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A Federal Long-run Projection Model for Germany

Oliver Holtemöller
Maike Irrek
Birgit Schultz

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Authors: *Oliver Holtemöller*
Martin-Luther-University Halle-Wittenberg and
Halle Institute for Economic Research,
Department of Macroeconomics
E-mail: Oliver.Holtemoeller@iwh-halle.de
Phone: +49 345 7753 800

Maike Irrek
Halle Institute for Economic Research
Department of Macroeconomics
E-mail: Maike.Irrek@iwh-halle.de
Phone: +49 345 7753 865

Birgit Schultz
Halle Institute for Economic Research
Department of Macroeconomics
E-mail: Birgit.Schultz@iwh-halle.de
Phone: +49 345 7753 818

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HALLE INSTITUTE FOR ECONOMIC RESEARCH – IWH
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Address: Kleine Maerkerstrasse 8, D-06108 Halle (Saale), Germany
Postal Address: P.O. Box 11 03 61, D-06017 Halle (Saale), Germany
Phone: +49 345 7753 60
Fax: +49 345 7753 820
Internet: http://www.iwh-halle.de

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A Federal Long-run Projection Model for Germany

Abstract

Many economic decisions implicitly or explicitly rely on a projection of the medium- or long-term economic development of a country or region. In this paper, we provide a federal long-run projection model for Germany and the German states. The model features a top-down approach and, as major contribution, uses error correction models to estimate the regional economic development dependent on the national projection. For the medium- and long-term projection of economic activity, we apply a production function approach. We provide a detailed robustness analysis by systematically varying assumptions of the model. Additionally, we explore the effects of different demographic trends on economic development.

Keywords: regional long-run projection, convergence, demographic change

JEL Classification: C53, E17, O10

Ein Modell für die langfristige Projektion der wirtschaftlichen Entwicklung in Deutschland und den deutschen Ländern

Zusammenfassung

Viele ökonomische Entscheidungen basieren implizit oder explizit auf einer Projektion der ökonomischen Aktivität in einem Land oder in einer Region. In dieser Studie wird ein Projektionsmodell für Deutschland insgesamt und die deutschen Länder vorgestellt. Dabei wird zunächst die wirtschaftliche Aktivität in Deutschland insgesamt mit Hilfe einer gesamtwirtschaftlichen Produktionsfunktion fortgeschrieben. Anschließend wird mit Fehlerkorrekturmodellen für die Länder die regionale Aktivität aus der deutschlandweiten Entwicklung und der regionalen Entwicklung der Produktionsfaktoren abgeleitet. Die Robustheit der Projektion wird mit Sensitivitätsanalysen untersucht, und die Auswirkungen des demographischen Wandels auf die Produktion in Deutschland insgesamt und in den einzelnen Ländern werden diskutiert.

Schlagwörter: regionale Wachstumsprojektion, ökonomische Konvergenz, demographischer Wandel

JEL-Klassifikation: C53, E17, O10

1 Introduction

Many economic decisions implicitly or explicitly rely on a projection of the medium or long-term economic development of a country or region. Governments have to set up medium-term budget plans including detailed projections of revenues and expenditures; public administration has to plan long-term public investments in infrastructure such as roads, railways, schools, universities and so on, and firms have to decide on investment in human and physical capital. Of course, economic projections for more than a few quarters are highly uncertain. This uncertainty cannot be resolved, but using a formal economic model can help to generate consistent scenarios and – even more important – make transparent the assumptions that are underlying a certain picture of the future economic development. In this paper, we provide a federal long-run projection model for Germany and for the German states. The model features a top-down approach and uses error-correction models to consistently relate the regional economic development to the national projection.

For the medium and long-term projection of economic activity we follow a standard approach and apply a production function similar to the method that the European Commission uses to identify potential output in its member states (D'Auria et al., 2010). In the first step, we project the total German real gross domestic product (GDP) using a production function of Cobb-Douglas type. We extrapolate labor, measured in hours worked, by combining the population forecast of the German Federal Statistical Office with assumptions about the future development of the employment rate (ratio of employed persons to the population between 20 and 65)[1] and about the working time per employed person. The capital stock is updated using univariate models for the share of gross investment in GDP and for depreciations. Total factor productivity (TFP) is predicted on the basis of a time series model for the Solow residual.

In a second step, we compute the regional projections. In the literature, shift-share analysis is often used for regional projection. Koops and Muskens (2005), for example, forecast industry-specific regional growth rates and explain the regional variation by a multivariate regression on national and regional economic indicators. We use a different approach and project the production factors on the state level and calculate GDP projections for the states from the states' production functions. We use the federal German projections as an anchor for the regional projections of the input factors by imposing the same deterministic or stochastic trend or by assuming long-run

[1] For the employment ratio, different age groups are used in the literature, mostly 15 to 65. We follow Fuchs and Zika (2010), e.g., and focus on the population between 20 and 65.

convergence of some key structural factors like total factor productivity. Finally, we take care that the sum of the states' production factors and GDPs is equal to the corresponding total German figures.

As an example of the various fields of application, we explore the effects of different demographic trends on real economic activity. For this purpose, several variants of the 12th Coordinated Population Forecast by the Federal Statistical Office, which differ with respect to the assumed development of life expectancy, fertility behavior and migration, are employed. Furthermore, we provide a detailed robustness analysis by systematically varying the most influential assumptions and parameters of the model. First, we use different assumptions about the development of the employment rate. Second, we employ varied values for the production elasticties. And third, we use different models for the projection of total factor productivity, including a model with declining growth rate. These scenarios make up the relevant space of possible future economic developments in Germany and the German states. The computed scenarios can be used to discuss politically important issues like the effects of demographic change (reduction and aging of the population) on regional economic activity or convergence of per capita GDP and productivity.

The rest of the paper is organized as follows: In the next section we motivate the model by pointing to and illustrating some of the most important fields of application, like the economic consequences of demographic change. Section three includes an overview of the relevant methodical literature. The fourth section contains a detailed description of the model. It is followed by the results of the model in general and for several demographic scenarios in section five. A robustness analysis is then given in section six. The paper concludes with a summary.

2 Fields of Application

The *demographic change* will have major effects on economic growth in Germany. The notion of *demographic change* refers to a decline in fertility rates and rising life expectancy leading to a shrinking and aging population. Despite a projected average population growth rate of 5 % for Europe until 2030, only two third of the regions will actually experience population growth during this period (Giannakouris, 2010). At the same time, there will be an increase in the median age in the vast majority of the regions. A disaggregated perspective shows that the population development in Germany will be so diverse, that German regions will have the widest range of relative population change in Europe. For the total of Germany, the Federal Bureau of Statistics (Statistische Ämter des Bundes und der Länder, 2011) projects a decrease in population by 5.7 %

until 2030. The population age structure will change as the shares of persons with an age of under 20 and over 20 but under 65 are expected to decrease by 17 % and 15 % respectively, accompanied by a rise of the share of persons with an age above 65 by 33 %. The strong variation of population change on the regional level is due, on the one hand, to internal migration, which favors the states in the south of Germany and has a negative impact on the states in the east of Germany. On the other hand, the eastern states will suffer from a higher birth deficit, i.e. a larger negative difference between births and deaths, relative to their population. Both effects account for the difference in population change rates between the western states, which will experience a population decrease between 0.4 % and 13.8 %, and the eastern states, where populations will decline by rates between 10.5 % and 21.3 %. Although both parts of Germany are expected to have increasing shares of persons above the age of 65, this share will be about 35 % in the eastern states, compared to 28 % in the western states in 2030. It is because of the economic implications that demographic change has been recognized as central for policy and therefore has gained a considerable amount of political and scientific attention (Enquête-Kommission Demographischer Wandel, 2002; Ragnitz et al., 2007; Sachverständigenrat, 2011).

The various direct and indirect effects of demographic change can be explained in the context of a neoclassical production function like the Cobb-Douglas,

$$Y = A \times L^{1-\alpha-\beta} \times K^{\alpha} \times H^{\beta}. \tag{1}$$

Output (Y) is produced by combining the input factors labor (L), capital (K), human capital (H) and the technological level, total factor productivity (A). In the context of demographic change, it is useful to consider output per capita, meaning the division of output by the population (P), as this value is related to the actual living standards of the individuals,

$$\frac{Y}{P} = \frac{A \times L^{1-\alpha-\beta} \times K^{\alpha} \times H^{\beta}}{P} \tag{2}$$

or, in growth rates

$$\hat{y} - \hat{p} = \hat{a} + \alpha(\hat{k} - \hat{\ell}) + \beta(\hat{h} - \hat{\ell}) + (\hat{\ell} - \hat{p}) \tag{3}$$

The growth rate of output per capita $\hat{y} - \hat{p}$ is decomposed into the growth rate of total factor productivity $\hat{a}$, the effect of an in- or decrease in capital intensity (capital deepening), $\alpha(\hat{k} - \hat{\ell})$, the effect of a rise or decline in human capital intensity (human capital deepening), $\beta(\hat{h} - \hat{\ell})$, and the change in the relationship between labor and population, $(\hat{\ell} - \hat{p})$. The demographic change may affect every term on the right hand side of the equation.

