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# Inflation Expectations: Does the Market Beat Professional Forecasts?

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# Inflation Expectations:

# Does the Market Beat Professional Forecasts?*

## Abstract

The present paper compares expected inflation to (econometric) inflation forecasts based on a number of forecasting techniques from the literature using a panel of ten industrialized countries during the period of 1988 to 2007. To capture expected inflation we develop a recursive filtering algorithm which extracts unexpected inflation from real interest rate data, even in the presence of diverse risks and a potential Mundell-Tobin-effect.

The extracted unexpected inflation is compared to the forecasting errors of ten econometric forecasts. Beside the standard AR(p) and ARMA(1,1) models, which are known to perform best on average, we also employ several Phillips curve based approaches, VAR, dynamic factor models and two simple model avering approaches.

**Keywords:** Inflation Expectations, Rational Expectations, Inflation Forecasting

**JEL classification:** E31; E37

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# Inflationserwartungen:

# Schlägt der Markt professionelle Prognosen?


## Zusammenfassung

Das vorliegende Papier vergleicht die auf dem Markt offenbarte Inflationserwartung mit den Inflationsprognosen auf der Grundlage aktueller ökonometrischer Schätz- bzw. Prognoseverfahren für ein Panel von zehn Industriestaaten seit 1980. Zu diesem Zweck entwickeln wir ein rekursives Filterverfahren, mit dessen Hilfe aus den Realzinsdivergenzen zwischen den Industriestaaten die unerwartete Inflation geschätzt werden kann.
Die verwendeten Inflationsschätzungen, mit denen die extrahierten Erwartungen verglichen werden, umfassen ARMA-Modelle, AR-Modelle unter Berücksichtigung von konstanten oder sich im Zeitablauf verändernden Phillipskurveneffekten, VAR-Modelle, dynamische Faktorenmodelle sowie auf diesen Schätzungen basierende Modellselektion- bzw. Modelaveraging-Prognosen.



**Schlüsselworte:** Inflationserwartungen, Rationale Erwartungen, Inflationprognosen

**JEL-Klassifikation:** E31; E37

# Inflation Expectations: Does the Market Beat Professional Forecasts?

## 1 Introduction

There are few phenomena where expectations played a prominent role in the economic debate for such a long time as in the case of inflation. The important role of expected inflation for the choice of a growth enhancing monetary policy has been an essential problem of monetary economics at least since the emergence of the dispute between the Keynesian and Monetarist schools of thought in the 1960s. The rationality of expectation formation in general has often been questioned, especially due to observations on the highly volatile financial markets.

However, the results presented in this paper show that the quality of inflation expectations matches the quality of the best available econometric forcasting techniques which are commonly employed to forecast inflation. Since these forecasting techniques are the best known methods to exploit the availabe information that is believed to be relevant for future inflation, this is a strong indicator for rational expectations, which are defined by the utilization of available information, considering the cost-benefit criterion.

For our analysis we compare unexpected inflation, which is estimated with a recursive filter mechanism based on real interest rate data, with the forecast errors of ten different econometric inflation forecasts. The data covers ten OECD countries for the period from 1980 to 2007 allowing the comparison of (reasonably precise) forecasts from 1988 to 2007.

Beside the development of the filtering algorithm, the main contribution of this paper to the literature is the comparison of expectations and forecasts which is made possible by this technique. Previous papers on this issue usually had to rely on survey data. However, the expected inflation manifested in the market results is unlikely to be an unweighted average of individual expectations. If labor-, money- and commodity markets are working, reasonably formed expectations of well informed agents should dominate the results. Survey data, which is not reflecting this heavy weight of some agents, thus is only a distorted picture of aggregate expectations.

The remainder of the paper is structured as follows: Section 2 discusses the relevant literature. In section 3 we devolop the filtering mechanism used to extract unexpected inflation and present the forecasting techniques used. In the section 4 the employed data is presented. Finally, section 5 presents the results and their interpretation. Section 6 concludes.

# 2 Expected Inflation, Inflation Forecasts and the Real Interest Rate

Especially in the theoretical literature the formation of inflation expectations, which are in turn affecting real values, plays a substantial role. There is a broad literature on information and learning applied to this issue.[1] The obvious problem when tackling these issues empirically is to measure expected and unexpected inflation. Nevertheless, there is a rich literature based on survey data. Especially in the United States several surveys, which focus partially on experts (Livingston, ASA-NBER-Survey of Professional Forecasters) and partially on private households (Michigan Survey), are performed annually for several decades by now. There is a large strand of literature comparing household surveys to expert surveys. Since experts are heavily influenced by econometric inflation forecasts of themselves and other experts the question of this strand of literature is closely related to this paper.

An early comparison of survey results is already found in Gramlich (1983) who finds that randomly selected households outperform experts. However, he finds substantially distorted expectations of households and experts and concludes that expectation formation is an irrational process. Bryan and Gavin (1986) show that this alleged bias of private households is caused by a misspecification of Gramlich's econometric model and that surprisingly only the biased expectations of experts persist in an improved version of the model.[2]

Similarily, Rich (1989) who is accounting for the heteroscedasticity of inflation cannot reject the hypothesis of rational inflation expections based on the Michigan Survey. Grant and Thomas (1999) confirm this result with more recent data for the Michigan Survey and find the same true for the Livingston Survey. Andolfatto, Hendy and Moran (2008) achieve the same result using a calibrated DSGE model.[3]

Thomas (1999) and Mehra (2002) find in their comparison of surveys that households and experts both are more precise than naive econometric forecasts. Contrary to many other papers Ang, Bekaert and Wei (2007) find that experts outperform households, however, they also find that both expert and household surveys are more

[1] Recent contributions include Orphanides and Williams (2005) and Adam, Evans and Honkapohja (2006).

[2] Thus, while criticizing Gramlich, the authors even strengthen his result that private households outperform professionals.

[3] All these papers use the rationality definition of Muth (1961). However, rational inflation expectations according to Muth's technical criteria do not necessarily match inflations expectations which are formed based on all availabe information when (rationally) considering information (and computing) costs. Branch (2004)shows that heterogeneous and imprecise expectations are well in line with rationality when information is not free of cost.

precise than econometric forecasts. This is interesting in so far, as these authors include quite sophisticated forecasting techniques in their analysis.

