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Basel III Capital Requirements and Heterogeneous Banks

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Abstract

I develop a theoretical model to examine the effect of capital requirements on risk taking and market structure of banks. Within a portfolio choice model, I allow for heterogeneous productivity among banks and consider the simultaneous capital regulation with a leverage ratio and a risk-weighted ratio. Regulators face a trade-off between the efficient allocation of resources and financial stability. In an oligopolistic market, risk-weighted requirements incentivise banks with high productivity to lend to low-risk firms. When a leverage ratio is introduced, these banks lose market shares to less productive competitors and react with risk-shifting into high-risk loans. While average productivity in the low-risk market falls, market shares in the high-risk market are dispersed across new entrants with high as well as low productivity.

Keywords: banking regulation, heterogeneous banks, banking competition, capital requirements, leverage ratio, Basel III

JEL classification: G11, G21, G28

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1 Introduction

Since the introduction of Basel III, banks are constrained by competing minimum capital requirements. Banks are subject to the revised risk-based capital framework of Basel II and the non risk-based leverage ratio. The intention of this dual approach was to curb model risk inherent in applied risk-weights and to counteract their pro-cyclicality (BCBS, 2010). This paper sheds light on unintended consequences, especially on the allocation of market shares.

Although new rules apply equally to all, competing capital requirements favor some banks at the expense of others. The simultaneity of both rules implies that the leverage ratio constraint binds only for some banks (BCBS, 2016). The question is, what kind of banks are affected. The rationale of capital requirements is to favor safe banks and charge risky banks. But being risky can be a feature of many traits. Still the question is, what kind of banks are risky.

To address this question, I develop a model with heterogeneous banks where differences in productivity determine banks' optimal strategies under competing capital constraints and hence riskiness. This paper leans on the idea, proposed in trade theory by Melitz (2003), that productivity differences play an important role in shaping firms' optimal strategies. Also empirical evidence by Delis et al. (2012) points out that banks' riskiness under capital requirements differs according to their productivity, size, and market share. I extend a portfolio choice model by adding heterogeneity in productivity among banks in the form of differences in marginal costs. Banks choose their strategy in a high-risk and a low-risk credit market with Cournot competition. I find that risk-weighted capital requirements incentivize banks with high productivity to specialize on low-risk loans. When the leverage ratio is introduced, these banks lose market shares in the low-risk market to less productive competitors and react with risk-shifting into high-risk loans as in Koehn and Santomero (1980), and Kim and Santomero (1988).

Theoretical work on capital requirements so far ignored the role of productivity in banks' decision about risk because studies focused on models with representative banks (VanHoose, 2007). Nevertheless, the relationship between productivity and risk taking received much attention in empirical work although the evidence is yet inconclusive. On the one hand, the efficiency-risk hypothesis[1] claims that more productive banks expect higher future profits and thus need a smaller cap-

[1]Note that empirical studies prefer the term efficiency over productivity, since most of them estimate the distance of a bank to the efficient production frontier. Nevertheless, it would be confusing to talk about efficiency in a theoretical context, since in a model every production decision is the result of an individual optimization.

ital buffer. Hence, they can afford a riskier strategy (Berger and di Patti, 2006; Altunbas et al., 2007). On the other hand, the charter-value hypothesis claims that more productive banks protect these higher profits by choosing less risky strategies (Fiordelisi et al., 2011). Therefore, it is unclear from the perspective of financial stability whether market shares should be allocated to the most productive banks. Due to frictions, e.g. asymmetric information and entry barriers, the banking industry is already prone to allocative inefficiency and X-inefficiency causing welfare losses (Vives, 2001a; Berger et al., 1993). If more productive banks were also safer banks, regulation should reallocate market shares to their favor. If not, a social planner might face a trade-off between an efficient allocation of resources and financial stability when setting new regulatory guidelines (Allen and Gale, 2004).

In this model, productivity creates positive charter value and market power. In the unregulated equilibrium, market shares are allocated according to productivity. The bank with the highest productivity is the market leader in the market for high-risk loans as well as in the market for low-risk loans. Since productivity differences are exogenous to the model, it can be categorized in the light of Efficient Structure theory pioneered by Demsetz (1973). The presence of risk-weighted capital requirements, however, introduces interdependence between both types of loans (Repullo and Suarez, 2004). As a consequence, banks with high productivity specialize on low-risk loans, and hence average productivity in the high-risk market is rather low. Capital requirements are not tailored to individual banks. On the contrary, they intend to provide a level playing field.[2] Therefore, banks with lower productivity do not have to provide more equity for taking the same risks, yet their default probabilities are higher due to lower charter values. Consequently, the Basel II equilibrium is characterized by concentration of high-risk investments in low-productivity banks. The introduction of the leverage ratio affects the allocation of market shares in both markets differently and tends to ameliorate this unwanted concentration. In the low-risk market, the most productive banks lose market shares to competitors with lower productivity so that average productivity falls. In the high-risk market, however, banks with low productivity lose market shares to banks with even lower productivity, that did not participate in any market previously, as well as to banks with higher productivity. In the Basel III equilibrium, the low-risk loan market is less concentrated.

I rely on the model of Repullo and Suarez (2004) used in Kiema and Jokivuolle

[2] At least Pillar 1 capital requirements are not tailored to individual banks depending on their productivity. Under the Supervisory Review and Evaluation Process in Pillar 2, regulators can levy additional bank specific requirements.