Total factor productivity can be supposed to be affected negatively by a declining population via its interaction with human capital, following endogenous growth theory (Lucas, 1988; Gruescu, 2007; Romer, 1990). A positive effect of a smaller or even negative growth rate of labor will come from capital deepening. However, this impact is considered to be only temporary, as the higher capital intensity will lower the interest on capital, which in turn will lead to less investment in capital in order to return to the world market interest rate. Effects of demographic change on human capital intensity can also be expected from the aging of labor, although the evidence on the relationship between age and productivity (age productivity pattern) is still ambigous (Skirbekk, 2004; Schneider, 2011; Börsch-Supan and Weiss, 2007). Turning to the last term on the right-hand side of equation (3), it is obvious, that an amelioration of the ratio of labor to population will have a positive effect on per capita income, while a worsening will have a negative effect. The model, which we present here, does not include human capital and human capital-connected effects, partly because of data constraints.[2] However, we keep these effects in mind when we specify the models for total factor productivity. Several studies have analyzed the impact of demographic change on economic development in Germany in the context of growth theory (Oliveira Martins et al., 2005; Ludwig, 2007; Leibfritz and Roeger, 2008; Eckey et al., 2009; Sachverständigenrat, 2011). However, solely the works by Ludwig and Eckey et al. deal with this issue on a regional level. Both predict a negative effect of demographic change on growth and convergence between regions for the period 2005 to 2020.[3] Yet they differ, among other things, with respect to the integration of interregional relationships in the model. Eckey et al. (2009) estimate one translog production function for all regions, using capital, labor and agglomeration level as inputs. Ludwig (2007), on the other hand, explains regional output as a function of national output and regional economic indicators like population.

Besides demographic change, the second major area in which our model can be applied is the debate on economic convergence. Article 72 of the Basic Constitutional Law of Germany motivates the political attention on making the living conditions in Germany equivalent throughout all regions.[4] The idea of economic convergence has its theoretical foundations in the works by Solow

[2]The existing estimations of human capital or human capital indicators in Germany (Werding et al., 2009; Irrek, 2010) do not provide regional data, but focus on East and West Germany.

[3]These analyses are carried out on the level of 97 spatial planning regions (Ludwig, 2007) or 150 self-constructed regional labour markets (Eckey et al., 2009).

[4]The actions taken by the Federal Government in order to achieve this are summarized on a yearly basis in the *Jahresbericht der Bundesregierung zum Stand der Deutschen Einheit*, see e.g. Bundesministerium des Innern (2011).

(1956) and Swan (1956). Convergence occurs in neoclassical growth models due to diminishing marginal returns on capital leading to higher growth rates for countries with a lower capital intensity. This concept has been empirically examined by, among others, Barro and Sala-i-Martin (1991) for the United States of America and the states in Western Europe. They found an average annual convergence rate of 2 %. Having this result in mind, Barro (1991) postulated that the East German states will experience a similar speed of convergence. Kosfeld et al. (2006) as well as Scheufele and Ludwig (2009) have analyzed the German convergence process. For the time period of 1992 to 2000, Kosfeld et al. apply methods from spatial econometrics to 180 regional labor markets and find a convergence process for the western and the eastern states. Scheufele and Ludwig consider the convergence of East and West Germany for 1991 to 2008 using time series methods. Their result confirms the hypothesis of Barro in finding a rate of convergence between annually 1 to 2 %. The model, which is presented here, allows for convergence, whereby the rate of convergence is estimated.

Another field, in which our model might be useful, is public budget planning. Since 2009, Germany has introduced a new fiscal rule, the debt brake.[5] It has replaced the previous ruling, which restricted net public borrowing not to exceed public investment in the same year and had been found ineffective in controlling the level of public debt (Föderalismuskommission II, 2010). The debt brake proscribes balanced structural budgets on the federal and the states' level. Forward looking budget plans have to be based on projections of public revenues and expenditures, and therefore economic activity. Our model may be extended by including fiscal variables and also by a module to identify potential output and the output gap – a variable that is necessary for the calculation of the structural deficit.[6]

3 Regionalization – A Short Literature Review

For regionalization of national growth projections, shift-share analysis is a widely-used instrument. Based on Creamer (1943), Dunn (1960) and Perloff et al. (1960) have developed this technique for decomposing regional employment growth in a specific industry into three interpretable parts. It was primarily a descriptive device for ex-post analysis, but has also been used

[5]For a detailed discussion of the various aspects of the debt brake see Kastrop et al. (2010).

[6]Potential output refers to a situation, in which the input factors are utilized to a normal extent. It therefore reflects the sustainable long-run development path. As potential output cannot be observed, because of short-term fluctuations, it has to be estimated. The output gap is the difference between estimated potential output and observed actual output. A proposal how to compute potential output of the federal states has been made by van Deuverden and Freye (2010), who suggest a regional partitioning of the national output gap on the basis of the regional share of total gross value added.

for forecasting regional development.

In equation (4) it is shown how shift-share analysis splits up a variable's regional growth rate in a specific industry into the general national growth rate (g^n), the difference between the growth rate, the variable would have, if the region's industry grew at the national rate of this industry and the general national rate ($g_i^n - g^n$), and the difference between the actual growth rate in the region and the growth rate of the region, if its industry grew at the national rate of this industry ($g_i^r - g_i^n$),[7]

$$g_i^r = g^n + (g_i^n - g^n) + (g_i^r - g_i^n). \tag{4}$$

The first term in equation (4) represents the share of national growth, whereas the second and the third term show the region's shifts in growth due to its industrial mix and its regional characteristics, respectively. While a popular method in science and practice, shift-share analysis was heavily criticized.[8] One crucial point of criticism was connected to its deterministic nature, which does not allow for statistical validation of the results.

In reply to this methodical shortcoming, shift-share analysis has been advanced to a statistical and econometric tool (Emmerson et al., 1975; Berzeg, 1978, 1984; Buck and Atkins, 1976; Patterson, 1991). In the following years, two strands of further modification evolved. Möller and Tassinopoulos (2000) were the first in Germany to apply Patterson's approach, which they turned into a panel-econometric estimation problem. They used employment data to examine the convergence of regional economic structures. Subsequently, their method has been used and enhanced in various studies (Blien and Wolf, 2002; Kowalewski, 2011; Klinger and Wolf, 2011), which were all based on employment data. The other strand has used the shift-share idea for regionalizing national growth in output and other variables (Koops and Muskens, 2005; Bassilière et al., 2008). Also using econometric methods, they explain the regional deviation from national growth in a specific industry with the help of national and regional economic indicators. A similar approach for explaining and forecasting regional growth has been applied by Ludwig (2007), who spares the industry-specific strategy and directly estimates the dependence of regional production growth on national production growth and further regional indicators. The model, which we present in this paper, can be assigned to this latter branch of research.

The explicit use of a shift-share approach and time series econometrics for regionalization in Germany is at the present time solely included in the model system PANTA RHEI REGIO, which

[7]The notation is based on Richardson (1978).

[8]Overviews of critique are given in Richardson (1978) and Knudsen and Barff (1991).

allows for projection and analysis on the national, federal and municipal level (Schnur and Zika, 2009; Distelkamp et al., 2009). However, apart from other distinctions like the general approach, the data used and the level of sectoral disaggregation, also the regionalization differs from our model. For the deduction of federal values from the national ones a dynamic shift-share approach is applied. It explains and forecasts the regional value of a specific sector depending on a constant, the national value of the sector and a time trend (Meyer et al., 1999). We instead use error correction models to specify the relationship between selected national and regional variables, thereby constituting an innovation in regionalization.

4 Model Description

The Federal Long-run Model (FLRM) is a model for long-term projections of real GDP in Germany and the states. As a basic principle, official population forecasts are used for the projection.[9] Whenever no official data are available, time series methods are used to project the single components. These methods are chosen on the basis of statistic selection criteria. The long-term projection uses a production function of the Cobb-Douglas type,

$$Y = A \times L^{1-\alpha} \times K^{\alpha}. \tag{5}$$

The value for the production elasticity α is chosen according to the factor income shares of labor and capital. Labor (hours worked, L) is projected using the exogenous population forecast of the Federal Bureau of Statistics, a time series model for the employment rate and a time series model for the hours worked per employed person. The projection of the capital stock (K) uses the perpetual-inventory method in order to update the capital stock by investment and depreciation, whereby these two components need to be projected. Finally, total factor productivity (A) is projected using a univariate time series model.