Some other papers discuss household and expert expectations in more detail and (partly) go beyond looking at mere averages of the polls. Mankiw, Reis and Wolfers (2003) show for example, that private households' expectations are much more widespread than expectations of professionals. Furthermore this spread seems to depend on inflation. Carroll (2003) finds some evidence that the expectations of private households are more rigid than those of professionals.[4]

Doepke, Dovern, Fritsche and Slacalek (2008) have similar findings based on European data. Lein and Maag (2008) have slightly different results, however, they analyse inflation perception and not inflation expectations.

To the best knowledge of the author the present paper is the first comparison of econometric forecasts and expected inflation that is not based on survey data but on an (econometric) estimate of expected inflation. The idea to identify unexpected inflation by looking at real interest rates is not new to the literature, and has already been attempted by Fama (1975). However, unlike previous approaches the present paper does not rely on the (very rigid) assumption of a constant planned real interest rate. Unexpected inflation is estimated taking differences in the real interest rate between countries (due to risk) and the development in the real interest rate over time into account.

Although the methodology is loosely based on the Fisher-effect (Fisher, 1930) in a sense that we assume that the nominal interest rate is the sum of the planned real interest rate and expected inflation, we do not rely on the strict Fisher-effect which implies that any change of the (realized) real interest rate is due to a change in unexpected inflation.

However, even this limited version of the Fisher-effect is controversial. Mishkin (1992) finds that inflation and the nominal interest rate (of the previous period) are only correlated in times where both, interest rate and inflation, follow a time trend. He concludes that a long term relation exists (causing the reoccuring correlation) but that there is no short term relation. However, Evans and Lewis (1995) show that short sample periods (like Mishkin's) cause substantially biased results if inflation is regressed on interest rates. This is mostly due to autocorrelation of errors which

[4] Furthermore there is a broad, mostly older, literature which only focusses on the Livingston Survey and thus on experts only. See e.g. Paquet (1992), Keane and Runkle (1990), Mullineaux (1978), Pesando (1975).

arises, when inflation does not change immediately after some (observed) event which is known to have an impact on inflation and thus affects expected inflation.[5]

In a recent paper Ang, Bekaert and Wei (2008) present clear evidence for a strong Fisher-Effect. According to their estimation, 80% of the fluctuations of US nominal interest rates can be explained by expected inflation and risk.

However, the results on the existence of a strict Fisher-effect are very heterogeneous. Many papers find a Fisher-coefficient (i.e. the correlation of inflation and nominal interest rates) of less than one (Weidmann, 1996; Herwartz and Reimers, 2006). This can indicate a negative impact of expected inflation on planned real interest rates, thus violating the Fisher's core assumption of a constant real interest rate.[6] Other authors (Panopoulou, 2005) using different data and methodology cannot reject the null hypothesis of a Fisher-coefficient of one. Crowder and Hoffman (1996) even find evidence for a Fisher-coefficient which is larger than one (which would be in line with the neutrality of money if taxation is taken into account), but they nevertheless also find a negative short run impact of expected inflation on the real interest rate.

To be on the save side, we control for a possible effect of expected inflation in the present paper. While theoretically and empircally controversial, there are several arguments for an impact of expected inflation on interest rates which cannot be ruled out.[7] By controling for expected inflation we also cover the possible feedback of inflation on interest rates that arises if the central bank follows a Taylor rule.

[5] The same applies when there is some unobserved shock causing a long term shift in inflation. If the persistence of the change is not realized immediately, inflation is over- or underestimated for several subsequent periods (Johnson, 1997).

[6] Weidman, however, interprets his results as a mere consequence of the fact that inflation expectations in industrialzed countries rarely exceed certain thresholds, even if inflation is high fore some time.

[7] E.g. Tobin (1969), Mundell (1963), Fama (1981), Fama and Gibbons (1982), Danthine and Donaldson (1986), Stulz (1986), to name just a few. For reasons of simplicity we will refer to any negative effect of expected inflation on real interest rates as "Mundell-Tobin"-effect in the remainder of the paper, without implying the theoretical reasoning presented by Tobin or Mundell.

# 3 Models and Methods

## 3.1 Extracting Unexpected Inflation from Real Interest Data

### 3.1.1 The Model

The analysis is based on a slightly modified version of the real interest parity, accounting for diverse risk and the Mundell-Tobin-effect. Due to imperfections of the global capital market, domestic factors such as a possible impact of expected inflation on interest rates are not prevented (or compensated) entirely by competition between international assets.

We employ two models which differ in the assumptions concerning country specific effects. These country specific effects are meant to control for unobservable risk factors and policy parameters like the capital taxation sytem that affect the real interest rate parity. The baseline model is a simple fixed effects model given by:

$$(r_{it} - \tilde{r}_{it}) = \beta^T R_{it} - \pi_{it}^{unexp} + \delta \pi_{it}^{exp} + u_i \tag{1}$$

where $r_{it}$ is the real interest rate and $\widetilde{r}_{it}$ the planned real interest rate which is proxied by a global reference interest rate, $R_{it}$ is a vector of risk indicators, including the variance of inflation, growth and relative GDP, $\pi_{it}^{unexp}$ is unexpected (annual) inflation, $\pi_{it}^{exp}$expected inflation for country $i$ at time $t$. $\beta$ and $\delta$ are regression parameters (or vectors of regression parameters).

The second model takes into account that - given the length of the sample - general political and economic conditions in a country can change. Since the changes are nonrandom, fixed effects might be an insufficient way to capture country specific effects. We thus use a state space representation given by:

$$(r_{it} - \tilde{r}_{it}) = \beta^T R_{it} - \pi_{it}^{unexp} + \delta \pi_{it}^{exp} + \phi_{it} \tag{2}$$

$$\phi_{it} = \phi_{i,t-1} + \varepsilon_{it}$$

The fixed country specific component $u_i$ is replaced by a country specific component $\phi_{it}$that follows a random walk. To avoid that high interest rate volatility is mistakenly attributed entirely to a high volatility of the unobserved component, we assume that the variance of $\phi$ is identical over countries. Essentially the resulting model is very similar to a standard regime switching or rather regime changing model, with the one major advantage that we use data from the cross country dimension of the panel to get information about the general structure of regime evolution.