(2014) and extend it by introducing heterogeneity and an oligopolistic market.[3] Kiema and Jokivuolle (2014) model banks' optimal portfolio choice with Basel III capital requirements. As in Repullo and Suarez (2004) and this paper, banks specialize under Basel II. After the leverage ratio is introduced, low-risk banks choose a mixed portfolio so that, overall, bank portfolios are more alike. They find that the role of the leverage ratio as a backstop to model risk is impeded by less diverse portfolio choices. A recent paper by Smith et al. (2017) also examines banks' risk choices under the competing rules and evaluates whether the leverage ratio effectively reduces the probability of insolvency. They contrast the risk-taking incentives of the leverage ratio with the increase of loss absorbing capital and show that the positive effect of higher capital outweighs the negative effect of increased risk-taking. They test their implications empirically and find that banks become more stable after the announcement of the leverage ratio. I find a similar result which indicates that the leverage ratio can contribute to financial stability. In switching from the Basel II to the Basel III equilibrium in my model, default probabilities of most banks decline, at least as long as realizations of a common systematic risk-factor not exceed a threshold.[4] Beyond this threshold, default rates in the high-risk market are so high that even the most productive banks are closer to default.

Thus my work contributes to the literature on capital requirements and risk, in particular to the recent literature on the interaction of competing capital requirements. Wu and Zhao (2016) and Blum (2008) show that the leverage ratio complements the risk-weighted ratio given that banks are opaque and able to misreport their actual risk level to the regulators. Brei and Gambacorta (2016) and Gambacorta and Karmakar (2016) study the joint effect of both requirements and demonstrate the countercyclical quality of the leverage ratio. Furthermore, I contribute to the literature which is using heterogeneous banks. Apart from macroeconomic models with heterogeneous agents, e.g. Choi et al. (2015), only few microeconomic banking models consider heterogeneity. Closest to my results, Barth and Seckinger (2018) demonstrate that stricter capital requirements in the form of a leverage ratio induce agents with lower monitoring ability to become banks and thus that the average ability in the banking market deteriorates. Other studies consider two distinct types of banks. Hakenes and Schnabel (2011) find that smaller banks take more risks if big banks have a competitive advantage by choosing the internal ratings-based

[3]In perfect competition with productivity differences the most productive bank which has the lowest marginal costs would effectively be a monopolist.

[4]For one subgroup of banks though, i.e. those that were specialized on high-risk loans in the Basel II equilibrium and switch to the mixed portfolio strategy in the new equilibrium, the reverse holds. Their default probabilities increase for moderate realizations of systematic risk.

over the standardized approach in the Basel II framework.

The remainder of this paper is organized as follows. Section 2 introduces the main assumptions and setting of the model. Section 3 gives the baseline equilibrium without regulation. In Section 4 banking regulation is introduced and the equilibria with risk-weighted and competing capital requirements as well as comparative statics are derived. Section 5 discusses the results and possible limitations. Section 6 concludes.

2 The model

Consider a Cournot-Nash game with N banks competing in two markets. There is a market for low-risk loans and a market for high-risk loans. Banks have different unit costs and no fixed costs. Unit costs of bank i are denoted as c_i. In what follows, we rank banks according to their costs such that the bank with the lowest unit costs is denominated as bank 1 whereas bank N has the highest unit costs.

$$c_1 < c_2 < \cdots < c_N \tag{1}$$

Each market represents one of two types of entrepreneurs, risky and less risky entrepreneurs. Once in the game, there is perfect information about types but these costs can be interpreted as screening costs that banks have to incur in order to discern high- and low-risk entrepreneurs. Further, these costs reflect monitoring and administrative costs, such as employment of loan officers, back-office administration of the loan portfolio, or maintenance of monitoring processes. Therefore, low costs represent a more efficient production technology. Banks that are able to operate their loan portfolio at lower costs are more productive. The model introduces productivity differences of banks in the simplest form of differing cost functions.[5] This leads to asymmetric Nash-equilibria where optimal strategies depend on marginal costs.[6]

Let the strategy of bank i be $q_i = (q_{h,i}, q_{l,i})$. Let $Q_{-i} = (Q_{h,-i}, Q_{l,-i})$ denote aggregate quantities of all banks except bank i and $Q = (Q_h, Q_l)$ the total aggregate supply of loans in the respective markets. Aggregate supply determines inverse

[5] Heterogeneous productivity is exogenous in the model. This is inspired by trade models with heterogeneous firms (Melitz, 2003). It is applicable since I do not want to study what constitutes productivity differences among banks but rather how they influence the portfolio decision and distribution of market shares. Caveats concerning this assumption are discussed in Section 5.

[6] I assume that banks are perfectly informed about their own as well as their rivals' marginal cost. It was shown at least for the case of linear demand functions that full disclosure of costs is optimal in Cournot games with initial uncertainty about rivals' costs (Shapiro, 1986).

demand $r_\eta(Q_\eta)$[7] from entrepreneurs of type $\eta = \{h, l\}$. Inverse demand functions are continuous, monotone, and concave:

$$\begin{aligned} r_h(Q_h) &= r_h\left(\sum_{i=1}^{N} q_{h,i}\right) , & r_l(Q_l) &= r_l\left(\sum_{i=1}^{N} q_{l,i}\right) , \\ r'_h(Q_h) &< 0 , & r'_l(Q_l) &< 0 , \\ r''_h(Q_h) &\leq 0 , & r''_l(Q_l) &\leq 0 . \end{aligned} \tag{2}$$

Entrepreneurs demand a loan of size 1 if the interest rate is lower than their expected payoff. I assume expected payoffs are distributed such that it entails inverse demand functions of the described kind. Entrepreneurs, however, have limited liability. They repay the interest rate only if their projects are successful. If their project defaults, entrepreneurs pay nothing to the bank, i.e. loss given default is 1. Banks use average probabilities of success for each type of loan to take this into account.