4.1 National Projection

In the part of the model, which is concerned with the projection of the national growth rate of real GDP, the future development of the variables is estimated as follows. The exogenous basis for the projection of total hours worked are three scenarios of the official 12th coordinated population

[9] Detailed information on the data used is given in table 4 in the appendix. In table 5, the notation of the model is summarized.

forecast of the Federal Bureau of Statistics for the population in the age-group from 20 to 65 years. These scenarios differ with respect to the assumptions on the future development of the number of children per woman, the life expectancies of men and women and migration. In the baseline scenario we use the variant 1W1 (V1W1), which is the lower limit of the middle projection. Further details on the population forecast scenarios are given in the following section.

The raw population data gives the number of persons at December 31. Therefore, the mean for two succeeding years is taken in order to interpolate the average population of a year. The size of the labor force (N) is constructed by multiplying average population (P) and employment rate (ρ),

$$N_t = \rho_t \times P_t \tag{6}$$

The trend of ρ is approximated by a logistic function; the saturation limit is given by the maximum attainable employment rate. In the baseline scenario its value is set to 90 % ($\overline{\rho} = 0.9$) of the population between 20 and 65 years. Additionally, an AR(1)-error term is incorporated in the model,

$$\rho_t = \frac{\overline{\rho}}{1 + \exp(\beta_1 + \beta_2 t)} + \varepsilon_t + \beta_3 \varepsilon_{t-1}. \tag{7}$$

Here, and in the rest of the paper, ϵ_t denotes an i.i.d. error term. The declining trend in hours worked per employed worker (ζ_t) is also captured by a logistic function added by an AR(1)-error term,

$$\zeta_t = \beta_4 - \frac{\beta_0}{1 + \exp(\beta_1 + \beta_2 t)} + \varepsilon_t + \beta_3 \varepsilon_{t-1}. \tag{8}$$

Total hours worked (L_t) then follow from population, employment rate and hours worked per employed person,

$$L_t = \zeta_t \times N_t = \zeta_t \times \rho_t \times P_t. \tag{9}$$

For the projection of the capital stock, the generally accepted and widely used perpetual-inventory-method is applied. This describes an iterative up-dating mechanism for the capital stock, which implies the forward projection of the capital stock (K) of the previous period by adding investment (I) in the form of gross fixed capital formation and subtracting depreciation (δK),

$$K_t = (1 - \delta_{t-1}) \times K_{t-1} + I_{t-1}. \tag{10}$$

Thus, the future developments of investment and depreciation have to be estimated in order to be able to project the capital stock. In our model, investment is determined by the share of investment in GDP, which is projected using an ARMA(1,1)-process,

$$I_t = \iota_t \times Y_t, \qquad \iota_t = \beta_0 + \beta_1 \iota_{t-1} + \varepsilon_t + \beta_2 \varepsilon_{t-1}. \tag{11}$$

The depreciation rate (δ) is assumed to follow an exponential trend,[10]

$$\delta_t = \beta_0 - \exp(\beta_1 + \beta_2 t). \tag{12}$$

Total factor productivity is projected using its growth rate, the Solow residual ($\widehat{a}_t = \Delta \ln A_t$), which is just that part of the growth rate, which is not explained by the change in production factors. It is empirically determined by subtracting the growth rates of the input factors, weighted by their factor income shares, from the growth rate of output,

$$\hat{a}_t = y_t - \alpha \times k_{t-1} + (1-\alpha) \times \ell_t. \tag{13}$$

In the baseline scenario, a random walk with drift for log TFP is used,

$$a_t = \beta_0 + \varepsilon_t. \tag{14}$$

Finally, the projection of gross domestic product (Y) is carried out on the basis of a Cobb-Douglas-production function, which is computed recursively. While the trends of hours worked and total factor productivity are independent of the development of GDP, the values for the capital stock imply the knowledge of the level of output in the previous year, as investment is determined as a share of GDP,

$$Y_t = A_t \times L_t^{1-\alpha} \times K_{t-1}^{\alpha}. \tag{15}$$

4.2 Regionalization

Regional GDPs are calculated from regional Cobb-Douglas production functions by inserting regional production factors labor and capital and regional total factor productivities. The regional input factors are computed using time series models which include the overall German development of the respective variable. The details are explained in the appendix. In particular, it is assumed that capital intensity and total factor productivity are converging across Germany. The

[10]The capital stock exhibits a structural break in 1991 leading to an outlier in the depreciation rate, which is corrected for by the inclusion of an impulse dummy variable.

joint speed of convergence for all states is estimated using pooled error correction models for capital intensity and total factor productivity.

The general structure of the regional models, however, is the same as on the national level. Hours worked in each state i are composed of the regional trends of the population in the age between 20 and 65 (P_i), the labor employment rate (ρ_i) and hours per worker (ζ_i). As before, the population development is taken from the official forecasts of the Federal Bureau of Statistics. The labor employment rate is set to develop in a constant difference to the projected national rate.[11] For the projection of the hours per worker an error correction model is used. Multiplied with each other, these regional variables yield estimations of the future development of hours worked in the states (L_i),

$$L_{i,t} = \zeta_{i,t} \times N_{t,i} = \zeta_{i,t} \times \rho_{i,t} \times P_{i,t}. \qquad (16)$$

The capital stock on the regional level (K_i) is not projected in the same way as on the national level. Instead, neoclassical growth theory and its implication of converging regional capital intensities are used. The convergence process of the regional capital intensity ($\kappa_i = K_i/L_i$) is estimated via an error correction model, which reflects the adjustment to the average German capital intensity. Total factor productivity on the state level is also estimated by using an error correction model. Finally, we compute regional GDP from the production function,

$$Y_{i,t} = A_{i,t} \times L_{i,t}^{1-\alpha} \times K_{i,t-1}^{\alpha}. \qquad (17)$$

It is taken care, that the sum of the projected values for the states is equal to the national projection (see appendix).

5 Results

Now, we present the baseline results of the model.[12] While the findings for the model on the national level are additionally given for three different population forecasts, the regional results are only reported for the baseline scenario.[13] The two available population forecasts for the states are similar, so that the presentation of both does not provide so much further insight. On the disaggregated level, the regional variation of future population developments is of interest, in

[11]We have chosen this specification based on the past relative development of the regional labor employment rates.

[12]The baseline assumptions are an employment rate limit of 0.9, a capital share of 0.3, a Random Walk with drift for log TFP; the population forecast V1W1 is used. Further information on these assumptions is given in section six.

[13]Further results on scenarios are available upon request.

	1995-2010	2010-2015	2010-2025
real GDP	1.2	1.8	1.2
capital stock	1.5	1.3	1.1
total factor productivity	0.8	1.3	1.1
hours worked	−0.0	0.0	−0.3
population (20-65)	−0.2	−0.2	−0.6
employment rate	0.7	0.5	0.4
hours per worker	−0.5	−0.3	−0.2
labor productivity (h)	1.3	1.7	1.6
labor productivity (n)	0.7	1.5	1.4
population	0.0	−0.2	−0.2
real GDP per capita	1.2	2.0	1.5

labor productivity (h) = real GDP per hour worked, labor productivity (n) = real GDP per worker

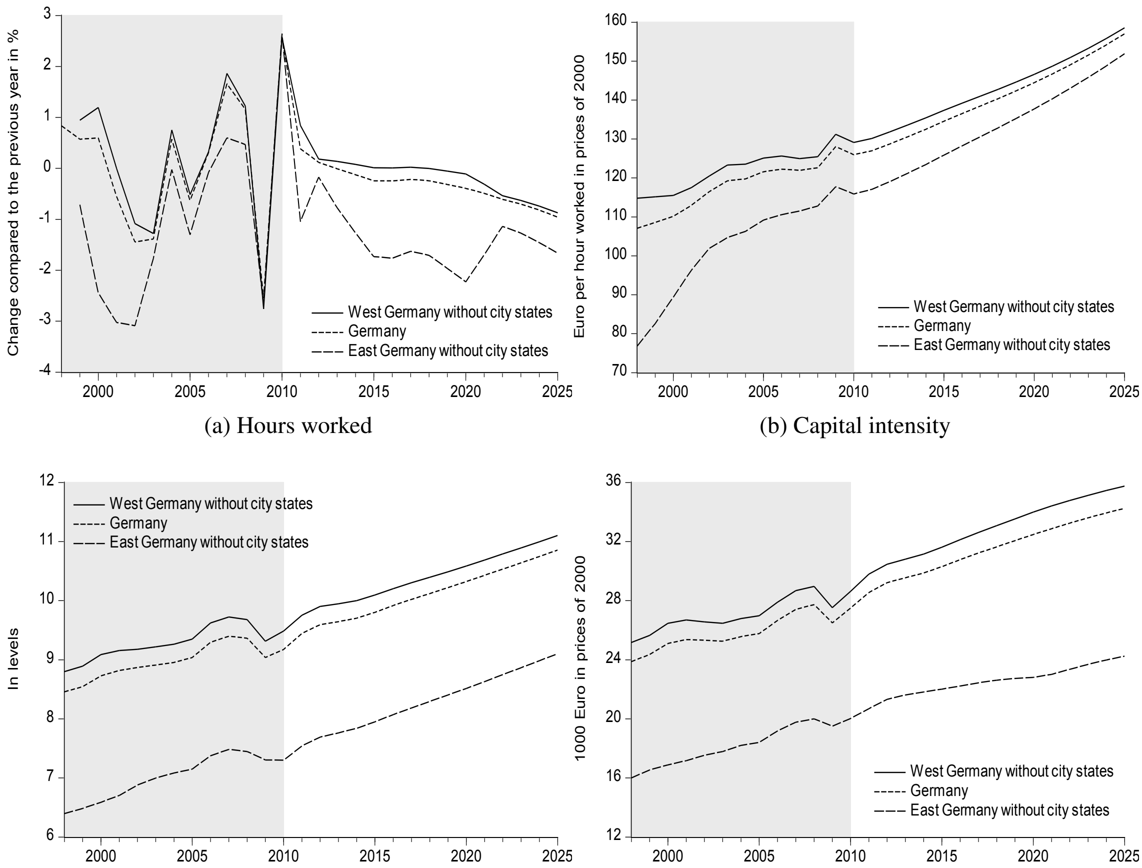
Table 1: Average annual GDP growth rates (in %) for Germany and its determinants in the baseline scenario

