | lw. bound | median | up. bound | | lw. bound | median | up. bound | | lw. bound | median | up. bound |
| 1 | 0.02535 | 0.115921 | 0.200022 | 1 | 0.314864 | 0.372206 | 0.442372 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 2 | 0.065545 | 0.155345 | 0.247634 | 2 | 0.245582 | 0.312362 | 0.394255 | 2 | -0.03955 | 0.000525 | 0.040169 | 2 | -0.0333 | 0.008432 | 0.05237 |
| 3 | 0.083224 | 0.170109 | 0.271344 | 3 | 0.156999 | 0.237959 | 0.341615 | 3 | -0.06312 | -0.00374 | 0.057699 | 3 | -0.06039 | 0.009715 | 0.083602 |
| 4 | 0.077071 | 0.159258 | 0.264852 | 4 | 0.063147 | 0.156831 | 0.282701 | 4 | -0.08194 | -0.01336 | 0.058992 | 4 | -0.07711 | 0.008312 | 0.101672 |
| 5 | 0.053232 | 0.132437 | 0.238686 | 5 | -0.02361 | 0.077473 | 0.21025 | 5 | -0.09413 | -0.02292 | 0.047741 | 5 | -0.09157 | 0.003989 | 0.101756 |
| 6 | 0.018142 | 0.096436 | 0.205344 | 6 | -0.09252 | 0.013103 | 0.155312 | 6 | -0.10285 | -0.02989 | 0.037533 | 6 | -0.0951 | 0.001491 | 0.103075 |
| 7 | -0.02432 | 0.057662 | 0.17083 | 7 | -0.14667 | -0.03318 | 0.102421 | 7 | -0.10765 | -0.03492 | 0.030509 | 7 | -0.09747 | 0.001743 | 0.100981 |
| 8 | -0.06734 | 0.01967 | 0.132181 | 8 | -0.1796 | -0.06094 | 0.066325 | 8 | -0.1095 | -0.038 | 0.026167 | 8 | -0.08808 | 0.004113 | 0.100521 |
| 9 | -0.10373 | -0.01198 | 0.098928 | 9 | -0.1876 | -0.07275 | 0.046627 | 9 | -0.10504 | -0.03789 | 0.020499 | 9 | -0.07591 | 0.005808 | 0.090593 |
| 10 | -0.13263 | -0.0343 | 0.064022 | 10 | -0.18665 | -0.0676 | 0.034751 | 10 | -0.09215 | -0.0356 | 0.019264 | 10 | -0.06739 | 0.005081 | 0.08568 |
| 11 | -0.15785 | -0.04976 | 0.043498 | 11 | -0.17274 | -0.05422 | 0.038676 | 11 | -0.0833 | -0.0305 | 0.014244 | 11 | -0.0614 | 0.004096 | 0.074093 |
| 12 | -0.16635 | -0.05833 | 0.020546 | 12 | -0.14812 | -0.0374 | 0.050799 | 12 | -0.07572 | -0.02468 | 0.012459 | 12 | -0.05273 | 0.002271 | 0.06691 |

# CHAPTER 2

# BEAR User's guide

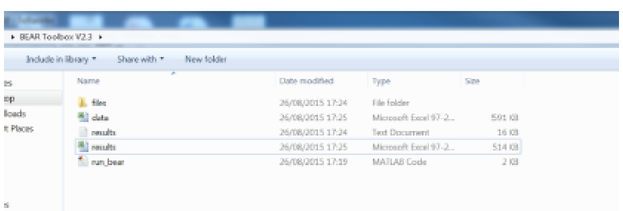
## 2.1 Preparing your project folder

The BEAR toolbox package comprises three elements:

- the folder named 'BEAR toolbox'
- the 'BEAR Toolbox Guide' pdf document (the document you are currently reading)

For econometric applications, the file of interest is the 'BEAR toolbox' folder. To create a new project, start by unzipping and copying this folder, and paste it in the directory of your choice. You may want to rename the folder to match your particular project, for instance 'my project', or 'project Japan', and so on. The first time you will open your project folder, it will look like this:

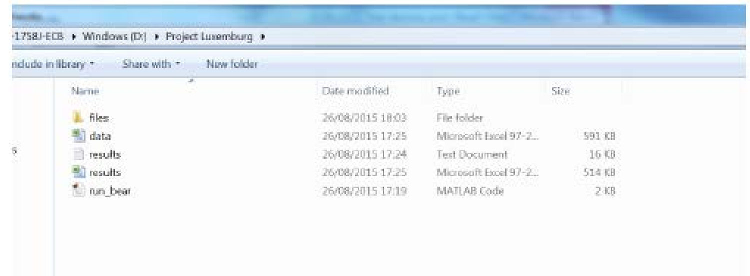
**Figure 2.1:** *Main directory*




For now, the folder comprises various elements. The first is the folder names 'files'. This folder contains all encoded Matlab functions used by the toolbox. The second is the Excel spreadsheet called 'data'. It is this spreadsheet which will be used by the code to read the data you want to use for the model (more details on this in the next section). The third is the results files (more provided below). Finally there is a matlab file called 'run_bear.m' and the guides.