It has to be emphasized that according to the models used, real interest rates do not have an own truly idiosyncratic component. Volatility in the difference between planned real interest rates and realized real interest rates is fully attributed to changes of (un)expected inflation, risk or the unobserved component. From what we know about real interest rates this is quite realistic. It is unlikely that planned real interest rates fluctuate strongly since this would imply frequent changes of the time preference rate. However, the non avoidable little inaccuracies in economic (and econometric) modelling, e.g. due to the necessity to proxy the planned real interest rate and risk premia, make it unlikely that this absence of a random component is fully reflected in the available data. Because the model we employ charges all the unexplained volatility in the data to unexpected inflation, the quality of expectation formation is rather underestimated than overestimated. Thus, it is very unlikely that the rationality hypothesis is wrongly accepted.

Both models are estimated using a recursive filter algorithm that is outlined in the following sections.

### 3.1.2 Initial Estimations

We begin estimation with slightly modified versions of the models specified above. In these initial estimations unexpected inflation is substituted by the forecast error of a simple ARMA(1,1)-model of inflation. Correspondingly, the forecasts themselves replace expected inflation. Unexpected inflation is not measured precisely by this proxy; thus a one unit change in forecast errors, which are used as a first approximation to unexpected inflation, does not necessarily correspond to a one unit change in interest rate deviation. Furthermore, this imprecision causes random deviations. Therefore, the models have to be respecified as follows:

$$(r_{it} - \tilde{r}_{it}) = \alpha + \beta^T R_{it} + \gamma(\pi_{it} - \hat{\pi}_{it}) + \delta\hat{\pi}_{it} + \theta v_t + \varepsilon_i + u_{it} \tag{3}$$

and

$$(r_{it} - \tilde{r}_{it}) = \beta^T R_{it} + \gamma(\pi_{it} - \hat{\pi}_{it}) + \delta\hat{\pi}_{it} + \theta v_t + \phi_{it} + u_{it}, \tag{4}$$

$$\phi_{it} = \phi_{i,t-1} + \varepsilon_{it}$$

where the "hat" indicates estimated values.[8]

The additional time effect ($\theta v_t$) is necessary due to the approximation of the planned real interest rate. A global average of (realized) real interest rates is a good measure of the planned real interest rate, only if unexpected inflation is independent between countries. Global shocks to unexpected inflation cause a distorted estimate of the planned real interest rate. The precise specification of the time specific effects used herein is found in section 3.1.4.

The non dynamic version of the model (3) is estimated using a simple FGLS approach with country specific fixed effects.[9] The model with time varying country specific effects (4) is estimated using a Kalman-Filter.

### 3.1.3 The recursive extraction algorithm

**The mechanism** Starting with the estimation presented above, in any following recursion unexpected inflation is proxied by the part of the real interest rate differential which has been explained by the previously used proxy and the unexplained part of the real interest rate differential (i.e. the error term of the previous recursion). The proxy for unexpected inflation ($\hat{\pi}^{unexp}_{n,it}$) in recursion $n$ thus is given by:

$$\hat{\pi}^{unexp}_{n,it} = -(\hat{\pi}^{unexp}_{n-1,it} * \gamma_{n-1} + u_{n-1,it}) \tag{5}$$

Using this new proxy for unexpected inflation the model of choice is estimated anew. This process is repeated until the estimates converge and the error term approaches the zero bound. By construction, the regression parameter $\gamma$ which describes the relation of unexpected inflation and real interst rate differential converges to one in this process.[10] This is important, since unexpected inflation should actually cause interest rate deviations of exactly the same size (if assets with a maturity of one yeare are considered). Since the error term converges to zero in this mechanism, we get closer to the original specifications (equations 1 and 2).

The logic behind this mechanism is as follows: Planned real interest rates most likely are not subject to many random shocks. Thus, most of the unexplicable fluctuation of the real interest rate differentials ($u_{it}$) can be attributed to unexpected inflation. A

[8] Since inflation forecasts are not measured but estimated, standard errors should actually be bootstrapped. However, significance of these inital results is not truly important for the following filtering algorithm which is working with the point estimates only. Thus, we do not bootstrap the standard errors here.

[9] Random effects are rejected by a Hausman-Test.

[10] A proof is found in the appendix as "proof of proposition 1".

critical estimate of unexpected inflation therefore includes these error terms and the share of the real interest rate differential that can be explained by the forecast error of an econometric inflation model, since the latter includes the truly unexplicable part of inflation - i.e. unpredictable shocks - and hence should be at least roughly correlated with unexpected inflation.

However, this interpretation is not consistent if applied to the initial estimate. Expected and unexpected inflation have to add up to inflation. Thus, if we find that the previously used proxy for unexecpeted inflation (i.e. the initial forecast) has to be adjusted, the same adjustment has to be applied to expected inflation, which is included as explanatory variable to capture the Mundell-Tobin-effect. This does in turn imply, that a part of the error term actually is not a part of unexpected inflation but has been caused by the bias in the proxy for expected inflation. However, the Mundell-Tobin-effect is not very strong. Therefore, the larger part of the error is correctly attributed to unexpected inflation. While we do not necessarily get the precise unexpected inflation by adding up the error term and the part of the real interest rate differential that is explicable by the previous proxy of unexpected inflation, the estimate improves by doing so. Thus, this adjusted measure of unexpected inflation is a reasonable starting point to repeat estimation and adjustment. By repetition until convergence is achieved we get the recursive mechanism described above.[11]

**Comments on the recursive filtering** Although the final estimate is perfectly consistent with theory, it remains an approximation. There is a small bias, since the unobservable country specific drivers of inflation are likely to affect real interest rates as well. This kind of multicollinerarity causes a bias in the inital estimates that can not be entirely corrected by the filtering mechanism.

Furthermore, the estimate is based on rigid assumptions about how much of the explicable part of the real interest rate differentials should be attributed to unexpected inflation or to between country differences in planned real interest rates.

As far as the controls for risk are concerned the interpretation is mostly uncontroversial. Risk premia to real interest rates seem much more likely than differences in unexpected inflation in these cases.