particular the contribution of demographic change to the relative economic growth of the federal states.

5.1 Baseline Results for Germany and the States

Based on table 1, the dynamics of the model shall be explained in detail. The growth rate of GDP is decomposed into the growth rates of labor, capital and TFP as in standard neoclassical growth accounting. The expected decline in the growth rate of output can be ascribed to the negative long-run growth rate of labor. The capital stock adapts slowly to demographic change, as future gross fixed capital formation is estimated as a share of future GDP. It is hence the effect of a declining GDP, which causes changes in the growth rate of the capital stock. TFP, on the other hand, is projected only on basis of its own past values and does not react to demographic changes in our model.

The breakdown of the projection of hours worked into single growth rates shows that the decrease in employable population and hours per worker is weakened by the estimated rise in the employment rate. The growth rates of GDP and GDP per capita both depend positively on the growth rates of capital intensity and labor, but the development of GDP per capita is additionally influenced by population change. Intuitively, a rising share of workers in population has a positive effect on



(a) Hours worked (b) Capital intensity

(c) TFP (d) Real GDP per capita

Figure 1: Baseline scenario for Germany, East Germany and West Germany

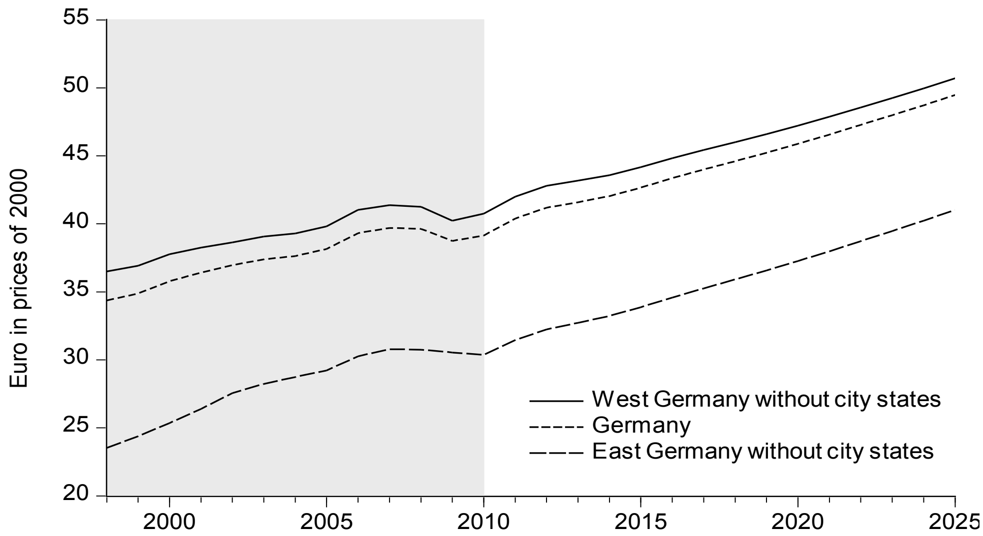


Figure 2: Real GDP per hour worked in the baseline scenario

GDP per capita. Since the decline in population is not directly influencing the growth rate of labor productivity, its growth rate is higher than the one of GDP per capita. These developments are presented in figure 1.

Additionally, these graphs show the projections for East and West Germany without city states, where the figures for the greater regions have been aggregated from the states.[14] This aggregation has been done in order to facilitate the reading of the figures. Figure 1a shows the projected growth rate of hours worked for East and West Germany. During the whole projection period the eastern states suffer from considerably more negative growth rates than the western states. This has to be ascribed to the stronger demographic change in East Germany. The capital intensity, which is the capital endowment per hour worked, is depicted in figure 1b. It grows in Germany as a whole as well as in each of its parts. Therewith it is continuing the past trend of a rising capital intensity and shows on the other hand the capital deepening effect of declining hours worked. The speed of convergence of the East German capital intensity results from the estimation with an error correction model and is therefore not exogenously given but extrapolated from the data. Another convergence process can be seen for TFP in figure 1c, although it does not exhibit such a high rate. Actually, the estimated rate of convergence for TFP in Germany is around 2 percent, which is just the speed of convergence for GDP per capita in neoclassical growth empirics. These two developments translate into a convergence of labor productivity, but are not able to dominate the relative future performance of GDP per capita. There, the negative effects of demographic change offset the positive development of capital intensity and TFP, as the difference between labor productivity and GDP per capita reflects the relation between hours worked and population. It can be concluded that a comparison between the future development of East and West Germany based on GDP per capita will not be able to provide any insight on the relative economic performance but only describe the stronger influence of demographic change in the eastern states.

Finally, figure 3 shows a comparison of the (projected) labor productivity levels of the states in 2010 and 2025, measured relative to the German average. The convergence process of East and West Germany is clearly seen. While the West German states will experience a slight decline in relative labor productivity, the East German states are predicted to have higher relative levels of labor productivity in 2025 compared to 2010. The gap closes, but does not disappear.

5.2 Demographic Scenarios

In order to elucidate the model dynamics, which are caused by demographic change, we have chosen three population forecasts on the national level. They comprise distinct assumptions concerning the development of the birth rate, the size of net-migration and future life expectancy. These three forecasts are the population scenarios V1W1, V4W2 and V1W5 (Statistisches Bundesamt,

[14]The average growth rate of GDP and its determinants are given for each state separately in the appendix.

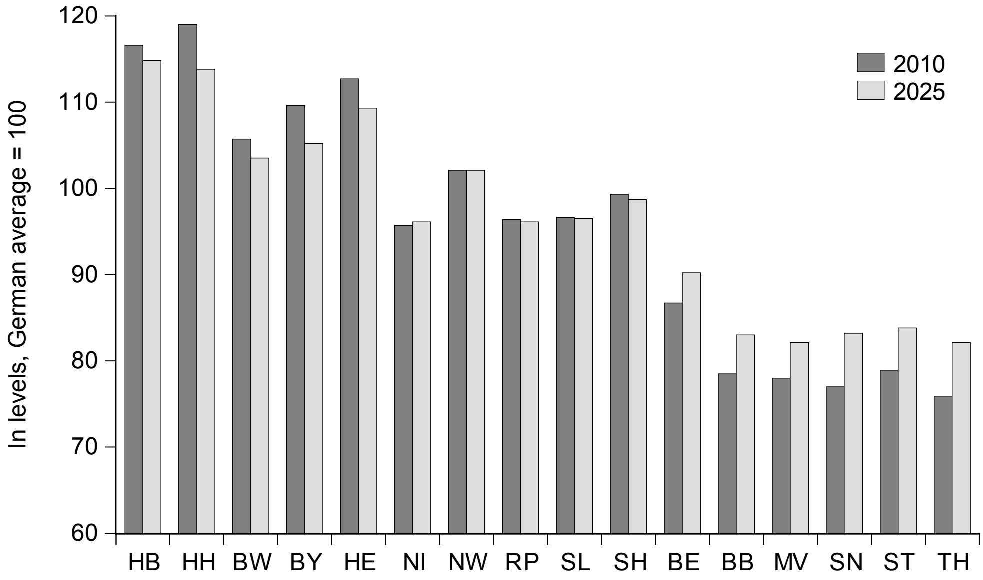


Figure 3: Labor productivity (h) in the German states in the baseline scenario

Scenario	Birth rate	Life expect. in 2060	Ann. net-migr. in 2020
V1W1	constant at 1.4	m 85.0 / f 89.2	100 000
V4W2	increase, 1.6 in 2025	m 87.7 / f 91.2	200 000
V5W1	decrease, 1.2 in 2060	m 85.0 / f 89.2	100 000

Table 2: Population forecasts (Federal Statistical Office)

2009). Table 2 summarizes their properties. Scenario V1W1 constitutes the lower bound of the middle population development by assuming a constant birth rate and a slight increase in life expectancy, while net-migration is supposed to be around 100,000 persons per annum. By contrast, scenario V4W2 implies a lower population decrease. This results from a presumably modest rising birth rate, a strong increase in life expectancy and an annual net-migration of 200,000 persons. Additionally we consider scenario V5W1, which resembles scenario V1W1 insofar as it assumes a mild rise of life expectancy and net-migration of about 100,000 persons a year. However, the birth rate is supposed to decline in the long-run. Figure 4 shows consequentially similar population movements for the scenarios V1W1 and V5W1.[15] We use the population forecast V1W1 for our baseline scenario. In figure 5 the corresponding projections for real GDP can be seen. Interestingly, the negative direct effect of a population decrease via a decline in total hours worked dominates the indirect, but positive effect of a higher capital intensity.