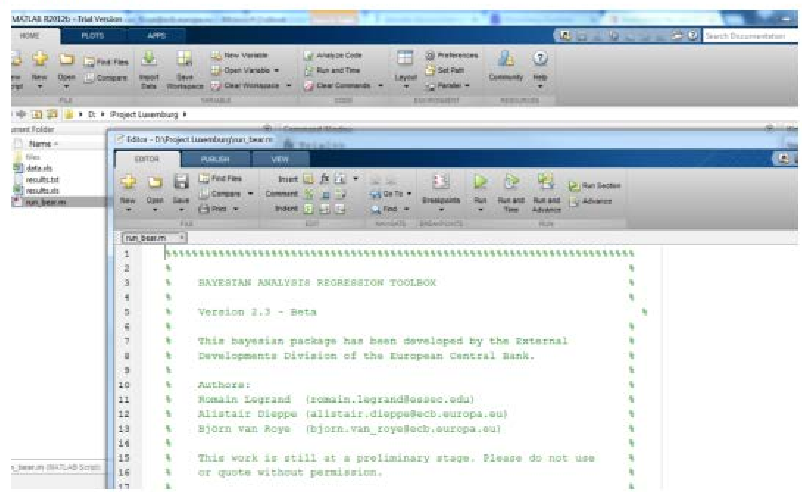


**Figure 2.2:** *Project directory*




To start your project application, all you have to do is now to double click on run_bear.m file and wait for Matlab to start. You should then navigate to your directory (in this case 'D:/Project Luxembourg').

**Figure 2.3:** *Run the code*




Then, to run the code, suffice is to press the 'run' button of the function window. However, before where $\sigma_i^2$ is again the OLS residual variance of an auto-regressive model previously estimated for variable $i$, and $\lambda_4$ is a large (potentially infinite) variance parameter.

$\Omega_0$ is thus a $q \times q$ diagonal matrix with three different types of variance terms on its main diagonal. For instance, for the VAR model with 2 variables, 2 lags and one exogenous variable specified above, $\Omega_0$ is given by:

$$\Omega_0 = \begin{pmatrix}
(\lambda_1)^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \left(\frac{\sigma_1^2}{\sigma_2^2}\right)(\lambda_1\lambda_2)^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \left(\frac{\lambda_1}{2^{\lambda_3}}\right)^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \left(\frac{\sigma_1^2}{\sigma_2^2}\right)\left(\frac{\lambda_1\lambda_2}{2^{\lambda_3}}\right)^2 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \sigma_1^2(\lambda_1\lambda_4)^2 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \left(\frac{\sigma_2^2}{\sigma_1^2}\right)(\lambda_1\lambda_2)^2 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & (\lambda_1)^2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \left(\frac{\sigma_2^2}{\sigma_1^2}\right)\left(\frac{\lambda_1\lambda_2}{2^{\lambda_3}}\right)^2 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \left(\frac{\lambda_1}{2^{\lambda_3}}\right)^2 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma_2^2(\lambda_1\lambda_4)^2
\end{pmatrix} \tag{3.1.3.8}$$

Different choices are possible for $\lambda_1$,$\lambda_2$,$\lambda_3$ and $\lambda_4$. However, values typically found in the literature revolve around:

$$\lambda_1 = 0.1 \tag{3.1.3.9}$$

$$\lambda_2 = 0.5 \tag{3.1.3.10}$$

$$\lambda_3 = 1 \text{ or } 2 \tag{3.1.3.11}$$

$$\lambda_4 = 10^2 \text{ to } \infty \tag{3.1.3.12}$$

Finally, since the Minnesota prior assumes that the variance-covariance matrix of residuals $\Sigma$ is known, one has to decide how to define it. The original Minnesota prior assumes that $\Sigma$ is diagonal which, as will be seen later, conveniently implies independence between the VAR coefficients of different equations. This property was useful at a time of limited computational power as it allows estimating the model equation by equation (this possibility is not used here). A first possibility is thus to set the diagonal of $\Sigma$ equal to the residual variance of individual AR models run on each variable in the VAR. A second possibility is to use the variance-covariance matrix of a conventional VAR estimated by OLS, but to retain only the diagonal terms as $\Sigma$. Finally, as the model estimates all the equations simultaneously in this setting, the assumption of a diagonal matrix is not required. Therefore, a third and last possibility consists in using directly the entire variance-covariance matrix of a VAR estimated by OLS.