To determine success probabilities of entrepreneurs, I use the representation by Repullo and Suarez (2004) and Kiema and Jokivuolle (2014) of the Vasicek model (Vasicek (1987),Vasicek (2002)). This risk model underpins the framework of risk-sensitive capital requirements of the Basel II accord. There is a common risk factor captured in z as well as idiosyncratic risk ϵ_j that are both standard normally distributed. Successes of high- and low-risk projects are correlated and ρ is the correlation parameter. The project of entrepreneur j is successful if a latent random variable $x_j \leq 0$, where

$$\begin{aligned} &x_j = \zeta_\eta + \sqrt{\rho}\, z + \sqrt{1-\rho}\, \epsilon_j \qquad \eta = \{h, l\} \\ &z \sim N(0,1), \quad \epsilon_j \sim N(0,1) . \end{aligned} \tag{3}$$

The two types differ in ζ_η which represents the financial vulnerability of entrepreneurs of type η and $0 < \zeta_l < \zeta_h$. If banks know the types of entrepreneurs, they know ζ_l and ζ_h. Consequently, the unconditionally expected probability to default of loans of type η is $\overline{PD}_\eta = \Phi(\zeta_\eta) = Pr(\zeta_\eta + \sqrt{\rho} z + \sqrt{1-\rho}\epsilon_j > 0)$, where Φ is the cumulative distribution function of the standard normal distribution. Let the expected probability of success be $p_\eta = 1 - \overline{PD}_\eta$, respectively. Note that $p_h < p_l$ since low-risk entrepreneurs are less likely to default. Assume that investing in the riskier project has a higher expected yield so that

$$1 < p_l r_l(Q_l) < p_h r_h(Q_h) \tag{4}$$

[7] All interest rates are absolute returns. Therefore, think of r_h as $1 + interest_h$, etc..

I assume depositors are insured and consequently ignorant of bank risk. They supply an inexhaustible amount of savings at an interest rate r_d. The deposit rate could be the value of an outside option of depositors, e.g. holding cash or a safe asset instead of investing their endowment in a bank. Depositors will then invest in banks whenever these offer a deposit rate at least as high as their outside option. For simpler notation, I define marginal costs as

$$MC_i = c_i + r_d \; . \tag{5}$$

Each banker is equally endowed with an amount of equity e. Let r_e denote the opportunity costs of equity capital and let it be higher than the opportunity costs of depositors, s.t. $r_d < r_e$.[8] Banks are only operated if expected profits from intermediation are higher than the outside option of bankers. Therefore, I assume that bankers have to invest their equity in the bank in order to employ the banking technology. Banks' balance sheet constraint is given by

$$e + d_i = q_{h,i} + q_{l,i} \; . \tag{6}$$

Let expected payoff of bank i be expected profits of intermediation minus opportunity costs given as

$$\Pi_i(q_i, d_i, e) = p_h r_h(Q_h) q_{h,i} + p_l r_l(Q_l) q_{l,i} - c_i(q_{h,i} + q_{l,i}) - r_d d_i - r_e e \; . \tag{7}$$

In addition, each bank has a capacity limit W_i which is finite but arbitrarily high so it cannot produce more than W_i in any market. This assumption ensures that banks' strategy sets are bounded in the unregulated case and is not crucial once regulation is introduced. Furthermore, banks are not allowed to take short positions in neither loans nor deposits, so that $q_i \geq 0$ and $d_i \geq 0$.

3 Unregulated equilibrium

Consider the optimization problem of a bank without capital requirements. By inserting eq. (5), and eq. (6) in eq. (7) and rearranging, the problem of bank i is

$$\begin{gathered} \operatorname*{Max}_{q_i} \Pi_i(q_i) \qquad \text{s.t.} \quad \Pi_i(q_i) \geq (r_e - r_d)e \quad \text{and} \quad 0 \leq q_i \leq W_i \quad \text{where} \\ \Pi_i(q_i) = (p_h r_h(Q_h) - MC_i) q_{h,i} + (p_l r_l(Q_l) - MC_i) q_{l,i} \end{gathered} \tag{8}$$

[8]This assumes that equity is costly contrary to the discussion in Admati et al. (2013).

Definition 1. A Cournot-Nash equilibrium in pure strategies is characterized by optimal strategy vector Q^* resulting in equilibrium interest rate vector $r^*(Q^*)$ with best-response correspondence such that

$$q_i^* = \arg\max \Pi_i(q_i, Q_{-i}^*) \quad \forall i \in \{1, \dots, N\}. \tag{9}$$

Given the assumption of concavity of inverse loan demand, the unregulated equilibrium exists. Similarly, the constrained equilibria, which are introduced in the next sections, must exist as well.

Lemma 1 (Existence of equilibria)**.** *The unregulated game, the game with a risk-weighted capital requirement, and the game with a leverage ratio and a risk-weighted capital requirement have at least one Nash-equilibrium in pure strategies.*

Proof. Proof is in the appendix. □

Because of the flat deposit rate due to the deposit insurance and the fact that debt financing is cheaper than equity financing, banks have strong incentives to increase their balance sheet size through levering. In a Cournot game though, competition ensures that bank size stays limited. If any bank expands its loan business the interest rates decrease for all banks so that competitors reduce their supply. Furthermore, the lower interest rates are, the fewer banks are able to participate in the loan market because some banks' marginal costs would be too high to make a profit. Consequently, the least productive banks do not provide loans in equilibrium and some less productive banks only provide loans in the high-risk market where expected revenues are higher.

By taking the first derivative of the profit function with respect to quantities $q_{h,i}$ and $q_{l,i}$, summing first-order conditions over all banks, and rearranging, the best response function of a bank i is[9]

$$q_{\eta,i}^* = \max\left[0, \frac{p_\eta r_\eta(Q_\eta^*) - MC_i}{\nu_\eta p_\eta r_\eta(Q_\eta^*) - \sum_{i=1}^{\nu_\eta} MC_i} Q_\eta^*\right], \tag{10}$$

where $\nu_\eta \in \{1, \dots, N\}$ denotes the least productive bank still active in market η, s.t. $q_{\eta,\nu_\eta} > 0$ while $q_{\eta,\nu_\eta+1} = 0$. Let $\kappa_{\eta,i}$ denote the market share of bank i in market η so Equation (10) can be written as $q_{\eta,i}^* = \kappa_{\eta,i}(MC_i)Q_\eta^*$.

