Estimating Non-Linear DSGEs with the Approximate Bayesian Computation: an application to the Zero Lower Bound

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Motivation

- In the aftermath of the Great Recession, empirical DSGEs were unprepared to deal with the new features of the data (asymmetries, Zero Lower Bound, stochastic volatility);
- Non-linear solution techniques developed;
- Non-linear models estimation is still perceived as impractical and computationally burdensome;
- The curse of dimensionality limited the diffusion of estimated non-linear DSGE models in academia and in central banking; ;
- Need for alternative methodologies $\rightarrow$ Approximate Bayesian Computation.;

This paper

Question

Is Approximate Bayesian Computation a valid (and easy-to-use) methodology to estimate non-linear DSGE models?

Goal of the paper

I study the use of the Approximate Bayesian Computation, in order to ease the estimation of non-linear DSGE models. I analyse the performance of the ABC methods along two dimensions:

- the small sample properties of ABC estimators $\rightarrow$ Montecarlo exercises to compare it with the Limited Information Method (Kim, 2002);
- the curse of dimensionality $\rightarrow$ ABC-Sequential Montecarlo.

Results

- With small samples, Approximate Bayesian Computation provides a better approximation of the full likelihood posterior distribution with respect to the Limited Information (Kim, 2002);
- This result lies on how the distribution of the moments is obtained;
- ABC-SMC tackles the curse of dimensionality and provides a good alternative to estimate non-linear DSGE models: an application to a new-keynesian model with Zero Lower Bound;

The literature

- The limited information methods: GMM, Hansen (1982) and SMM, Duffie and Singletone (1993) Ruge Murcia (2010), BLI, Kim (2002), Christiano at al. (2010), (2014), Indirect Methods: ILI, Creel and Kristensen (2011);
- ABC techniques: Pritchard (2000), Beaumont et al. (2002), Beaumont et al. (2010), Blum et al. (2010), Calvet and Czellar (2012);
- The ZLB: Fernandez-Villaverde et al. (2012), Gust et al. (2012), Guerrieri and Iacoviello (2014);

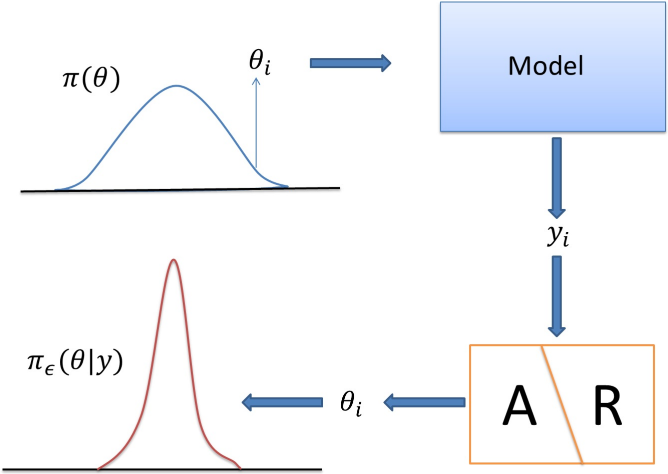
What is the Approximate Bayesian Computation?

- It's a methodology developed in natural sciences, epidemiology, population genetics;
- It's a limited information method, where inference is based on matching moments;
- The moments are computed by simulating the model and taking into account the size of the sample (differently from GMM-style estimators);
- In order to simulate the model and obtain the simulated moments, vectors of parameters are drawn from the prior;
- The prior is updated by an accept/reject algorithm based on the distance bewteen the simulated moments and observed moments;

The algorithm

1. Compute the moments (i.e. autocovariances) of the observed sample;
2. Draw from the prior distribution and simulate the model conditional on the vector of parameters drawn: each simulation has the size of the observed sample;
3. Compute the moments of the simulated sample (i.e. simulated autocovariances);
4. Compute the Euclidean distance between the vector of simulated moments and of the observed moments;
5. Accept and store the vector of parameters if the Euclidean distance is below a small threshold, go back to step 2;

The basic idea of ABC



A comparison with the Limited Information Method (BLI)

- Bayesian version of the GMM-style estimators;
- By the CLT, the likelihood of the moments is:

$$P(\gamma|\hat{\theta}, V) = \frac{1}{(2\pi)^{\left(\frac{N}{2}\right)}} |V|^{-\frac{1}{2}} \exp\left\{-\frac{T}{2}\left(\hat{\gamma} - \gamma(\theta)\right)' V^{-1} \left(\hat{\gamma} - \gamma(\theta)\right)\right\} \tag{1}$$

where $\hat{\gamma}$ is the sample moment, $\gamma(\theta)$ is the analytical moment, V is the variance covariance matrix.

