Measuring nonfundamentalness for structutural VARs

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Nonfundamentalness: a problem for validating business cycle models (DSGE) with structural VAR.

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- Resurgence of interest with anticipated shocks: news (Beaudry and Portier (AER, 2006)), fiscal foresight (Leeper et al. (Ecta, 2013))
- E. Sims (AiE, 2012): not necessarily a pitfall for SVAR
- Forni and Gambetti (JME, 2014): test for nonfundamentalness
- Beaudry et al. (NBER, 2015): nonfundamentalness may be mild but significant

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- A population measure for the *nonfundamental bias*
- ...taking lag truncation (Christiano et al. (NBER Macro, 2006), Chari et al.(JME, 2008), Erceg et al.(JEEA, 2005)) into account

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NONFUNDAMENTAL BIAS

FUNDAMENTALNESS

The moving average $y_t = \psi(L)\varepsilon_t$ is **fundamental** iff present and past of observables and shocks span the same space: $\mathcal{H}_t^y = \text{span}(y_{t-k}, k > 0)$

$$\mathcal{H}_t^y = \mathcal{H}_t^\varepsilon \Leftrightarrow \psi(z) \neq 0 \;\; \forall |z| < 1$$

and **nonfundamental** iff $\mathcal{H}_t^y \subset \mathcal{H}_t^\varepsilon$.

The bias:

- $$y_t = (a_1 - L)(a_2 - L)\dots(a_p - L)\,u_t$$

  for simplicity all roots are real.
- Suppose nonfundamentalness: $\exists\, j,\ 1 \leq j \leq p \ :\ |a_j| < 1$.

Nonfundamental bias

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NONFUNDAMENTAL BIAS

Many representations of y_t, the fundamental one is:

$$\begin{aligned} y_t &= \underbrace{(a_1 - L)(a_2 - L)\dots(a_p - L)\frac{a_j^{-1} - L}{a_j - L}}_{\widetilde{a}(L)} \underbrace{\frac{a_j - L}{a_j^{-1} - L} u_t}_{v_t} \\ &= \widetilde{a}(L)\, v_t \end{aligned}$$

$$|u_t - v_t| = \left| u_t \left(\frac{a_j^{-1} - a_j}{a_j^{-1} - L} \right) \right|$$

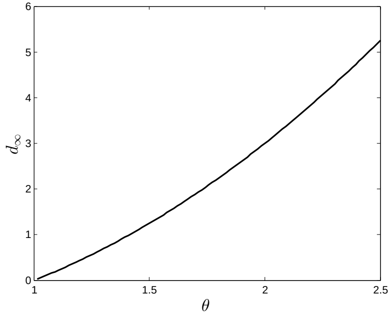
goes to zero as $a_j \to 1$.

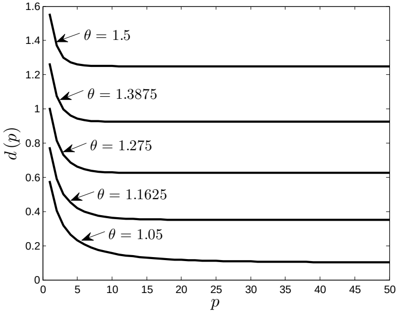
Nonfundamental bias:

$$d_\infty = \frac{E\,(u_t - v_t)^2}{E u_t^2}$$

MA(1) example: $y_t = (1 - \theta L)\, u_t$

- $d_\infty = \frac{\theta - \theta^{-1}}{1+\theta^2}$
- Truncation and nonfundamental bias: $d(p) = \frac{E\left(u_t - v_t^{(p)}\right)^2}{E u_t^2}$ where $v_t^{(p)} = y_t - \text{Proj}\left(y_t \,|\, y_{t-1}, y_{t-2}, \ldots, y_{t-p}\right)$





I prove that $d(p)$ always decreases with p.

TRUNCATION AND NONFUNDAMENTALNESS

State-space for the equilibrium conditions of a DSGE:

$$\begin{aligned} X_t &= A(\theta) X_{t-1} + B(\theta) w_t \\ Y_t &= C(\theta) X_{t-1} + D(\theta) w_t \end{aligned}$$

$$F \equiv A - BD^{-1} C$$

1. **Nonfundamentalness**: eigenvalues of F greater than one in absolute value (Fernandez-Villaverde et al. (AER, 2007): *poor man's invertibility condition*)
2. **Lag truncation**: eigenvalues of F different from zero (Ravenna (JME,2007), Franchi and Vidotto(EconLett,2013))

- Structural VMA representation:

$$y_t = G(L)\,u_t$$

where $u_t = Dw_t$ and

$$G(z) \quad = \quad \left(I - C\,(I - Fz)^{-1}\,BD^{-1}z\right)^{-1}$$

- Suppose nonfundamental: $\exists\ |z_0| < 1$ st $G(z_0) = 0$

- Fundamental representation: root flipping

$$y_t = \underbrace{G(L)\mathscr{B}(L,z_0)}_{\widetilde{G}(L)} \underbrace{\mathscr{B}(L,z_0)^{-1} u_t}_{v_t}$$

where $\mathscr{B}(L,z_0) = \frac{1-L\bar{z}_0}{L-z_0}$

- I prove that the zeros of

$$G(z) = \left(I - C(I-Fz)^{-1} BD^{-1}z\right)^{-1}$$

are those of $I - Fz$

- It is equivalent to find $\widetilde{F}$ st $\left(I - \widetilde{F}z\right)$ is the fundamental root flipping solution of $(I - Fz)$ and

$$\widetilde{G}(z) = \left(I - C\left(I - \widetilde{F}z\right)^{-1} BD^{-1}z\right)^{-1}$$