A problem can arise concerning the regression parameter describing the correlation of forecasts errors, which are used as initial proxies of unexpected inflation, and real interest rate differentials. As explained in the model specification, a correlation of expected inflation and real interest rates or real interest rate differentials can be

[11] We tested the mechanism thoroughly by Monte-Carlo-experiments as described in the appedix.

caused by a Mundell-Tobin-effect if capital markets are imperfect. However, it is also possible (albeit unlikely) that economic agents systematically underestimate inflation in high inflation periods. Then, high econometric inflation forecasts would coincide with high unexpected inflation. This would cause a correlation of inflation forecasts and real interest rate differentials that is not due to a Mundell-Tobin-Effect, but is a consequence of the unexpected inflation of market participants which is involuntarily included in the forecast.

However, a closer examination of the inital estimation results allows to outrule this problem: If the expectations of market participants are systematically biased in high inflation periods this essentially means that unexpected inflation is either rising faster than the error term of inflation forecasts or independent of inflation forecasts in times when inflation is increasing. The former implies a parameter $\gamma$ of more than one (in absolute terms) in the initial estimates. The latter implies a low significance of the parameter estimate, since the forecast error would include a substantial component which is not strongly correlated with unexpected inflation. Neither is found in the results. $\gamma$ is close to one in the inital estimates and highly significant. Anyhow, the outlined problem is only relevant in the initial estimates, since $\gamma$ equals minus one in the adjusted final estimate by construction.

Another difficulty is the interpretation of the unobserved country specific effect. Interpreting fixed effects as consequence of differences in planned real interest rates, e.g. due to different risk, seems very plausible. Even, if the possibility of irrationality is taken into account it is very unlikely that market participants have a country specific systematic bias in expected inflation. The distinction is not as clear if the alternative specification of unobserved effects (according to equation 2) is used. Persistent changes in the real interest rate differential then are fully attributed to planned real interest rates. This is plausible since the general economic conditions of a country change over time. However, mistakes in expectations which persist for some time are theoretically also possible and have been found in some empirical studies. Such persistent mistakes can even be rational if some shock has been observed which is known to affect inflation after some unknown time, but which has not yet affected inflation. Empirically it is impossible to distinguish persistent changes of planned real interest rates and persistent changes of unexpected inflation.

Essentially the results derived from the fixed effects model can be considered to be a pessimistic estimate, while the results derived from the model with time varying country specific effects can be considered to be an optimistic estimate. Actual unexpected inflation most likely is somewhere in between.

### 3.1.4 Remarks on time specific effects

If an unexpected shock affects inflation in all countries simultaneously, this affects the average realized real interest rate. The difference between real interest rate and planned real interest rate then is not equal any more to the difference between the real interest rate and the average real interest rate. Since the inflation shock does nevertheless cause unexpected inflation in the analyzed countries, which is not reflected in real interest rate differentials, it is essential to correct for these time specific effects.

However, in a small n, large t sample as used here, time dummies potentially create biased results due to the substantial reduction in degrees of freedom. Thus, instead of using time dummies we employ an alternative approach to correct for time specific effects. In a sample of 30 industrialized countries covering 40 years we analyze the error terms of a simple AR(1) model of inflation. The global error is computed as the unweighted average of these error terms. We interpret the global error as a shock, if it exceeds a certain threshold. This threshold is chosen in way that it is sufficiently high to guarantee, that any value exeeding the threshold is not drawn from the distribution of global errors which would arise if national errors were uncorrelated with 90% probability.

The variable $v_t$ is zero if the deviation of the global error from this hypothetical distribution is insignificant and takes the value of the global average error otherwise.

## 3.2 Inflation forecasting

### 3.2.1 Univariate time series models

Following Ang et al. (2007) we use two traditional time series models of inflation as benchmark models: an AR(p) model were the lag order p is separately determined for each country in every year based on the Akaike Information Criterion (AIC) [12] and an ARMA(1,1) model as proposed by Hamilton (1985).

[12] While the AIC (Akaike, 1973) does not perform as well as the later developed Bayesian Information Criterion (BIC) (Schwarz, 1978)in determining the "true" model, it usually indicates models with better forecasting performance (Kuha, 2004). Therefore, in this paper the choice of the lag number is always based on the AIC if the choice of the forecasting model is concerned, but on the BIC otherwise, i.e. for choosing the right model in stationarity tests, etc.

### 3.2.2 Phillips-curve models

In the 1990s Phillips-curve[13] based forecast models gained new popularity, mostly due to the emerging New-Keynesian school. Most of these models do not rely on a naive Phillips-curve but at least partially include the Phelps-Friedman-critique by concentrating on the correlation of the change of inflation and (the level of) unemployment instead of the correlation of inflation and unemployment (or other real activity indicators) itself.

However, Atkeson and Ohanian (2001) show that Phillips-curve based forecasts do not perform better than naive univariate inflation forecasts for the period from 1985 to 2000 in the US. Due to their widespead use, Phillips-based forecasts are nevertheless included in this paper. Anyhow, Stock and Watson (2008) who essentially confirm the results of Atkeson and Ohanian (2001) note that there are episodes where Phillips-forecasts have an excellent fit while they perform very poorly in others. This makes them potentially interesting candidates to be included in model selection or model averaging approaches as they are also used in this paper.

The Phillips curve approach selected for this paper is loosely based on Stock and Watson (1999) and includes further lags of both inflation and the employed real acitivty indicator:

$$(\pi^a_{t+4} - \pi_t) = \alpha + \beta(L)x_t + \gamma(L)(\pi_t - \pi_{t-1}) + \varepsilon_{t+4} \tag{6}$$

$\beta(L)$ and $\gamma(L)$ are polynomials of the lag operator, $\pi^a$ is the annual inflation rate in quarter $t$, $\pi$ itself the quarterly inflation rate expressed in percent per year.[14] $x$ is the employed real activity indicator. Lacking the data which allows Stock and Watson (1999) to compute a generalized real activity index out of 85 different variables, we stick to the standard indicators used in the literature, i.e. unemployment and output gap. Although usually performing worse than output gap in inflation models, unemployment is included since the estimates of the output gap are often substantially distorted at the end of the sample (Orphanides and Norden, 2002; Watson, 2007).

The output gap is estimated using to different approaches: First, with the well known Hodrick-Prescott (HP) filter, second, using a bandpass filter developed by Christiano and Fitzgerald (2003). The advantage of a bandpass filter is that it sorts out fluctuations of a certain frequency. This allows to identify output movements

[13] The term “Phillips-curve” is used in a broad sense here, i.e. it is not restricted to the original relation between unemployment and wages or prices as discussed by Phillips (1958), but includes any relation of inflation and real activity indicators.