Proposition 1 (Unregulated equilibrium)**.**
In an unregulated equilibrium more productive banks gain higher market shares in

[9] More details on the derivation of equilibrium are provided in the proof of Proposition 1.

both markets and are therefore bigger than less productive banks.

$$\kappa_{\eta,i}(MC_i) > \kappa_{\eta,i+1}(MC_{i+1}) \quad \forall i \in \{1, \dots, \nu_\eta\} \quad \eta \in \{h, l\} \,. \tag{11}$$

Proof. Proof is in the appendix. □

The fraction $\kappa_{\eta,i}$ equals the ratio of the rent that bank i can earn on a loan of type η relative to total rents earned in the market. All banks weight their revenue with the same unconditional success probabilities p_η and earn in equilibrium the same market interest rates. Therefore, bank 1 with the lowest marginal costs MC_1 has the highest market share in the market for low-risk loans and the market for high-risk loans, whereas bank ν_h has the lowest market share in the market for high-risk loans and its marginal costs MC_{ν_h} are only slightly smaller than or equal to the market interest rate $r_h(Q_h)$. Consequently, bank 1 has the biggest balance sheet and the highest debt-to-equity ratio. Therefore, in the unregulated equilibrium with Cournot competition and heterogeneous cost functions productivity advantages translate into scale and market power.[10]

A bank i defaults if the realization of systematic risk z is higher than the critical value $z_{i,crit}$ defined as

$$\pi_i(c_i, q_i, r(Q), z_{i,crit}) - r_d d_i(q_i, e) = 0 \tag{12}$$

where

$$\begin{aligned}\pi_i(c_i, q_i, r(Q), z) = &(1 - PD_l(z))\ r_l(Q_l) q_{l,i} + \\ &(1 - PD_h(z))\ r_h(Q_h) q_{h,i} - c_i(q_{h,i} + q_{l,i})\end{aligned}$$

and $PD_\eta(z)$ is the default probability of projects of type η conditional on the realization of systematic risk z. In a portfolio with many loans of type η with roughly equal size, the fraction of defaulting loans in such a portfolio converges to $PD_\eta(z)$ (Elizalde et al., 2005). Rearranging $\Pr(\zeta_\eta + \sqrt{\rho}\, z + \sqrt{1-\rho}\, \epsilon_j > 0)$ gives

$$PD_\eta(z) = \Pr\left(\epsilon_j > -\frac{\zeta_\eta + \sqrt{\rho} z}{\sqrt{1-\rho}}\right) = \Phi\left(\frac{\zeta_\eta + \sqrt{\rho}\, z}{\sqrt{1-\rho}}\right) \,. \tag{13}$$

Lemma 2 (Risk-taking in the unregulated equilibrium)**.** *In the unregulated equilibrium, banks with higher productivity have lower default probabilities. Formally,*

$$\hat{z}_i > \hat{z}_{i+1} \quad \forall i \in \{1, \dots, N\} \,. \tag{14}$$

[10] Since productivity differences are exogenous to the model, it can be categorized in the light of Efficient Structure theory pioneered by Demsetz (1973).

Proof. Proof is in the appendix. □

Under any continuous distribution of risk, here it is the standard normal distribution, extreme realizations of systematic or idiosyncratic risk are possible, so that default cannot be prevented with absolute certainty no matter how much loss absorbing capital is available to a bank. A regulator would try to avoid bank failures and banking crisis because these are associated with costs and loss of economic output (Laeven and Valencia, 2013). The micro-prudential approach of the Basel Committee is to set a maximal admissible default probability.

Assumption 1 (Necessity of regulation)**.** Given that the regulator sets maximal admissible default probability at α, I assume

$$Pr(z \leq \hat{z}_1) < 1 - \alpha \ . \tag{15}$$

Therefore, in the unregulated equilibrium the probability that all banks default is unacceptably high. Given that critical values are ordered according to Lemma 2, if systematic risk z realizes higher than critical value of some bank i, then bank i is expected to default and all banks with marginal costs higher than MC_i are expected to default as well. Hence, for any realization of z above $\hat{z}_1$, the whole banking system is expected to default. According to Eq. 15, the probability of this event happening is higher than α.

4 Regulating heterogeneous banks

4.1 Basel II equilibrium

The Basel II accord introduced risk-sensitive capital requirements to avoid the risk-shifting phenomenon described by Koehn and Santomero (1980), Kim and Santomero (1988) and others. They show that if capital requirements are not risk-sensitive, banks have incentives to shift their portfolio towards riskier assets. Following the Basel II approach for credit risk, banks must categorize their assets with respect to their riskiness into different buckets for which different risk-weights are applied. In the Standard Approach these weights are set by the regulator. In the Internal Ratings based Approach banks are allowed to use internal risk-models to provide expected default probabilities or more inputs, e.g. loss given default, for the calibration of the weights.

This model describes the IRB approach where default probabilities of loans of a certain type are used to calculate capital requirements. The model is static so that the maturity of all loans is one period. The risk-weighted requirement is constructed such that the probability that unexpected losses of the asset portfolio exceed available equity is lower than a threshold α, i.e. the admissible probability of default set by the regulator.[11] Let us assume the regulator sets α for some representative bank. As a results, equity is insufficient to cover unexpected losses with probability α for that bank.