[15]The time horizon of this and the following figures is extended to 2060 in order to allow for a comprehensive comparison between the results for different szenarios.

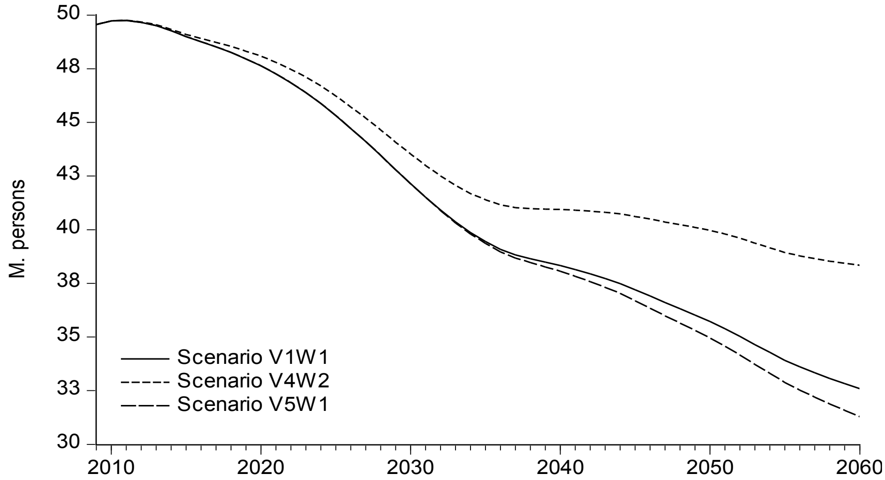


Figure 4: Population between 20 and 65 in Germany for different population forecasts

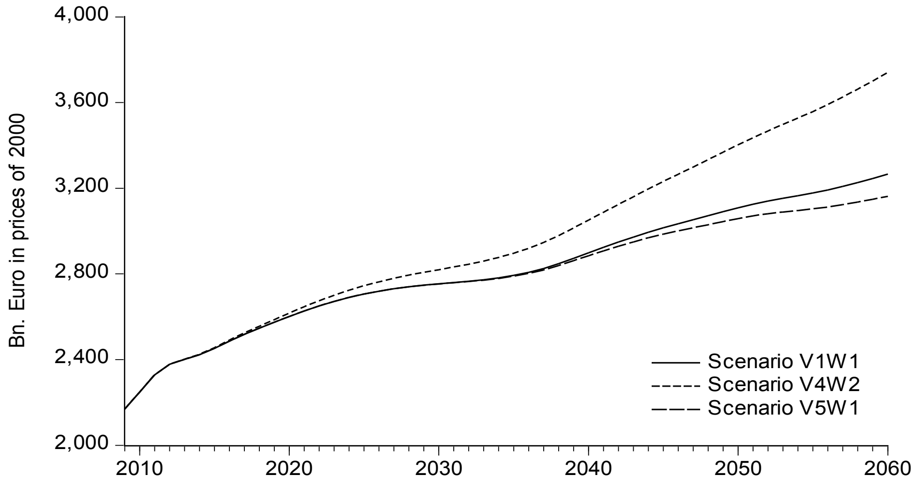


Figure 5: Real GDP in Germany for different population forecasts

6 Sensitivity Analysis

The following sensitivity analysis comprises the change of the exogenously set parameter values, being in this model the employment rate limit and the capital share. Additionally, we show the results for two alternative models of total factor productivity. Therewith, it shall be illustrated, how robust the outcomes of the model are with respect to changes in its assumptions. It is always the set-up of the baseline-model, in which the one parameter, or method, of interest is varied. The different assumptions are summarized in 3.

As described above, we project the number of workers by applying an estimated employment rate to the official population forecast of the age group between 20 and 65. The employment rate in Germany shows an increase in the last decade, which can be expected to continue for further years because of the rising labor participation of women and the raised pensionable age. The estimation of the employment rate uses a logistic function with a limit, which is set exogenously. Closely related to the projection of the employment rate by the Institute of Employment Research (Fuchs and Zika, 2010), which estimates the employment rate to be 86,5 % in 2025, we have chosen 0.85, 0.9 and 0.95 for the robustness check. The middle value is used in the baseline scenario. Figure 6 shows the resulting projections for the hours worked. A higher employment rate limit results, unsurprisingly, in a higher number of hours worked. The differences between the estimated developments are distinct. The corresponding projections of real GDP in figure 7 exhibit these distances as well. Although obviously not negligible, they nevertheless allow for a general statement about the future economic development.

There are various values for the production elasticies of capital and labor for Germany to find in the literature. The German economic research institutes use 0.35 for the capital share and 0.65 for the labor share in their joint medium-term projection, following a recommendation of the European Commission. The latter's suggestion is based on the average factor income shares of both input factors in the EU-15 over the past decades. This value is set to be the upper bound of the sensitivity

	Baseline Scenario	Alternative Scenarios
Employment rate limit ($\overline{\rho}$)	0.9	0.85, 0.95
Capital share (α)	0.3	0.25, 0.35
Total factor productivity	Random walk with drift ($\ln A_t$)	Linear trend (A_t)