[14] This notation is used in the entire section on forecasting. $\pi$ denotes annual inflation in the section on filtering unexpected inflation

which are likely to be true business cycle movements, freed from the seasonal component and the trend component at the same time. Contrary to this, the HP filter only identifies a trend component and all the rest. Anyhow, since only seasonally adjusted data is used in forecasts in this paper, this effect should be negligible.

Both filters are used in the implementation of Baum (2006).

#### 3.2.3 Time-variant Phillips curves

Even if a Phillips-curve exists in the short run, it is by no means guaranteed that its parameters remain constant over time. Learning of economic agents can lead to a shift in expected inflation and thus to the necessity of even more expansionay monetary policy to "use" the Phillips effect. Learning can even change general economic conditions and by that the alleged relation between unemployment and unexpected inflation. By modelling the parameters of a Phillips-curve as states in a state space model which can in turn be estimated using a Kalman-Filter it is possible to take these changes into account. The model we use is based on the proposition of Nadal-De Simone (2000):

$$\pi_t = \alpha_t + \beta_t \pi_{t-1} + \gamma_t x_t + \varepsilon_t \tag{7}$$

$$\begin{pmatrix} \alpha_t \\ \beta_t \\ \gamma_t \end{pmatrix} = \begin{pmatrix} \alpha_{t-1} \\ \beta_{t-1} \\ \gamma_{t-1} \end{pmatrix} + u_t,$$

where $\alpha$, $\beta$, and $\gamma$ are the time-variant parameters, $x$ is the real activity indicator and $u$ the vector of errors, i.e. the change of the parameters. Since this kind of model is quite data intensive and the official unemployment statistics partially do not cover our full sample, the real activity indicator chosen for this approach is the output gap (estimated with the HP-filter).

#### 3.2.4 VAR forecasts

The VAR forecast used in this paper goes back to Mishkin (1990) but is also found more recently in Ang et al. (2007). The model tries to capture the interaction of inflation, interest rates, interst rate spreads (between short and long term interest rates) and growth. The resulting model has the form:

$$\begin{pmatrix} \pi \\ r_s \\ r_l - r_s \\ \hat{y} \end{pmatrix}_t = \alpha_t + A \begin{pmatrix} \pi \\ r_s \\ r_l - r_s \\ \hat{y} \end{pmatrix}_{t-1} + \varepsilon_{t+1}, \tag{8}$$

where $r_l$and $r_s$ denote long and short term interest rates. $A$ is the coefficient matrix.

### 3.2.5 Dynamic factor models: An Automatic Leading Indicators (ALI) approach

In recent years dynamic factors models became increasingly popular in inflation forecasting and are nowadays widespreadly used as auxiliary forecasting models at central banks (Eickmeier and Ziegler, 2006). These models do not regress inflation on the common set of economic indicators but instead on a number of factors which are extracted from a large dataset using dynamic factor analysis.[15]

In this paper we use a so called ALI model (see Camba-Mendez, Kapetanios, Smith and Weale, 2001; Qin, Cagas, Ducanes, Magtibay-Ramos and Quising, 2006). Similar methods are used by Stock and Watson (2002) and Banerjee, Marcellino and Masten (2005).

The inflation forecast itself is made using an VAR model. However, instead of the large number of variables which are partially used in standard VAR approaches this setup does only include inflation and a small number of extracted factors. This allows to include the information contained in a great lot of variables, which could not be simultaneaously included in a VAR without risking substantial multicollinearity and by that a reduction in forecast performance.

To extract the factors of interest we use a dynamic factor model in state space form given by:

$$Z_t = BF_t + e_t \tag{9}$$

$$F_t = AF_{t-1} + u_t$$

[15] The core idea to extract the key determinants of the economy that way is aready found in Stock and Watson (1990).

, where $Z$ is the vector of potentially interesting variables and $F$ the vector of extracted factors, $A$ and $B$ are coefficient matrices.

Based on the factors extracted with this model we estimate the VAR:

$$\begin{pmatrix} \pi \\ F \end{pmatrix}_t = \beta_i(L) \begin{pmatrix} \pi \\ F \end{pmatrix}_{t-1} + \varepsilon_t \tag{10}$$

The factor time series which are meant to represent economic key developments are estimated using a Kalman filter algorithm. However, for the Kalman filter to be (reasonably) applied, the coefficient matrices $A$ and $B$ have to be known. To obtain a first estimate of $A$ and $B$, a preliminary estimate of the factors $F$ is generated using a simple principal components analysis. Inital values for $A$ and $B$ are computed based on this first estimate of $F$. The estimate of $F$ then is refined using the Kalman filter algorithm. New values of $A$ and $B$ can now be computed using this new esimate of $F$. Based on these new coefficient matrices the Kalman filter algorithm can be applied again to refine the estimate of $F$ even further. This process is repeated until convergence is achieved.[16]

The number of factors is chosen based on the initial results of the principal component extraction for the first forecast period. Factors are selected so that every factor with an Eigenvalue of more than one is included and that more than 75% of the total variance of all variables is explained by the chosen factors. The lag order is determined using an AIC for each forecast period separately.

### 3.2.6 Model Averaging

It has already been shown by Granger and Bates (1969) that model combination approaches usually outperform single model approches, due to the inavoidable simplification of any econometric model and the resulting model uncertainty. In recent years this general result has been reconfirmed several times considering inflation forecasts in particular (see Wright (2003) and Dijk (2004)).

Representative for the class of model combination approaches we use an unweighted model averaging. In contrary to the more commonly employed Bayesian model averaging where the individual forecasts have to be from the same class of forecasts and may only differ by the variables considered, our more general approach allows to consider all the forecasts used in this paper.

[16] Simplified versions of this approach work with the initial estimate of $F$ based on principal component analysis. If the fluctiaton of factor loadings over time is low, this simplification is unproblematic as shown by Stock and Watson (2002).

Since the forecasts are partly only available for a brief period of time, two averages are computed. The forecast AVG3 includes the ARMA(1,1) forecast, the Phillips-curve forecast with an HP-based output gap estimation and the VAR forecast. All of these are available for almost 20 years. The forecast combination AVG5 additionally includes the moving Phillips-curve forecast and the ALI forecast. This reduces the forecast sample roughly by half.