The regulator infers the critical value of systematic risk $z_\alpha = \Phi^{-1}(1-\alpha)$ from eq. (12) such that $Pr(z \leq z_\alpha) = 1 - \alpha$. Consequently, if the representative bank holds at least $PD_\eta(z_\alpha)$ equity for each loan of type η, it is able to cover losses with probability $1-\alpha$. In detail, the capital requirement has two components: loan loss provisions for expected losses ($\overline{PD}_\eta$) and equity capital for unexpected losses ($PD(z) - \overline{PD}_\eta$). In this model the risk-adequate capital requirement for a loan of type η simplifies to

$$\beta_\eta = PD_\eta(z_\alpha) = \Phi\left(\frac{\zeta_\eta + \sqrt{\rho}\,\Phi^{-1}(1-\alpha)}{\sqrt{1-\rho}}\right) . \tag{16}$$

The requirement is additive for both types of loans given that banks hold a well-diversified portfolio within each class of loans (Vasicek, 2002). Since high-risk firms have a higher financial vulnerability ($\zeta_h > \zeta_l$), the capital requirement for high-risk loans is higher than for low-risk loans. The risk-weighted capital constraint of Basel II is given by

$$e \geq \beta_h q_{h,i} + \beta_l q_{l,i} \quad \textit{where} \quad 0 < \beta_l < \beta_h < 1 . \tag{17}$$

Adding the risk-weighted capital constraint to bank i's optimization problem and introducing μ_i as the shadow price of being constrained by the requirement gives

$$\begin{aligned} \underset{q_i,\mu_i}{\text{Max}} \quad & \Pi_i(q_i,\mu_i) = (p_h r_h(Q_h) - MC_i)q_{h,i} + (p_l r_l(Q_l) - MC_i)q_{l,i} \\ & \qquad\qquad\qquad - \mu_i\left(\beta_h q_{h,i} + \beta_l q_{l,i} - e\right) \\ \text{s.t.} \quad & \Pi_i(q_i,\mu_i) \geq (r_e - r_d)e \ , \quad 0 \leq q_i \leq W_i \ , \quad 0 \leq \mu_i \end{aligned} \tag{18}$$

Let $\Pi_i^s(q_{h,i}^s, q_{l,i}^s, \mu_i^s)$ denote the expected payoff of bank i implementing strategy s where $s = h$ when bank i specializes on high-risk loans, $s = l$ when the bank specializes on low-risk loans, $s = rw$ when the bank chooses a mixed strategy under a risk-weighted (rw) requirement, $s = uc$ when the bank chooses an unconstrained strategy, and $s = 0$ if the bank stays out of both markets. Figure 2 illustrates the

[11] Confer Kiema and Jokivuolle (2014) for a detailed account of how default probabilities are effectively restricted by Basel II capital requirements in a representative bank model.

notation and feasible strategies are

$$(q^s_{h,i}, q^s_{l,i}, \mu^s_i) = \begin{cases} \left(0, \dfrac{e}{\beta_l}, \mu^l_i\right) & \text{if} \quad s = l \ , \\ \left(\dfrac{e}{\beta_h}, 0, \mu^h_i\right) & \text{if} \quad s = h \ , \\ \left(q^{rw}_{h,i}, q^{rw}_{l,i}, \mu^{rw}_i\right) & \text{if} \quad s = rw \ , \\ \left(q^{uc}_{h,i}, q^{uc}_{l,i}, 0\right) & \text{if} \quad s = uc \ , \\ (0, 0, 0) & \text{if} \quad s = 0 \ , \end{cases} \tag{19}$$

where $q^{uc}_{\eta,i}$ is defined in eq. (10) and

$$\begin{aligned} q^{rw}_{h,i} &= \frac{(p_l r_l(Q_l) - MC_i) - \frac{\beta_l}{\beta_h}(p_h r_h(Q_h) - MC_i) + p_l r'_l(Q_l)\frac{e}{\beta_l}}{\frac{\beta_h}{\beta_l} p_l r'_l(Q_l) + \frac{\beta_l}{\beta_h} p_h r'_h(Q_h)} \\ q^{rw}_{l,i} &= \frac{(p_h r_h(Q_h) - MC_i) - \frac{\beta_h}{\beta_l}(p_l r_l(Q_l) - MC_i) + p_h r'_h(Q_h)\frac{e}{\beta_h}}{\frac{\beta_h}{\beta_l} p_l r'_l(Q_l) + \frac{\beta_l}{\beta_h} p_h r'_h(Q_h)} \ . \end{aligned} \tag{20}$$

Whereas in the unregulated equilibrium competitive pressures are the main force limiting bank size and determining the bank portfolio composition, under assumption 1 capital requirements pose much stricter limits on size and composition. They introduce interdependence between both types of loans. Because the requirement in eq. (17) is additive, banks enjoy no immediate advantage by diversifying their portfolio between asset classes. Therefore, a specialized portfolio is always better than a mixed portfolio strategy if it is feasible (Repullo and Suarez, 2004; Kiema and Jokivuolle, 2014). Moreover, by comparing Π^h_i and Π^l_i, whenever

$$p_l r_l(Q_l) - MC_i > \frac{\beta_l}{\beta_h}(p_h r_h(Q_h) - MC_i) \tag{21}$$

bank i has incentives to fully specialize on low-risk loans. Rearranging eq. (21) for MC_i gives the cutoff marginal costs of the bank with the lowest productivity which specializes on low-risk loans. It is therefore the cutoff of the low-risk market, denoted as $\widetilde{MC}^l$, and defined s.t.

$$\begin{aligned} &\Pi^l_i\left(0, \frac{e}{\beta_l}\right) \geq \Pi^h_i\left(\frac{e}{\beta_h}, 0\right) \qquad \forall\, i \in \{1, ..., N\} : MC_i \leq \widetilde{MC}^l \\ &\text{where} \quad \widetilde{MC}^l = \frac{\beta_h p_l r_l(Q_l) - \beta_l p_h r_h(Q_h)}{\beta_h - \beta_l} \ . \end{aligned} \tag{22}$$

An equilibrium can only exist if this cutoff is positive and there are banks that specialize on low-risk loans as well as banks that specialize on high-risk loans. It follows that in equilibrium capital requirements pose an upper bound on the interest rate on high-risk loans relative to the interest rate of low-risk loans, i.e.