Once $\beta_0$ and $\Omega_0$ are determined, and that proper values re-attributed to $\Sigma$, one may compute the prior distribution of $\beta$. The normality assumption implies that its density is given by:

$$\pi(\beta) = (2\pi)^{-nk/2}|\Omega_0|^{-1/2} \exp\left[-\frac{1}{2}(\beta-\beta_0)'\Omega_0^{-1}(\beta-\beta_0)\right] \tag{3.1.3.13}$$

Relegating terms independent of $\beta$ to the proportionality constant, 3.1.3.13 rewrites:

$$\pi(\beta) \propto \exp\left[-\frac{1}{2}(\beta-\beta_0)'\Omega_0^{-1}(\beta-\beta_0)\right] \tag{3.1.3.14}$$

Now, directly applying 3.1.2.2, that is, combining the likelihood 3.1.3.2 with the prior 3.1.3.14, the posterior distribution for $\beta$ obtains as:

$$\begin{aligned}
\pi(\beta\,|y) &\propto f(y\,|\beta)\pi(\beta) \\
&\propto \exp\left[-\frac{1}{2}\left(y-\bar{X}\beta\right)'\bar{\Sigma}^{-1}\left(y-\bar{X}\beta\right)\right] \times \exp\left[-\frac{1}{2}(\beta-\beta_0)'\Omega_0^{-1}(\beta-\beta_0)\right] \\
&= \exp\left[-\frac{1}{2}\left\{\left(y-\bar{X}\beta\right)'\bar{\Sigma}^{-1}\left(y-\bar{X}\beta\right) + (\beta-\beta_0)'\Omega_0^{-1}(\beta-\beta_0)\right\}\right]
\end{aligned} \tag{3.1.3.15}$$

Equation 3.1.3.15 represents the kernel of the posterior distribution, but it does not have the form of a known distribution. Yet, it is possible to show that after some manipulations, it reformulates as:

$$\pi(\beta\,|y) \propto \exp\left[-\frac{1}{2}\left\{\left(\beta-\bar{\beta}\right)'\bar{\Omega}^{-1}\left(\beta-\bar{\beta}\right)\right\}\right] \tag{3.1.3.16}$$

with:

$$\bar{\Omega} = \left[\Omega_0^{-1} + \Sigma^{-1} \otimes X'X\right]^{-1} \tag{3.1.3.17}$$

and:

$$\bar{\beta} = \bar{\Omega}\left[\Omega_0^{-1}\beta_0 + (\Sigma^{-1} \otimes X')y\right] \tag{3.1.3.18}$$

This is the kernel of a multivariate normal distribution with mean $\bar{\beta}$ and covariance matrix $\bar{\Omega}$. Therefore, the posterior distribution of $\beta$ is given by:

$$\pi(\beta\,|y) \sim \mathcal{N}(\bar{\beta}, \bar{\Omega}) \tag{3.1.3.19}$$

# Current applications at the ECB

- Spill-overs from US monetary tightening to EMEs
- The role of monetary tightening in China's growth slowdown
- Estimating oil impact on economic activity using sign-restrictions
- Medium-term growth projections for EMEs using mean-adjusted BVAR's
- Drivers of inflation in Japan using sign-restrictions
- Assessing the US term premia and the risks of an abrupt upward adjustment in US interest rates.
- Toolbox is used for Low Inflation Task Force WS1 – what are the drivers of low inflation in euro area countries?

# On-going toolbox extension

## Panel BVAR

Structured as in Canova and Ciccarelli (2013)

- Dynamic interdependency
  - Interaction between units
- Static interdependency
  - Residuals are correlated across units
- Cross-sectional heterogeneity
  - Coefficients are allowed to vary across units
- Dynamic heterogeneity
  - Coefficients are time-varying

# Envisaged toolbox extensions

- **Batch jobs**
  - Looping processes for the BEAR toolbox
  - Developers version (no interface)
- **Time Varying Parameter BVAR**
  - Time-varying with stochastic volatility: Carter and Kohn / Primiceri , corrigendum of Del Negro and Primiceri (2013)
  - Time-varying sparse matrices : Chan et al (2015)
  - Algorithm for state space models : Durbin and Koopman (2002)
- **Other possibilities:**
  - Markov Switching, Threshold VAR, Mixed frequency, Factor-Augmented BVARs…