# 4 Data

## 4.1 The sample

The sample consists of ten industrialized nations: Australia, Canada, Denmark, Japan, New Zealand, Norway, Switzerland, Sweden, the UK and the US. Essentially these are most OECD members which are neither part of the European Monetary Union (EMU) nor were part of the Warsaw Pact. The exclusion of EMU member is necessary since differences in expected inflation do not cause nominal interest rate differences between member states. The former Warsaw Pact countries are excluded because for most of these data is not available for a sufficiently long period. Furthermore, their development is strongly affected by the transition process. Thus, it is problematic to compare them to fully industrialzed nations. Mexico and Turkey similarily are excluded because of their significantly different level of development. In the case of Korea data is insufficient. Data on inflation (measured as percentage change of the GDP deflator) and GDP are available for all countries but New Zealand since 1980, and since 1987 in the case of New Zealand. This allows rolling forecasts over 20 years for nine out of the ten countries.

All predictor variables are available quarterly. Besides lagged inflation they include a number of real economic and monetary indicators, including alternative (lagged) measures of inflation based on consumer or producer prices. Data on price levels, wages, monetary aggregats and production is usually used in the form of growth rates, while unemployment rates, interest rates and the like are included as level variables.

The long and short term interest rates used in the VAR forecast refer to government bonds since the IMF reports interest rates seperated by maturity for these for most countries. The precise definitions and maturities differ by country. Since the inflation forecasts are made for the individual countries and are not based on panel data, these differences do not matter substantially. However, it has to be kept in mind that the varying precision of the VAR estimates is partly due to these differences. There is no sufficient data for Norway and Denmark. The VAR thus covers only eight countries.

The real interest rate differentials which are used to identify unexpected inflation are computed based on interbank rates for 12 month maturity on January the first of every year. The sample ends in 2007. The last interest rate thus is from January the first, 2007, i.e. from before the current financial crisis.

## 4.2 Seasonal adjustment

Every method employed is based on seasonally adjusted data. Seasonal adjustment is performed for every forcast separately, only using data from before this point in time. This procedure is necessary since seasonal adjustment is problematic at the end of a sample. So it is guaranteed, that the date used to create the forecast includes this bias as actual forecasts from this time did.

This repeated seasonal adjustment is not necessary when the mere objective is the comparison of different forecast methodologies if there is no reason to believe that the different methods are differently affected by the end of sample bias. However, since we want to compare forecasts to expectations which where formed based on the data available at this time, it is necessary to recreate this situation for the econometric forecasts.

Seasonal adjustment is done using the X12 algorithm which is also used by the US Census Bureau.

## 4.3 Stationarity

For most of the methods employed stationarity is an essential prerequisite. Although forecasts are made on the national level, stationarity is tested for the entire panel. Even if some indicator shows a trendlike behavior in one country, it is unlikely that this reflects true nonstationarity if this behavior is not found in other countries. Consequently, a correlation of these seemingly trended variables is most likely not spurious.

Stationarity is tested using a test developed by Fisher (1932) as recommended by Maddala and Wu (1999). The test statistic of the Fisher-test is computed based on the significance levels at which the null hypothesis of stationarity is rejected in individual stationarity test for every country. The resulting indicator is $\chi^2$-distributed, the degrees of freedom depend on the number of series included in the panel but not on their length. This is the source of the two major advantages of the Fisher test: First, it can be applied to unbalanced panels, contrary to the most widespread panel stationarity test developed by Im, Pesaran and Shin (2003). Second, any unit root test can serve as basic test for the actual Fisher test. In the present paper we use augmented Dickey-Fuller tests. The number of lagged first differences used in the test is chosen based on the BIC for every country separately.

All variables are found to be stationary.

# 5 Results

## 5.1 Inflation forecasts vs. inflation expectations

Most forecasting techniques (including the well performing AR(p) and ARMA(1,1) approaches) are slightly outperformed by market expectations. However, the most of the differences are insignificant. Significance levels reported are based on commony Morgan-Granger-Newbold (MGN) tests. The tests are not performed on the panel level but they are instead run countrywise and aggregated to the panel level using the panel testing approach developed by Fisher (1932). By that we account for country heterogeneity and do not risk to accept the null hypothesis of identical forecast accuracy mistakenly, because of some countries with difficult prediction conditions that might increase the average error of forecasts and expectations.

In table 1 the root mean squared errors (RMSE) of the forecasts are compared to the "root mean squared unexpected inflation" (RMSUE) filtered from the real interest data. Since the forecast techniques partially need a different amount of data to allow forecasting they are not all available for the same time period. Therefore, expectations are separately compared to each forecast within the respective sample of this forecast. This guarantees, that certain forecasting techniques are not advantaged or disadvantaged due to stable or highly volatile inflation during the period where forecasts are available using these forecasting techniques.

It can clearly be seen in the data that only model averaging outperforms expectations. Both filtering techniques indicate highly precise expectations. However, as indicated above, neither do market expectations significantly outperform most forecast techniques, nor do the averaging approaches outperform market expectations significantly.

We use encompassing tests in the style of Harvey, Leybourne and Newbold (1998). Again, these are run for each country separately and aggregated using the Fisher approach.Table 2 on page 23 summarizes the results of these tests. In no case the null-hypothesis that the expectations encompass the forecast of interest can be rejected significantly. That is, it seems very likely that market participants actually use a broad range of indicators to form their expectations. If any, the VAR forecast might include information that is not used by the public, assuming that the pessimistic estimation of inflation expectations was true.