Table 3: Assumptions in the baseline and alternative scenarios

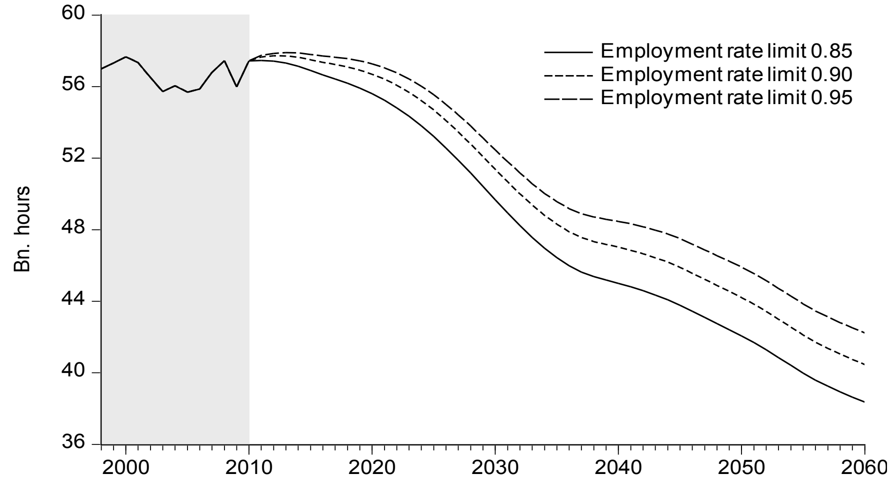


Figure 6: Hours worked in Germany for different employment rate limits

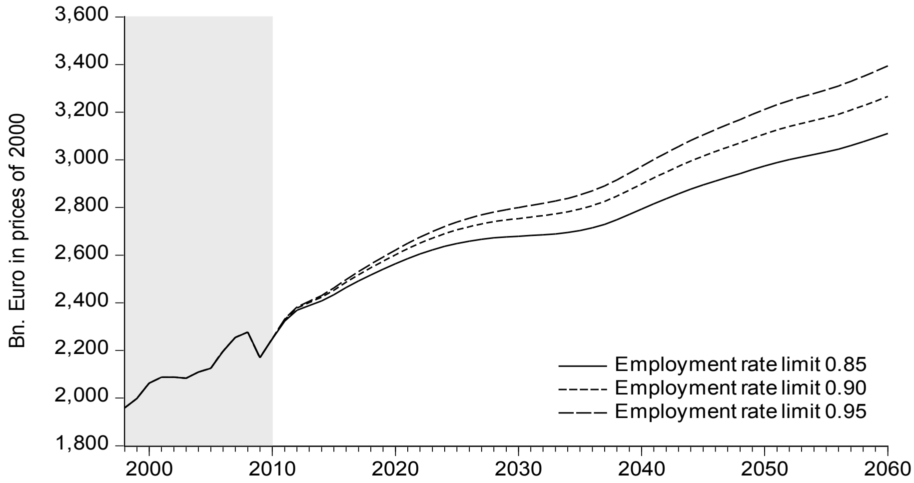


Figure 7: Real GDP in Germany for different employment rate limits

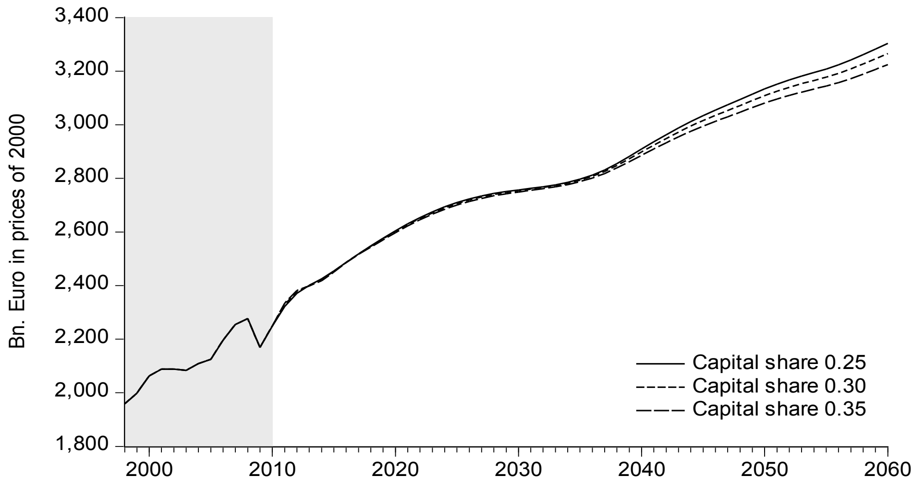


Figure 8: Real GDP in Germany for different capital shares

analysis with respect to the production elasticities. The middle value 0.3 is used in the majority of growth studies for Germany and represents an international and intertemporal mean. For the lower bound, we use 0.25, which has been used by Scheufele (2008). It is an average of current econometric estimates and values, which are derived from the factor income shares. In the baseline scenario we use 0.3. The projections of real GDP for these three scenarios are depicted in Figure 8. The differences are hardly visible from this perspective. The sensitivity of the model with respect to differing parameter values for the production elasticities is therefore small.

For the future growth rate of total factor productivity we analyze two different scenarios. In the baseline scenario, we project TFP using a random walk with drift for log TFP. The alternative scenario is based on a linear trend for TFP, which implies a decreasing rate of technological progress. These two TFP-paths are shown in figure 9. The resulting projections of real GDP are presented in figure 10. As TFP has a weight of one in the production function, varying methods and therefore TFP-developments have an unmitigated effect on GDP. The robustness of the model with respect to this assumption is thus limited.

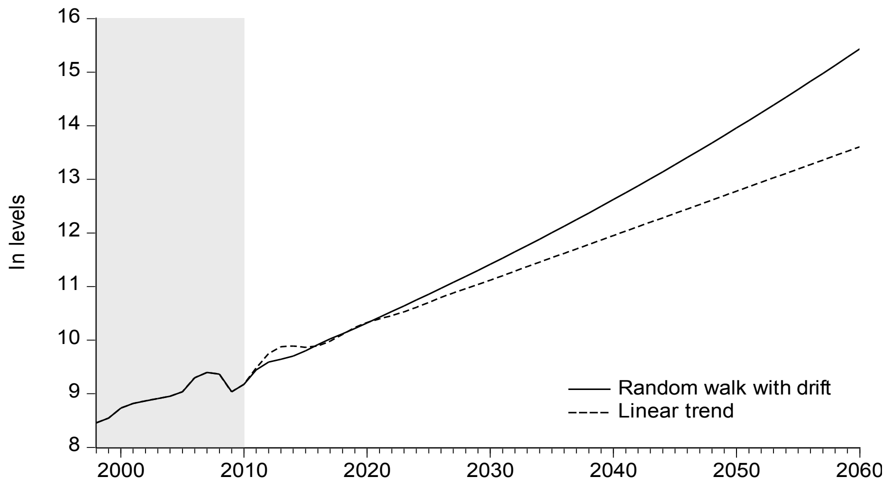


Figure 9: TFP in Germany for different TFP-estimation methods

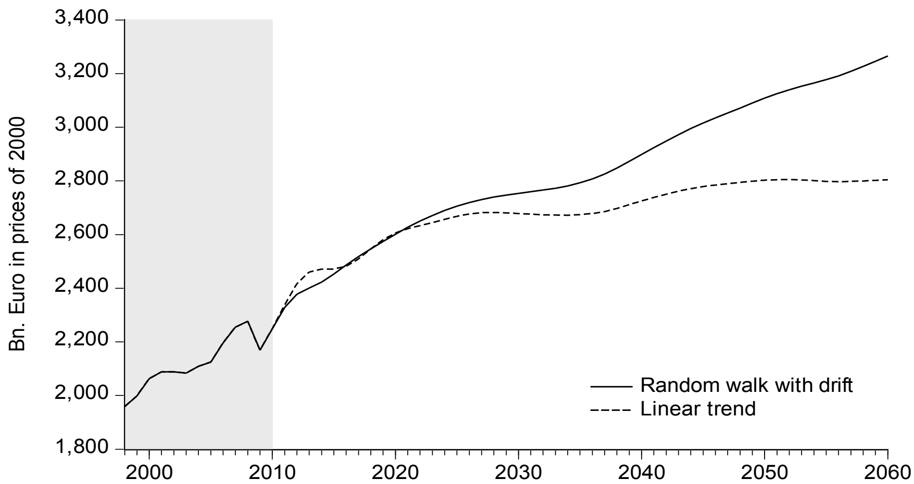


Figure 10: Real GDP in Germany for different TFP-estimation methods

7 Summary

In this paper, we have presented a new approach for regional long-term projections, using a structural production function approach combined with time series models for the relation between national and regional input factors. As illustrated, the model is relatively robust with respect to changes in the underlying assumptions like for example the imposed employment rate limit or the elasticity of substitution in production. The results show, that, not surprisingly, the demographic change in Germany will have a negative impact on economic growth on the national level and in all federal states. Since the eastern states suffer from a higher decrease in population and therewith labor supply, their growth rates of output are even stronger affected. It is due to this circumstance that the convergence of total factor productivity does not fully translate into convergence of output (per capita). The rate of convergence is therefore lower than in the case of equally distributed demographic change, hereby delaying full convergence. According to our projections, East German total factor productivity may increase from 77.6 % of the German average in 2010 to 82.9 % in 2025. However, East German GDP per capita may decrease from 72.8 % of the overall German average in 2010 to 70.8 % in 2025.

The model will be augmented by the public budgets on the national and regional level in order to broaden the field of application to public finance. Besides the estimation of potential output on the regional level, which is of crucial importance for the implementation of the debt brake, this extension will allow for an analysis of the consequences of a possible restructuring of the federal financial equalization system. Additionally, the employment rate will be split up in future research such that the work force and the unemployment rate can be modeled separately.

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A Data and notation

Table 4 contains the data sources, while the symbols used throughout the text are explained in table 5.

Germany[a b]	
Population (age 20-65)	1970-2009, Federal Statistical Office
Projected Population (age 20-65)	12^{th} coordinated population projection, 2009-2060, Federal Statistical Office
Working population	Persons in employment (workplace concept), 1991-2010, Regional Account VGRdL
Hours worked	Hours worked of persons in employment (workplace concept), Federal Statistical Office
Investment	Gross fixed capital formation (EURO, price adjusted, Chain-linked index, 2000=100), 1970-2010, Federal Statistical Office
Capital stock	Net stock of fixed assets (EURO, constant prices of 2000), 1970-2010, Federal Statistical Office
GDP	Real Gross Domestic Product (EURO, price adjusted, Chain-linked index, 2000=100), 1970-2010, Federal Statistical Office
German states[b]	
Population (age 20-65)	1970-2009, Federal Statistical Office
Projected Population (age 20-65)	$12^t h$ coordinated population projection, 2009-2060, Federal Statistical Office
Working population	Persons in employment (workplace concept), 1991-2010, Regional Account VGRdL
Hours worked	Hours worked of persons in employment (workplace concept), 1998-2010, Regional Account VGRdL
Investment	Gross fixed capital formation (EURO, price adjusted, Chain-linked index, 2000=100), 1991-2010, Regional Account VGRdL
Capital stock	Net stock of fixed assets (EURO, constant prices of 2000), 1991-2010, Regional Account VGRdL
GDP	Real Gross Domestic Product (EURO, price adjusted, Chain-linked index, 2000=100), 1991-2010, Regional Account VGRdL

[a] 1970-1990 former territory of the Federal Republic of Germany, since 1991 inclusive East Germany.
[b] Data status is June 2011 (before major revision 2011).