Remark

ABC relies on the simulated distribution of the moments, the BLI relies on their asymptotic distribution.

Another helicopter tour

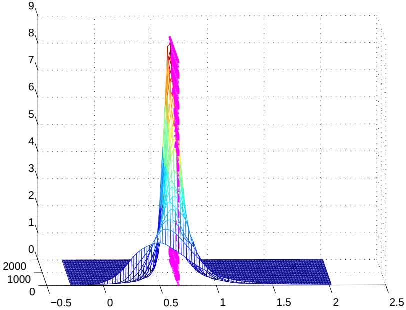


Figure : Distribution of the sample autocovariance for an AR(1) process with $\phi = 0.50$ for different sample sizes: from 50 to 2000 observations. The pink plane represents the population autocovariance. When the sample is small, the distribution of the moment is not normal and is not centred around the population moment. This is a comparative advantage of ABC with respect to the GMM-style estimators.

Comparison with the BLI: Montecarlo experiments

- Two Montecarlo experiments: on the AR(1) and on a RBC model;
- A Bayesian perspective;
- First criterion: RMSE with respect to the Full likelihood posterior mean;
- Second criterion: Overlapping Ratio of the 90% credible intervals of the Approximate posterior distribution with the full likelihood posterior distribution; RMSE-OR

A Montecarlo experiment: the AR(1)

- We estimate the autocorrelation of an AR(1) process

$$y_t = \phi y_{t-1} + \epsilon_t \tag{2}$$

with

$$\epsilon_t \sim N(0,1) \tag{3}$$

- Different sample sizes: $T = [100, 300, 1000]$
- Autocorrelations $\phi = [0.10, 0.20, ..., 0.90, 0.95, 0.99]$
- Limited information: Autocovariance
- Prior:

$$P(\theta) \sim U[0,1] \tag{4}$$

- 1000 runs;

Results: RMSE for 100 Obs.

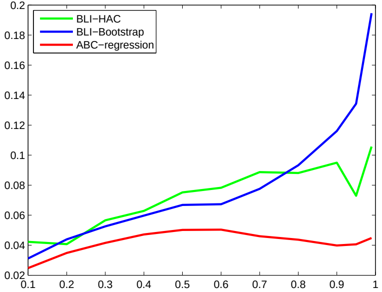


Figure : RMSE of the Montecarlo experiment: AR(1) process. Comparison among the ABC, HAC-BLI, Bootstrapping-BLI estimators, sample size=100. Different autocorrelations on the horizontal axis. With small samples, ABC estimators outperform BLI estimator in terms of RMSE with respect to the full likelihood posterior mean. The larger the persistence of the process, the better the relative performance of ABC.

Results: OR for 100 Obs.

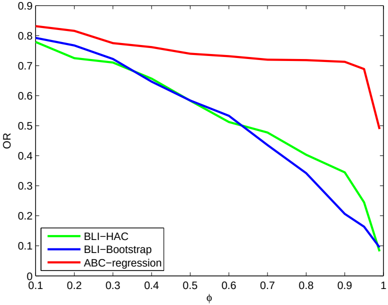


Figure : Overlapping Ratios of the Montecarlo experiment: AR(1) process. Comparison among the ABC, HAC-BLI, Bootstrapping-BLI estimators, sample size=100. Different autocorrelations on the horizontal axis. Overlapping ratio is generally larger for ABC than for BLI posteriors. The larger the persistence of the process, the better the relative performance of ABC.

ABC and DSGE

- ABC can be applied to the estimation of non-linear macroeconomic models (DSGE with ZLB, stochastic volatility and so forth);
- ABC-rejection subject to the curse of dimensionality;

ABC-SMC (Sisson, 2007)

- Initial ABC-rejection
- In subsequent iterations, new parameters are drawn from the accepted parameters of the previous iteration;
- Until convergence to the target distribution;
- Perturbation step, Kernel weighting and Resampling step, as in standard SMC.

ABC-SMC

An application to the ZLB

- ABC-SMC is applied to the estimate of a model with an occasionally binding ZLB;
- The observables: income, consumption, wages, hours, inflation interest rate
- Moments: variances, covariances, autocovariances;
- 4 subsamples:
  - Baseline: 1966Q1-2014Q3;
  - Great Moderation + ZLB: 1983Q1-2014Q3;
  - Great Moderation: 1983Q1-2007Q4;
  - The Great Volatility II: 2001Q1-2014Q3;

ABC-SMC: iterated approximate posteriors

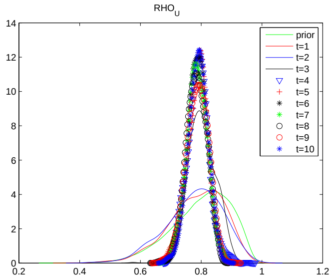


Figure : Approximate posterior distributions obtained for the first 10 iterations of the ABC-SMC for the parameter ρ_U in on of the estimation exercise. The sample spans from 1983Q1 to 2014Q3. The ZLB binds for approximately one fifth of the sample.