Tabelle 1: RMSE compared to RMSUE

| Forecast technique | RMSE | RMSUE1 | RMSUE2 | Observations |
|---|---|---|---|---|
| ALI | 2.823318 | 2.597916 (0.694) | 2.596024 (0.722) | 154 |
| ARMA | 2.718137 | 2.503834 (0.830) | 2.563792 (0.864) | 256 |
| AR(p) | 2.920998 | 2.503834 (0.754) | 2.563792 (0.802) | 256 |
| Phillips (CF) | 8.422603 | 2.506840 (0.061) | 2.570296 (0.077) | 234 |
| Phillips (HP) | 3.119216 | 2.508821 (0.726) | 2.517955 (0.785) | 212 |
| Phillips (UE) | 2.802333 | 2.550794 (0.746) | 2.440192 (0.743) | 164 |
| Phillips (moving) | 3.406498 | 2.758586 (0.801) | 2.695753 (0.807) | 142 |
| VAR | 2.085415 | 1.857981 (0.619) | 2.015178 (0.687) | 169 |
| AVG3 | 1.800701 | 1.839609 (0.929) | 2.028555 (0.956) | 145 |
| AVG5 | 1.671664 | 1.810983 (0.944) | 1.925857 (0.958) | 108 |

Unexpected inflation:
** RMSUE1 - estimation of unexpected inflation using the fixed effects specification*
** RMSUE2 - estimation of unexpected inflation using time variant country specific effects*
Specification of the Phillips-curve based forecasts:
** CF - Output gap estimated using a Christiano-Fitzgerald-Filter*
** HP - Output gap estimated using a Hodrick-Prescott-Filter*
** UE - Unemployment as business cycle indicator*
** moving - Time variant Phillips-curve, Output gap estimated using a Hodrick-Prescott-Filter*
AVG3 - Average of ARMA, Phillips(HP) und VAR
AVG5 - Average of ARMA, Phillips (HP), VAR, Phillips (moving) and ARMA

The significance levels of Fisher-aggregated MGN-tests are given in parantheses.

## 5.2 When do expectations outperform forecasts?

While there are times when expectations outperform forecasts, forecasts yield more precise results in other periods. Since the differences in performance between expectations and most forecasts are not significant, this general result is not surprising. However, the clustering of years in which expectations outperform forecasts indicates that the alternation in superiority is not randomly driven, but rather follows a structure.

Plots of inflation, expectations and an ARMA-forecast are given in the appendix, to gain a visual understandig of the relevant kind of systematic behavior of relative performance. Most notably, expectations seem to catch turning points in the inflation process more precisely than forecasts. At the same time, very high volatility seems to detoriate the qualitiy of expecations stronger than the quality of forecasts.

Table 2: Encompassing tests on expectations and forecasts

| | Expectations according to fixed effects estimation | Expectations according to time variant country specific effects |
|---|---|---|
| ALI | 0.989114 | 0.978244 |
| ARMA | 0.998199 | 0.979515 |
| AR(p) | 0.998665 | 0.988653 |
| Phillips (CF) | 0.999284 | 0.991622 |
| Phillips (HP) | 0.992998 | 0.981023 |
| Phillips (UE) | 0.918891 | 0.953259 |
| Phillips (moving) | 0.986975 | 0.958157 |
| VAR | 0.909000 | 0.784302 |
| Significance level of Harvey-Leybourne-Newbold-test, Null hypothesis: Expectations encompass forecast | | |

In the present section we analyze the impact some of these factors on the relative quality of expectation and forecasts quantitatively. This is achieved by testing whether the differences in absolute error terms are partially explicable by other indicators. While the structure of this kind of test is seemingly similar to forecast accuracy and encompassing test, the differences are substantial: Since the difference of absolute errors and not the difference of errors itself is used as left hand side variable, significant correlations do not necessarily imply that the variable of interest also has got additional forecasting power.

We consinder the inflation level and the change of inflation:

Thus, the test equation has the form:

$$(|\pi_{i,t}^{exp} - \pi_{i,t}| - |\pi_{i,t}^{est} - \pi_{i,t}|) = \alpha + \beta X_{it} + u_i + \varepsilon_{i,t}, \tag{11}$$

where $X_{it}$ can be $\pi_{it}$, or $\Delta\pi_{it}$.

Table 3 summarizes the $\beta$ coefficients and their significance. Note that a negative $\beta$ indicates improving relative quality of expectations if the variable interest is increasing.

Concerning inflation, we find significant negative $\beta$s for most tests. That means that the expectations of market participants deal significantly better with high inflation. Possibly, most forecasting techniques overestimate the mean reverting process in high inflation periods due to some nonlinearity that is not taken into account by traditional inflation forecast models.

Table 3: The impact of inflation and the change of inflation on relative performance

| | $X_{it} = \pi_{it}$ | | $X_{it} = \Delta\pi_{it}$ | |
|---|---|---|---|---|
| | Expectations according to fixed effects estimation | Expectations according to time variant country specific effects | Expectations according to fixed effects estimation | Expectations according to time variant country specific effects |
| ALI | -0.001979 | -0.075254 | 0.032069 | -0.075254 |
| ARMA | -0.193557*** | -0.191279 *** | -0.154639 *** | -0.191279 *** |
| AR(p) | -0.173836** | -0.121094 *** | -0.134917 *** | -0.121094 *** |
| Phillips (CF) | -0.241458** | 0.458013 *** | -0.201422 * | 0.458013 *** |
| Phillips (HP) | -0.019174 | 0.140607 ** | 0.009013 | 0.140607 ** |
| Phillips (UE) | -0.232528** | -0.060294 | -0.229952 ** | -0.060294 |
| Phillips (moving) | -0.190711*** | -0.111031 *** | -0.176679 *** | -0.111031 *** |
| Var | -0.090151 | -0.243908 *** | -0.008383 | -0.243908 *** |

The table includes the estimates of $\beta$ in Equation 11.
* = significant at 10%-level
** = significant at 5%-level
*** = significant at 1% level

This is in line with the results on the impact of the change of inflation. Especially the time series approaches and the VAR forecast - that perform well overall - evidently perform substantially worse than expectations in times of rising inflation. However, some Phillips-Curve approaches improve substantially in terms of relative performance in times of rising inflation. This might be due to the fact, that the Phillips-relationship is most reliable in times of accelerating inflation, when expectations are "outrun" by policy.

Inflation expectations of market participants seem to be based on a surprisingly complex implicit model of inflation behavior. Market participants capture the behavior of inflation over the business cycles quite well while at the same time avoiding the problems of Phillips-curve forecasts that usually perform worse than univariate time series forecasts in the long run.

# 6 Conclusions

It could be shown very clearly that the expectations prevailing in the market results outperform most econometric forecasts slightly but not significantly. Even the best performing forecasts that are derived from model averaging approaches do not outperform expectations significantly. Essentially this means that the rational expectations hypothesis cannot be rejected.