$$\frac{p_h r_h(Q_h^*)}{p_l r_l(Q_l^*)} < \frac{\beta_h}{\beta_l} \; . \tag{23}$$

Furthermore, strategies l or h are only feasible as long as shadow prices are non-negative, i.e. $\mu_i \geq 0$ according to eq. (18). This is the case for all banks with marginal costs above $\widetilde{MC}^l$ if

$$-\frac{\beta_l}{\beta_h - \beta_l}(p_h r_h(Q_h^*) - p_l r_l(Q_l^*)) < \frac{e}{\beta_l} p_l r_l'(Q_l^*). \tag{24}$$

There is a negative relation between MC_i and μ_i. More productive banks are able to produce the highest quantities in an unregulated equilibrium, hence they face higher shadow prices of being constrained by capital requirements. Therefore, of all banks with marginal costs below $\widetilde{MC}^l$ only those with non-negative shadow prices are able to choose strategy h. Let the cutoff marginal costs of the least productive bank that specializes on high-risk loans and is fully constrained by the capital requirement be denoted as $\widetilde{MC}^{\mu_h}$ and defined as

$$\begin{aligned} &\Pi_i^l\left(0, \frac{e}{\beta_l}\right) < \Pi_i^h\left(\frac{e}{\beta_h}, 0\right) \;\wedge\; \mu_i^h \geq 0 \qquad \forall\, i \in \{1, ..., N\} : \widetilde{MC}^l < MC_i \leq \widetilde{MC}^{\mu_h} \\ &\text{where} \quad \widetilde{MC}^{\mu_h} = p_h r_h(Q_h) + \frac{e}{\beta_h} p_h r_h'(Q_h) \; . \end{aligned} \tag{25}$$

Still, banks with marginal costs higher than $\widetilde{MC}^{\mu_h}$ could be active in equilibrium. However, if

$$(p_h r_h(Q_h) - p_l r_l(Q_l)) < -\frac{e}{\beta_h} p_h r_h'(Q_h) \tag{26}$$

their shadow costs are negative when choosing strategy h as well as strategy rw. Hence, they choose an unconstrained strategy, i.e. $e > \beta_h q_{h,i} + \beta_l q_{l,i}$, as long as they meet their participation constraint. These banks specialize on high-risk loans since these offer a higher return. Let the cutoff marginal costs of the least productive bank that specializes on high-risk loans but is not constrained by the capital requirement be denoted as $\widetilde{MC}^h$ and defined as

$$\begin{aligned} &\Pi_i^{uc}\left(0, q_{h,i}^{uc}\right) - (r_e - r_d)e \geq 0 \;\wedge\; \mu_i^h < 0 \qquad \forall\, i \in \{1, ..., N\} : \widetilde{MC}^{\mu_h} < MC_i \leq \widetilde{MC}^h \\ &\text{where} \quad \widetilde{MC}^h = p_h r_h(Q_h) - ((-p_h r_h'(Q_h))(r_e - r_d)e)^{\frac{1}{2}} \; . \end{aligned} \tag{27}$$

We can call this also the cutoff in the high-risk market, since all banks with marginal

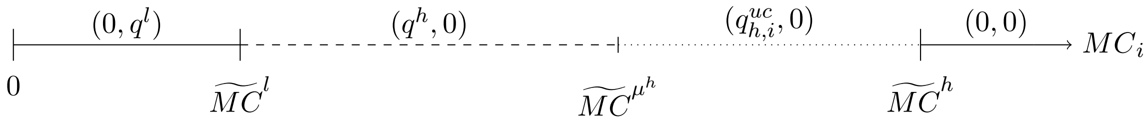

Figure 1: Optimal strategies and cutoff marginal costs in the Basel II equilibrium.

costs below the cutoff in the low-risk market specialize on high-risk loans while some are fully constrained by the risk-weighted capital requirements and some are not constrained.

The equilibrium is illustrated in Figure 1 and is summarized in Proposition 2.

Proposition 2 (Basel II equilibrium)**.**
Consider the case with additive risk-weighted capital requirements. If eq. (23)*, (24), and (26) hold, then banks with marginal costs* $MC_i \leq \widetilde{MC}^l$ *specialize on low-risk loans while less productive banks with marginal costs* $\widetilde{MC}^l < MC_i \leq \widetilde{MC}^h$ *specialize on high-risk loans in equilibrium.*

Proof. It follows from eq. (22) and the arguments above. Detailed proof is in the appendix □

Given these equilibrium strategies, it is possible to determine default probabilities. The direct effect of productivity advantages on the critical value of systematic risk $z_{i,crit}$ which is defined in eq. (12) is positive, i.e. banks with lower marginal costs *ceteris paribus* have higher profits. Positive profits constitute positive charter value and add to loss absorbing capacity. Therefore, when comparing banks that specialize on the same type of loans, the relationship between productivity and default probability is straightforward. These banks have the same strategy and earn the same interest rate. Hence, banks with lower marginal costs have lower default probabilities than banks with higher marginal costs that are active in the same loan market.

When comparing specialists on the high-risk and low-risk market, the relationship between productivity and default probabilities is not straightforward. On the one hand, high-risk specialists have a riskier investment strategy and higher costs. On the other hand, they are less levered and earn a higher return on their non-defaulting loans. If we impose a stricter limit on the upper bound of the high-risk market interest rate than eq. (23) and therewith limit the influence of the interest rate channel, a relationship can be clearly stated. In that case, the direct cost channel and the portfolio channel outweigh the leverage channel, so that banks with higher productivity are definitely less likely to default. Lemma 3 summarizes.