Table 4: Data Definitions

Variable/Parameter	Letter
Population (age 20-65)	P
Working population	N
Hours worked	L
Investment	I
Capital stock	K
Total factor prodivity	A
Real Gross Domestic Product	Y
Employment rate	ρ
Hours per worker	ζ
Investment rate	ι
Depreciation rate	δ
Production elasticity of capital	α
Capital intensity	κ

Table 5: Notation used in model description

B Regional Models

In this appendix, we describe the regional models that are used to project the regional production factors which are then inserted into regional Cobb-Douglas production functions. In general, the sum of the regional projections does not necessarily equal the national totals. Therefore, we define for a local variable x_{it} the sum over all states: $x_t^i = \sum_i x_{it}$. Adjusted local variables are calculated as $x_{it}^* = x_t \times (x_{it}/x_t^i)$; these adjusted variables sum up to the national total by definition. Below, the latest observation period is denoted by T. Estimates and forecasts are indicated by a hat.

Ratio of employees to population (employment rate). The local ratio ρ_{it} of employed persons (N_{it}) to population (P_{it}) is assumed to exhibit a constant difference to the average German ratio in the projection period:

$$\rho_{it} - \rho_t = \alpha_i + \varepsilon_{it}, \qquad \forall t > T: \ \widehat{\rho}_{it} = \widehat{\rho}_t + \widehat{\alpha}_i, \quad \widehat{N}_{it} = \widehat{\rho}_{it} \times P_{it}.$$

Working time per employee and total hours worked. Local hours worked per employee (ζ_{it}) are specified to adjust to the German average (ζ_t) with an estimated common adjustment speed:

$$\Delta \ln \zeta_{it} - \Delta \ln \zeta_t = \gamma_\zeta \left(\ln \zeta_{i,t-1} - \ln \zeta_{t-1} \right) + \varepsilon_{it},$$

and therefore

$$\forall t > T: \; \Delta \ln \widehat{\zeta}_{it} = \Delta \ln \zeta_t + \widehat{\gamma}_\zeta \left(\ln \zeta_{i,t-1} - \ln \zeta_{t-1} \right).$$

Total hours worked in state i follow from the product of working time per employee (ζ_{it}) and number of employees (N_{it}):

$$\widehat{L}_{it} = \widehat{\zeta}_{it} \times \widehat{N}_{it}.$$

Capital intensity and capital stock. The local capital intensities $\kappa_{it} = K_{it}/L_{it}$ are assumed to converge to the average German capital intensity κ_t according to the following adjustment process:

$$\Delta \ln \kappa_{it} - \Delta \ln \kappa_t = \gamma_\kappa \left(\ln \kappa_{i,t-1} - \ln \kappa_{t-1} \right) + \varepsilon_t + \gamma_1 \varepsilon_{t-1}.$$

Therefore, local capital stock (K_{it}) is given by

$$\widehat{K}_{it} = \widehat{\kappa}_{it} \times \widehat{L}_{it}.$$

Total factor productivity. Local total factor productivities are computed as follows

$$A_{it} = Y_{it} \times K_{it}^{-\alpha} \times L_{it}^{(\alpha-1)}.$$

It is assumed that local factor productivities converge to the average German total factor productivity

$$\Delta \ln A_{it} - \Delta \ln A_t = \gamma_a \left(\ln A_{i,t-1} - \ln A_{t-1} \right) + \varepsilon_t + \gamma_1 \varepsilon_{t-1}.$$

C Detailed Results for East and West Germany and for States

Tables 6 and 7 show detailed projection results for West and East German states (without city states).

	2000-2010	2010-2015	2010-2025
real GDP	0.9	1.9	1.3
capital stock	1.2	1.5	1.2
total factor productivity	0.4	1.2	1.1
hours worked	0.1	0.2	−0.1
population (20-65)	−0.2	−0.1	−0.4
employment rate	0.6	0.5	0.4
hours per worker	−0.3	−0.2	−0.1
labor productivity (h)	0.8	1.6	1.5
labor productivity (n)	0.5	1.4	1.4
population	0.1	−0.1	−0.2
real GDP per capita	0.8	2.0	1.5
level labor productivity (h)[a]	104.1	103.5	102.5
level real GDP per capita[a]	104.1	104.3	104.4

[a] in relation to the German average which is set to 100, in the final year of the period, respectively.

Table 6: Average annual growth rates (in %) for West German GDP and its determinants (without the city states) in the baseline scenario

	2000-2010	2010-2015	2010-2025
real GDP	1.0	1.2	0.6
capital stock	1.8	0.6	0.4
total factor productivity	1.0	1.7	1.5
hours worked	−0.8	−1.0	−1.4
population (20-65)	−0.9	−0.9	−1.5
employment rate	0.7	0.3	0.4
hours per worker	−0.6	−0.4	−0.3
labor productivity (h)	1.8	2.2	2.0
labor productivity (n)	1.2	1.8	1.8
population	−0.7	−0.7	−0.7
real GDP per capita	1.7	1.9	1.3
level labor productivity (h)[a]	77.6	79.4	82.9
level real GDP per capita[a]	72.8	72.6	70.8

[a] in relation to the German average which is set to 100, in the final year of the period, respectively.

Table 7: Average annual growth rates (in %) of East German GDP and its determinants (without the city states) in the baseline scenario

In the following the detailed results are presented for each state individually. While the table 8 shows the projected growth rates for the city states, the tables 9 and 10 are concerned with the East and West German territorial states, respectively.

	2000-2010	2010-2015	2010-2025
real GDP	0.6	1.1	1.2
capital stock	0.2	0.3	1.2
total factor productivity	0.6	1.5	1.3
hours worked	−0.0	−0.7	−0.6
population (20-65)	−0.2	−0.3	−0.6
employment rate	0.6	0.5	0.4
hours per worker	−0.7	−0.4	−0.3
labor productivity (h)	0.7	1.8	1.8
labor productivity (n)	−0.0	1.4	1.6
population	0.2	0.0	−0.1
real GDP per capita	0.5	1.1	1.3
level labor productivity (h)[a]	86.7	87.1	90.2
level real GDP per capita[a]	88.1	84.7	86.3

(a) Berlin

	2000-2010	2010-2015	2010-2025
real GDP	1.2	1.9	1.4
capital stock	0.9	2.6	2.2
total factor productivity	1.0	0.9	0.8
hours worked	−0.2	0.3	−0.1
population (20-65)	−0.3	−0.1	−0.4
employment rate	0.4	0.4	0.3
hours per worker	−0.2	−0.2	−0.1
labor productivity (h)	1.4	1.6	1.5
labor productivity (n)	1.1	1.4	1.4
population	−0.0	−0.1	−0.1
real GDP per capita	1.2	2.1	1.5
level labor productivity (h)[a]	116.6	115.9	114.8
level real GDP per capita[a]	137.0	137.7	138.1

(b) Bremen

	2000-2010	2010-2015	2010-2025
real GDP	0.8	1.5	1.3
capital stock	3.9	1.9	1.7
total factor productivity	−0.8	0.9	0.8
hours worked	0.7	0.0	0.0
population (20-65)	0.2	0.3	0.1
employment rate	0.5	0.4	0.3
hours per worker	−0.2	−0.4	−0.3
labor productivity (h)	0.1	1.5	1.3
labor productivity (n)	−0.1	1.1	1.0
population	0.4	0.4	0.3
real GDP per capita	0.4	1.2	1.0
level labor productivity (h)[a]	119.0	117.8	113.8
level real GDP per capita[a]	160.3	154.2	149.9

(c) Hamburg

[a] in relation to the German average which is set to 100, in the final year of the period, respectively.