Furthermore it could be shown that expectations are not only based on the past development of inflation, but that further economic indicators are obviously considered by market participants. It cannot be rejected that expectations include all the information that is used in Phillips-Curve models or an ALI-model - that includes a very broad range of economic factors.

Furthermore, expectations seem to capture certain nonlinearities in inflation behavior very well compared to forecasts, leading to far better results in times of high inflation and in times of increasing inflation.

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# Appendix

## Appendix A: Proof of proposition 1

If the values converge, we know that:

$$\gamma_n \approx \gamma_{n-1}$$

and

$$\pi_{n,it}^{unexp} \approx \pi_{n-1,it}^{unexp}$$

etc., i.e. the iteration number $n$ can be ignored.

By substituting this into equation 5, we get

$$\pi_{it}^{unexp} = -\pi_{it}^{unexp} * \gamma - u_{it}$$

or

$$(1+\gamma)\pi_{it}^{unexp} = -u_{it}$$

We assume that $\gamma \neq -1$ and by that $1+\gamma \neq 0$. The equation above then says that the error term $u$ of the real interest differential model is determined by an explanatory variable of this very model. Since the error term is not correlated with an explanatory variable by construction the assumption $\gamma \neq -1$ causes an inconsistency. Thus, it is proven that $\gamma = -1$.

This does in turn imply that $u_{it} = 0$ for any combination of $i$ and $t$.

## Appendix B: Monte-Carlo Simulations

The recursive filtering algorithm has been tested with artificially created time series of expected inflation and real interest rate differentials. Inflation has partially been simulated, partially real time series have been used. The following problems considering the data have been taken into account:

- country specific effects in the real interest rates

- period specific effects in real interest rates
- Multicollinearity due to fixed effects in the inflation time series which affect the fixed effects in the real interest rate series

The quality of the estimate is evaluated using the benchmark regression:

$$\pi_{it}^{unexp} = \beta * \hat{\pi}_{it}^{unexp} + \varepsilon_{it} \tag{12}$$

The coefficient $\beta$and the $R^2$-value of this regression are used as quality indicators. If unexpected inflation is correctly filtered, $\beta$ is close to one and $R^2$ close to 100 percent.

In all tested scenarios there is a strong convergence towards the simulated "true" unexpected inflation. However, there is a small (and non systematic) bias in small samples.

Unexpected inflation is initialized in the filtering algorithm with a random number series drawn from a standard normal distribution.

**Baseline specification**

Real interest rates are simulated by the process:

$$(r_{it} - \tilde{r}_{it}) = -\delta\pi_{it}^{exp} - (\pi_{it} - \pi_{it}^{exp}) \tag{13}$$

Inflation is simulated by the mean-reversion process:

$$\pi_{it} = \pi_{it-1} + \beta_i(\bar{\pi}_i - \pi_{it-1}) + \varepsilon_{it} \tag{14}$$

$$\varepsilon \sim N(0, 1)$$

Both average inflation $\bar{\pi}$ and the parameter $\beta$ differ between countries. They are drawn from the following distributions:

$$\beta \sim N(0.7, 0.1)$$

$$\varepsilon \sim N(3, 0.5)$$

Tabelle 4: Results of the control regressions using the baseline specification

| | $\delta$=0.2, t=100, n=100 | $\delta$=0.4, t=100, n=100 | $\delta$=0.6, t=100, n=100 | $\delta$=0.2, t=20, n=10 |
|---|---|---|---|---|
| $Mean(\beta)$ | 0.996 | 0.992 | 0.989 | 0.983 |
| $\sigma(\beta)$ | 0.009 | 0.010 | 0.015 | 0.080 |
| $Mean(R^2)$ | 0.979 | 0.975 | 0.970 | 0.835 |
| $\sigma(R^2)$ | 0.002 | 0.003 | 0.003 | 0.041 |
| Regression runs | 1000 | 1000 | 1000 | 2500 |

**Large samples** Tests are made for three time series of simulated inflation in a sample of 100 countries over 100 years using the parameters $\delta$=0.2, $\delta$=0.4, and $\delta$=0.6. For each inflation series 20 series of unexpected inflation are simulated. Each of them is filtered 50 times using different starting values.

The $R^2$-statistics all lie between 0.96 and 0.99. The coefficient $\beta$ is very close to one without exception. The following table summarizes the key indicators for all 3000 test runs. It can clearly be seen that all results have been very precise.

**Small sample** Using the inflation time series with $\delta = 0.2$ the algorithm was tested for small samples of 20 years and 10 countries. 50 panels of unexpected inflation have been simulated, each has been tested 50 times. On average the $\beta$ coefficient is still close to 1. However, the dispersion is larger. The bias is sample specific. That is, all $\beta$ estimates are very close to each other for a given simulated sample. The broader distribution is almost exclusively due to variations between samples. Obviously, the algorithm still converges, but there is a sample specific distortion in small samples.

**Specification including fixed effects**

Further tests were made based on following simulation processes for the real interest series:

First with country specific effects:

$$(r_{it} - \tilde{r}_{it}) = -\delta\pi_{it}^{exp} - (\pi_{it} - \pi_{it}^{exp}) + u_i \tag{15}$$

Second, with country and time specific effects:

$$(r_{it} - \tilde{r}_{it}) = -\delta\pi_{it}^{exp} - (\pi_{it} - \pi_{it}^{exp}) + u_i + v_t \tag{16}$$

Third, with country and time specific effects, the country specific effects being correlated with the country specific effect of the inflation time series:

$$(r_{it} - \tilde{r}_{it}) = -\delta\pi_{it}^{exp} - (\pi_{it} - \pi_{it}^{exp}) + u_i + v_t \tag{17}$$

$$u_i = \varepsilon_i + 0.3\bar{\pi}_i$$

Using a simulated inflation series with $\delta = 0.2$ each of these specifications has been tested 1000 times for a big sample (n=100, t=100) and 2500 times for a small sample (n=10, t=20). Big and small sample test were performed using 50 different time series of unexpected inflation. The results do almost precisely match what we know from the tests without fixed effects. The mechanism obviously is not substantially affected by fixed effects of any kind.

# Appendix C: Country plots of inflation, expectations and ARMA(1,1)-Forecasts