Lemma 3 (Risk taking in the Basel II equilibrium)**.** *In equilibrium, more productive banks have lower default probabilities than less productive banks in the same market, i.e.*

$$
\begin{aligned}
z_{i,crit} &> z_{i+1,crit} \quad \forall\, i \in \{1, ..., \nu_l\} : \; q^*_{l,i} > 0 \\
z_{i,crit} &> z_{i+1,crit} \quad \forall\, i \in \{\nu_l + 1, ..., \nu_h\} : \; q^*_{h,i} > 0 \, .
\end{aligned} \tag{28}
$$

If $p_h r_h(Q^*_h) < \frac{\beta_h}{\beta_l} p_h r_l(Q^*_l)$*, more productive banks have lower default probabilities even across markets, i.e.*

$$
z_{i,crit} > z_{i+1,crit} \quad \forall\, i \in \{1, ..., N\} : \; q^*_i > 0 \, . \tag{29}
$$

Proof. Proof is in the appendix. □

4.2 Basel III equilibrium

Among other measures aimed at capital adequacy, the Basel III accord introduced the leverage ratio. The motives of the regulator were driven by macro- as well as micro-prudential considerations. In order to comply, banks need to back up 3% of their total exposure with Tier 1 equity capital. Total exposure includes on-balance as well as off-balance sheet assets. The leverage ratio capital constraint of Basel III is given by β according to

$$
e \geq \beta\,(q_{h,i} + q_{l,i}) \quad \textit{where} \quad 0 < \beta_l < \beta < \beta_h < 1 \, . \tag{30}
$$

Adding the leverage ratio to the risk-weighted capital constraint in bank i's optimization problem and introducing λ_i as the shadow price of being constrained by the leverage ratio gives

$$
\begin{aligned}
\underset{q_i, \mu_i, \lambda_i}{\text{Max}} \quad & \Pi_i(q_i, \mu_i, \lambda_i) = (p_h r_h(Q_h) - MC_i) q_{h,i} + (p_l r_l(Q_l) - MC_i) q_{l,i} \\
& \qquad - \mu_i\,(\beta_h q_{h,i} + \beta_l q_{l,i} - e) - \lambda_i\,(\beta\,(q_{h,i} + q_{l,i}) - e) \\
\text{s.t.} \quad & \Pi_i(q_i, \mu_i, \lambda_i) \geq (r_e - r_d) e \, , \quad 0 \leq q_i \leq W_i \, , \quad 0 \leq \mu_i \, , \quad 0 \leq \lambda_i
\end{aligned} \tag{31}
$$

The additional constraint reduces the set of feasible strategies. The shaded area including the bounding line segments in fig. 2 illustrates the set of feasible strategies of bank i. Full specialization on low-risk loans as given in strategy l according to eq. (19) is not feasible under the leverage ratio. Adjusting strategy l and adding a mixed portfolio strategy lr for banks constrained by the leverage ratio as well as strategy v for banks constrained by both ratios simultaneously, feasible strategies

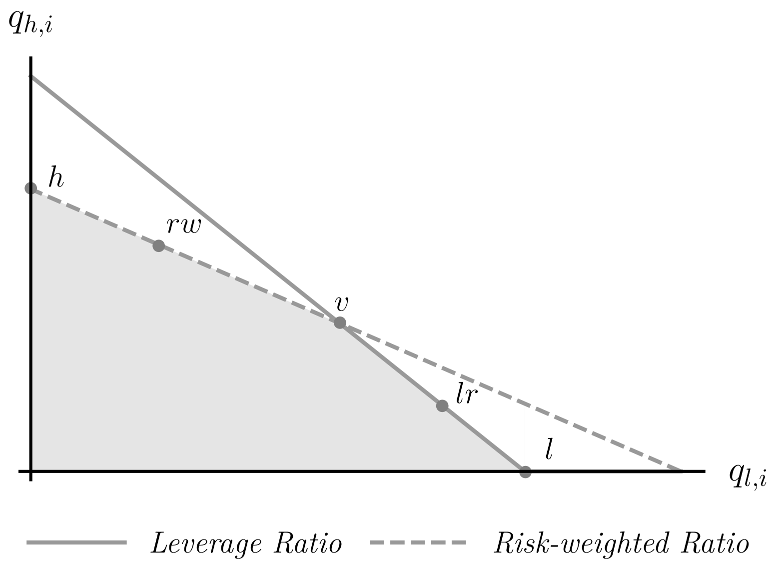

Figure 2: Feasible quantities under both capital requirements.

are denoted by

$$(q_{h,i}, q_{l,i}, \mu_i, \lambda_i) = \begin{cases} \left(0, \dfrac{e}{\beta}, 0, \lambda_i\right) & \text{if} \quad s = l \ , \\ \left(\dfrac{e}{\beta_h}, 0, \mu_i^h, 0\right) & \text{if} \quad s = h \ , \\ \left(q_{h,i}^{rw}, q_{l,i}^{rw}, \mu_i^{rw}, 0\right) & \text{if} \quad s = rw \ , \\ \left(q_{h,i}^{lr}, q_{l,i}^{lr}, 0, \lambda_i\right) & \text{if} \quad s = lr \ , \\ \left(\dfrac{(\beta - \beta_l)e}{\beta(\beta_h - \beta_l)}, \dfrac{(\beta_h - \beta)e}{\beta(\beta_h - \beta_l)}, \mu_i^{rw}, \lambda_i\right) & \text{if} \quad s = v \ , \\ \left(q_{h,i}^{uc}, q_{l,i}^{uc}, 0, 0\right) & \text{if} \quad s = uc \ , \\ (0, 0, 0, 0) & \text{if} \quad s = 0 \ , \end{cases} \tag{32}$$

where $q_{\eta,i}^{uc}$ is defined in eq. (10), $q_{\eta,i}^{rw}$ is defined in eq. (20), and

$$\begin{aligned} q_{h,i}^{lr} &= \frac{-\beta\left(p_h r_h(Q_h) - p_l r_l(Q_l)\right) + p_l r_l'(Q_l)e}{\beta\left(p_l r_l'(Q_l) + p_h r_h'(Q_h)\right)} \\ q_{l,i}^{lr} &= \frac{\beta\left(p_h r_h(Q_h) - p_l r_l(Q_l)\right) + p_h r_h'(Q_h)e}{\beta\left(p_l r_l'(Q_l) + p_h r_h'(Q_h)\right)} \end{aligned} \tag{33}$$