Impulse responses for 4 different sub-samples.

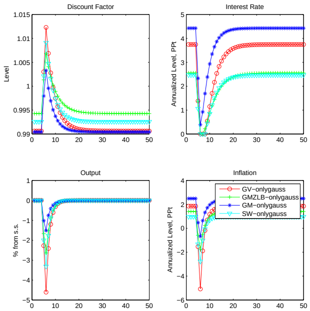


Figure : 2-standard deviation Impulse responses of preference shock for 4 different sub-samples: Baseline (SW, 1966Q1-2014Q3), Great Moderation (GM, 1983Q3-2008Q3), Great Moderation+Zero Lower Bound (1983Q1-2014Q3), Great Volatility-II (GV, 2001Q1-2014Q3). ABC-SMC is performed using covariances and autocovariances of income, wages, consumption, investments, interest rates, inflation.

Conclusion

- With small sample, Approximate Bayesian Computation outperforms the Bayesian Limited Information method, in terms of approximating the full likelihood posterior;
- ABC can be adopted to estimate medium-scale DSGE models with occasionally binding constraints;
- ABC-SMC presents several advantages to tackle multi-modality, numerical efficiency, curse of dimensionality.
- Including the period of the ZLB affects the estimated probability of hitting the ZLB of the model.

Comparison with the BLI: Montecarlo experiments

back

- I run two Montecarlo experiments: on the AR(1) and on a RBC model.
- First criterion: RMSE with respect to the Full likelihood posterior mean;
- Second criterion: Overlapping Ratio of the 90% credible intervals of the Approximate posterior distribution with the full likelihood posterior distribution;

$$RMSE = \frac{1}{N}\left(\sum_{i=1}^{N}(\hat{\theta}_{App} - \hat{\theta}_{Fl})^2\right)^{\frac{1}{2}} \quad (5)$$

$$OR = \frac{CI_{90\%,App} \cap CI_{90\%,Fl}}{CI_{90\%,App} \cup CI_{90\%,Fl}} \quad (6)$$

Overlapping Ratio

back

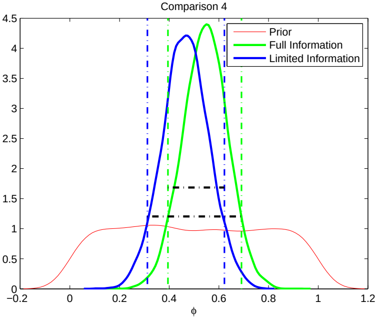


Figure : Full likelihood posterior distribution (Green), BLI Posterior Distribution (Blu), Prior Ditribution (Red), The 90% Credible Intervalsa are defined by the dash-dotted lines.

ABC-SMC: Drawing and accept/reject

back

- If $t = 1$, draw the swarm of particles $\{\theta_1\theta_2...\theta_N\}$ from the importance distribution μ_1.
- If $t > 1$, sample the new swarm $\left\{\theta_{i,t-1}^{**}\right\} i = 1^N$ with weights $\left\{W_{i,t-1}^{**}\right\}_{i=1}^{N}$ ad perturb each particle according to a transition kernel $\theta^{**} \sim K_t(\theta|\theta^*)$
- Simulate the model to obtain x^{**} conditional on each particle : if $\rho(S(x^{**}, S(x_0)) < \epsilon_t$ accept the particle, otherwise reject.
- If accepted, assign the particle a weight;

ABC-SMC: Weighting and resampling

back

- If $t = 1$, $W_{i,1} = \frac{\pi(\theta_{i,1})}{\mu_1(\theta_{i,1})}$.
- If $t > 1$,

$$W_{i,t} = \frac{\pi(\theta_{i,t})}{\sum_{j=1}^{N} W_{t-1}(\theta_{t-1,j}) K_t(\theta_{t,i}|\theta_{t-1,j})} \tag{7}$$

  where $\pi(\theta)$ is the prior distribution for θ.
- Normalize the weights such that $\sum_{i=1}^{N} W_{t,i} = 1$.
- Compute the Effective Sample Size (ESS):

$$ESS = \left[\sum_{i=1}^{N} (W_{t,i})^2\right]^{-1} \tag{8}$$

  If the ESS is below $N\frac{1}{2}$, resample with replacement the particles according to the weights $\{W_{i,t}\}_{i=1}^{N}$ and obtain the new population with new weights $W_{t,i} = \frac{1}{N}$.
- If $t < T$, return to (2).