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ROOT FLIPPING

Let $z_1, \ldots, z_{n_{NF}}$ be the roots in the unit disk of $F(z) := I - Fz$

1. Spectral decomposition

$$F(z_k) = U_k V_k U_k^{-1}$$

with eigenvectors $U_k = \left[\mathscr{U}_{k,1}, \mathscr{U}_{k,2}, \ldots, \mathscr{U}_{k,n_x}\right]$

2. Letting $j \in [1, n_x]$ be st $V_{k,jj} = 0$, define the orthogonal matrix

$$M_k = \left[\begin{array}{cc} \mathscr{U}_{k,j} & \ker\left(\mathscr{U}_{k,j}'\right) \end{array}\right]$$

3. Calculate $F^{(k)}(z) = F(z) M_k$. Notice that $f_{11}^{(k)}(z) = 0$

ROOT FLIPPING

4.
$$\widetilde{F}(z)^{(k)} = \begin{bmatrix} \mathscr{B}_k(z) f_{11}^{(k)}(z) & f_{12}^{(k)}(z) & \dots & f_{1n_x}^{(k)}(z) \\ \mathscr{B}_k(z) f_{21}^{(k)}(z) & f_{22}^{(k)}(z) & \dots & f_{2n_x}^{(k)}(z) \\ \vdots & \vdots & & \vdots \\ \mathscr{B}_k(z) f_{n_x1}^{(k)}(z) & f_{n_x1}^{(k)}(z) & \dots & f_{n_xn_x}^{(k)}(z) \end{bmatrix}$$
$$\mathscr{B}_k(z) = \frac{1 - z\,\bar{z}_k}{z - z_k}$$
5. Repeat 2-4 for the multiplicity of z_k
6. Repeat 1-5 for $k = 1, \dots, n_{NF}$: $F^{\dagger}(z) := \widetilde{F}(z)^{(1,\dots,n_x)}$
7. As $F^{\dagger}(z) = F_0^{\dagger} + F_1^{\dagger} z$, compute
$$\widetilde{F}(z) = I - \widetilde{F}\, z$$
where $\widetilde{F} = -F_1^{\dagger} F_0^{\dagger^{-1}}$

VMA representations:

$$\begin{aligned} y_t &= G(L)\,u_t && \text{nonfundamental, structural} \\ y_t &= \widetilde{G}(L)\,v_t && \text{fundamental} \end{aligned}$$

Infinite order VAR representations:

$$\begin{aligned} y_t &= H(L)\,y_t + u_t && \text{noncausal} \\ y_t &= \widetilde{H}(L)\,y_t + v_t && \text{causal} \end{aligned}$$

where:

$$\begin{aligned} H(z) &= C(I - Fz)^{-1} BD^{-1} z \\ \widetilde{H}(z) &= C\left(I - \widetilde{F}z\right)^{-1} BD^{-1} z \end{aligned}$$

$$u_t - v_t = \left(\widetilde{H}(L) - H(L)\right) y_t$$

Given that $u_t - v_t = \left(\widetilde{H}(L) - H(L)\right) y_t$, can calculate:

$$\Sigma_{u-v} = \int_{-\pi}^{\pi} \left(\widetilde{H}\left(e^{-i\omega}\right) - H\left(e^{-i\omega}\right)\right) \Sigma_y(\omega) \left(\widetilde{H}\left(e^{i\omega}\right) - H\left(e^{i\omega}\right)\right)' d\omega$$

where

$$\Sigma_y(\omega) = \frac{1}{2\pi} G\left(e^{-i\omega}\right) \Sigma_u G\left(e^{i\omega}\right)'$$

is the (unique) spectral density matrix of the data and $\Sigma_u = D\Sigma_w D'$.

My measure of nonfundamental bias is:

$$d_\infty = \frac{\|\Sigma_{u-v}\|}{\|\Sigma_u\|}$$

Application: news shocks

A natural experiment along the lines of Sims (2012):

- Generate data from a DSGE with nominal (price stickiness) and real rigidities (habit in consumption and investment adjustment cost) and a RBC model
- News shocks in technology

$$\ln a_t = g_a + \ln a_{t-1} + \xi_t + \eta_{t-q}$$

- Bivariate VAR with output and TFP growth

d_∞

	q=1	q=2	q=3	q=4	q=5	q=6	q=7	q=8
				d_∞				
full	0	0.1356	0.2123	0.2477	0.2593	0.2620	0.2664	0.2693
RBC	0	0.0266	0.0480	0.0665	0.0827	0.0970	0.1097	0.1208

- Estimate IRFs with VAR(p) for $p = 5, 6, \ldots, 20$
- Identification: Choleski with TFP growth ordered first
- Mean absolute percentage errors (MAPE) averaged across the 4 IRFs.

Main results:

- Both MAPE and d_∞ grow with q
- Relatively more truncation in the RBC case: smaller d_∞ similar MAPE
- In the large sample exercise under fundamentalness ($q = 1$) the MAPE goes to zero as p grows
- In the nonfundamental case ($q \geq 2$) high-order VARs never more accurate, not even in large samples

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- A measure of the *nonfundamental bias*
- Inference on d_∞ considering parametric uncertainty
- Mild nonfundamental bias in news shocks models
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Thank you!