Table 8: Average annual growth rates (in %) of the city states' GDPs and their determinants in the baseline scenario

	2000-2010	2010-2015	2010-2025
real GDP	0.8	1.2	0.7
capital stock	2.2	0.2	0.3
total factor productivity	0.6	1.7	1.5
hours worked	−0.6	−0.9	−1.2
population (20-65)	−0.6	−0.6	−1.2
employment rate	0.4	0.6	0.5
hours per worker	−0.7	−0.4	−0.3
labor productivity (h)	1.4	2.1	2.0
labor productivity (n)	0.7	1.6	1.7
population	−0.4	−0.3	−0.4
real GDP per capita	1.2	1.5	1.1
level labor productivity (h)[a]	78.5	79.8	83.0
level real GDP per capita[a]	70.6	69.1	67.0

(a) Brandenburg

	2000-2010	2010-2015	2010-2025
real GDP	0.8	1.3	0.5
capital stock	1.6	0.3	−0.1
total factor productivity	1.0	1.8	1.5
hours worked	−1.0	−0.7	−1.4
population (20-65)	−0.8	−0.9	−1.6
employment rate	0.6	0.6	0.4
hours per worker	−0.7	−0.4	−0.3
labor productivity (h)	1.8	2.1	1.9
labor productivity (n)	1.1	1.7	1.7
population	−0.8	−0.7	−0.7
real GDP per capita	1.6	2.0	1.2
level labor productivity (h)[a]	78.0	79.2	82.1
level real GDP per capita[a]	71.9	72.2	68.9

(b) Mecklenburg-Pomerania

	2000-2010	2010-2015	2010-2025
real GDP	1.3	1.4	0.8
capital stock	1.6	1.2	0.8
total factor productivity	1.3	1.7	1.4
hours worked	−0.7	−0.9	−1.3
population (20-65)	−0.9	−0.8	−1.3
employment rate	0.7	0.5	0.4
hours per worker	−0.6	−0.4	−0.3
labor productivity (h)	2.0	2.3	2.1
labor productivity (n)	1.4	1.9	1.8
population	−0.7	−0.6	−0.6
real GDP per capita	2.0	2.0	1.4
level labor productivity (h)[a]	77.0	79.3	83.2
level real GDP per capita[a]	75.5	75.4	74.8

(c) Saxony

	2000-2010	2010-2015	2010-2025
real GDP	0.6	0.8	0.2
capital stock	1.1	0.2	−0.0
total factor productivity	1.0	1.7	1.4
hours worked	−1.1	−.3	−1.8
population (20-65)	−1.4	−1.2	−1.9
employment rate	0.8	0.6	0.5
hours per worker	−0.7	−0.4	−0.3
labor productivity (h)	1.7	2.1	2.0
labor productivity (n)	1.0	1.8	1.7
population	−1.2	−1.1	−.1
real GDP per capita	1.7	1.9	1.3
level labor productivity (h)[a]	78.9	80.5	83.8
level real GDP per capita[a]	71.0	70.8	68.7

(d) Saxony-Anhalt

	2000-2010	2010-2015	2010-2025
real GDP	1.0	1.2	0.4
capital stock	2.4	1.0	0.3
total factor productivity	1.0	1.7	1.5
hours worked	−1.0	−1.2	−1.7
population (20-65)	−1.0	−1.1	−1.8
employment rate	0.5	0.5	0.4
hours per worker	−0.6	−0.5	−0.3
labor productivity (h)	2.1	2.4	2.1
labor productivity (n)	1.4	2.0	1.8
population	−0.8	−0.9	−0.9
real GDP per capita	1.9	2.1	1.3
level labor productivity (h)[a]	75.9	78.4	82.1
level real GDP per capita[a]	73.0	73.7	71.3

(e) Thuringia

[a] in relation to the German average which is set to 100, in the final year of the period, respectively.

Table 9: Average annual growth rates (in %) of the East German territorial states' GDPs and their determinants in the baseline scenario

	2000-2010	2010-2015	2010-2025
real GDP	0.8	2.1	1.4
capital stock	1.5	1.9	1.3
total factor productivity	0.3	1.2	1.0
hours worked	0.1	0.4	−0.0
population (20-65)	−0.0	−0.0	−0.3
employment rate	0.6	0.5	0.4
hours per worker	−0.3	−0.2	−0.1
labor productivity (h)	0.7	1.7	1.4
labor productivity (n)	0.4	1.5	1.3
population	0.2	−0.0	−0.1
real GDP per capita	0.6	2.2	1.5
level labor productivity (h)[a]	105.7	105.4	103.5
level real GDP per capita[a]	109.4	110.5	109.5

(a) Baden-Wuerttemberg

	2000-2010	2010-2015	2010-2025
real GDP	1.3	1.8	1.3
capital stock	1.7	1.1	0.8
total factor productivity	0.6	1.2	1.0
hours worked	0.3	0.3	−0.0
population (20-65)	0.1	0.1	−0.2
employment rate	0.5	0.4	0.4
hours per worker	−0.3	−0.3	−0.2
labor productivity (h)	1.0	1.5	1.3
labor productivity (n)	0.7	1.2	1.1
population	0.3	0.1	0.0
real GDP per capita	1.0	1.7	1.2
level labor productivity (h)[a]	109.6	108.2	105.2
level real GDP per capita[a]	118.6	117.1	114.6

(b) Bavaria

	2000-2010	2010-2015	2010-2025
real GDP	0.8	1.5	1.0
capital stock	1.1	1.4	1.2
total factor productivity	0.5	1.1	0.9
hours worked	0.0	−0.0	−0.3
population (20-65)	−0.3	−0.2	−0.5
employment rate	0.7	0.5	0.4
hours per worker	−0.3	−0.3	−0.2
labor productivity (h)	0.8	1.5	1.4
labor productivity (n)	0.5	1.2	1.2
population	0.0	−0.1	−0.2
real GDP per capita	0.8	1.7	1.2
level labor productivity (h)[a]	112.7	111.5	109.3
level real GDP per capita[a]	119.2	117.4	114.5

(c) Hesse

	2000-2010	2010-2015	2010-2025
real GDP	0.7	1.7	1.3
capital stock	0.9	1.0	1.2
total factor productivity	0.4	1.4	1.2
hours worked	0.2	0.0	−0.3
population (20-65)	−0.3	−0.2	−0.5
employment rate	0.8	0.5	0.4
hours per worker	−0.3	−0.2	−0.1
labor productivity (h)	0.6	1.7	1.6
labor productivity (n)	0.2	1.5	1.5
population	0.0	−0.3	−0.3
real GDP per capita	0.7	2.0	1.6
level labor productivity (h)[a]	95.7	95.4	96.1
level real GDP per capita[a]	88.8	88.8	90.9

(d) Lower Saxony

	2000-2010	2010-2015	2010-2025
real GDP	0.6	2.0	1.4
capital stock	0.9	2.0	1.8
total factor productivity	0.4	1.2	1.0
hours worked	−0.0	0.3	−0.2
population (20-65)	−0.3	−0.1	−0.5
employment rate	0.7	0.5	0.4
hours per worker	−0.3	−0.2	−0.1
labor productivity (h)	0.7	1.7	1.6
labor productivity (n)	0.4	1.5	1.5
population	−0.1	−0.3	−0.3
real GDP per capita	0.7	2.2	1.7
level labor productivity (h)[a]	102.1	101.9	102.1
level real GDP per capita[a]	98.7	99.9	101.9

(e) North Rhine-Westphalia

	2000-2010	2010-2015	2010-2025
real GDP	0.7	2.0	1.5
capital stock	1.0	1.1	0.9
total factor productivity	0.3	1.5	1.2
hours worked	0.2	0.4	−0.1
population (20-65)	−0.2	−0.1	−0.4
employment rate	0.8	0.5	0.4
hours per worker	−0.4	−0.2	−0.1
labor productivity (h)	0.5	1.7	1.6
labor productivity (n)	0.2	1.5	1.5
population	−0.1	−0.2	−0.2
real GDP per capita	0.8	2.2	1.6
level labor productivity (h)[a]	96.4	96.1	96.1
level real GDP per capita[a]	88.8	89.9	90.9

(f) Rhineland-Palatinate

	2000-2010	2010-2015	2010-2025
real GDP	0.8	1.8	0.9
capital stock	0.3	1.2	0.7
total factor productivity	0.9	1.4	1.2
hours worked	−0.2	0.1	−0.7
population (20-65)	−0.7	−0.5	−1.0
employment rate	0.9	0.5	0.4
hours per worker	−0.2	−0.2	−0.1
labor productivity (h)	1.0	1.7	1.6
labor productivity (n)	0.8	1.5	1.5
population	−0.5	−0.7	−0.7
real GDP per capita	1.3	2.5	1.6
level labor productivity (h)[a]	96.6	96.5	96.5
level real GDP per capita[a]	95.5	97.9	97.1

(g) Saarland

	2000-2010	2010-2015	2010-2025
real GDP	0.8	1.8	1.4
capital stock	0.7	1.0	1.1
total factor productivity	0.6	1.4	1.2
hours worked	−0.1	0.2	−0.1
population (20-65)	−0.4	−0.0	−0.3
employment rate	0.7	0.5	0.4
hours per worker	−0.4	−0.2	−0.1
labor productivity (h)	0.8	1.6	1.5
labor productivity (n)	0.5	1.4	1.4
population	0.2	−0.0	−0.1
real GDP per capita	0.6	1.9	1.6
level labor productivity (h)[a]	99.3	98.7	98.7
level real GDP per capita[a]	89.9	89.5	90.9

(h) Schleswig-Holstein

[a] in relation to the German average which is set to 100, in the final year of the period, respectively.

Table 10: Average annual growth rates (in %) of the West German territorial states' GDPs and their determinants in the baseline scenario