Since the leverage ratio poses extra costs on banks specializing on low-risk loans, it sets incentives to shift the portfolio toward riskier assets. Therefore, a mixed strategy is better for banks that previously specialized on low-risk loans. These banks change their strategy to strategy v which is the mixed portfolio exactly on the vertex in fig. 2 where both constraints are binding. For the remainder of banks it is still optimal to specialize on high-risk loans as long as it is feasible. Let $\widetilde{MC^l}$ denote the cutoff marginal costs between banks choosing strategy v and banks choosing strategy h. Since only banks that choose strategy v offer loans to low-risk entrepreneurs, $\widetilde{MC^l}$ defines the marginal costs of the bank with the lowest productivity that still participates in the low-risk market. Let $\widetilde{MC^l}$ be defined by

$$\begin{aligned} &\Pi_i^v\left(q_h^v, q_l^v\right) \geq \Pi_i^h\left(q_h^h, 0\right) \qquad \forall\, i \in \{1, ..., N\} : MC_i \leq \widetilde{MC^l} \\ &\text{where} \quad \widetilde{MC^l} = \frac{\beta_h p_l r_l(Q_l) - \beta_l p_h r_h(Q_h)}{\beta_h - \beta_l}\ . \end{aligned} \tag{34}$$

But strategy v is only feasible as long as shadow prices are non-negative, i.e. $\mu_i^v \geq 0$ according to eq. (18). This is the case for all banks with marginal costs above $\widetilde{MC^l}$ if

$$\tfrac{\beta(\beta_h-\beta_l)}{\beta_h(\beta_h-\beta)}\left(\beta_l p_h r_h(Q*_h) - \beta_h p_l r_l(Q*_l)\right) + \tfrac{\beta_l(\beta-\beta_l)}{\beta_h(\beta_h-\beta)} p_h r_h'(Q_h^*)e < p_l r_l'(Q_l^*)e\ . \tag{35}$$

Strategies l and lr that both have a higher share of low-risk loans than strategy v are strictly dominated by strategy v irrespective of banks' marginal costs. Furthermore, $\Pi_i^h \geq \Pi_i^{rw}$ for all banks with marginal costs higher than $\widetilde{MC^l}$. Therefore, these banks specialize on high-risk loans. Let the least productive bank that specializes on high-risk loans and is fully constrained by capital requirements define the cutoff $\widetilde{MC}^{\mu_h}$ as

$$\begin{aligned} &\Pi_i^v\left(q_h^v, q_l^v\right) < \Pi_i^h\left(\frac{e}{\beta_h}, 0\right) \ \wedge\ \mu_i^h \geq 0 \qquad \forall\, i \in \{1, ..., N\} : \widetilde{MC^l} < MC_i \leq \widetilde{MC}^{\mu_h} \\ &\text{where} \quad \widetilde{MC}^{\mu_h} = p_h r_h(Q_h) + \frac{e}{\beta_h} p_h r_h'(Q_h)\ . \end{aligned} \tag{36}$$

As in the Basel II equilibrium, banks with marginal costs above $\widetilde{MC}^{\mu_h}$ participate in the market if they are able to make a profit. If

$$-\beta_h\left(p_h r_h(Q_h^*) - p_l r_l(Q*_l)\right) < p_h r_h'(Q_h^*)e < -\beta\left(p_h r_h(Q*_h) - p_l r_l(Q*_l)\right) \tag{37}$$

their shadow costs when choosing strategy rw, l, or lr would be negative as well. Furthermore, it implies that $MC_i > p_l r_l(Q_l)$. Therefore, banks with marginal costs higher than $\widetilde{MC}^{\mu_h}$ specialize on high-risk loans as well. Let the marginal costs of the least productive bank to do so denote the cutoff marginal costs in the high-risk market as $\widetilde{MC}^h$ which is defined by

$$\Pi_i^{uc}\left(0, q_{h,i}^{uc}\right) - (r_e - r_d)e \geq 0 \ \wedge \ \mu_i^h < 0 \qquad \forall\, i \in \{1, ..., N\} : \widetilde{MC^{\mu_h}} < MC_i \leq \widetilde{MC}^h$$
$$\text{where} \quad \widetilde{MC}^h = p_h r_h(Q_h) - ((-p_h r_h'(Q_h))(r_e - r_d)e)^{\frac{1}{2}} \ . \tag{38}$$

The Basel III equilibrium is illustrated in the lower half of Figure 3 and is summarized in the following proposition.

Proposition 3 (Basel III equilibrium).
Consider the case with additive risk-weighted capital requirements and a leverage ratio. If eq. (23)*, (35), and (37) hold, then banks with marginal costs* $MC_i \leq \widetilde{MC^l}$ *hold a mixed portfolio while less productive banks with marginal costs* $\widetilde{MC^l} < MC_i \leq \widetilde{MC}^h$ *specialize on high-risk loans in equilibrium.*

Proof. Proof is in the appendix. □

Note that the cutoffs defined above for the Basel III equilibrium are only formally the same as for the Basel II equilibrium in eq. (22), (25), and (27). Because the interest rates in both equilibria are not necessarily the same, the values of these cutoffs differ between the Basel II and Basel III equilibrium. In fact, the number of banks in the low-risk market can only increase and therefore the number of active banks in the high-risk market increases as well.

Corollary 1 (Change in market cutoff marginal costs)**.** Comparing the portfolio choices in the Basel II and Basel III equilibrium, the cutoffs for marginal costs increase, i.e.

$$\widetilde{MC^l}^{BaselII} < \widetilde{MC^l}^{BaselIII} \tag{39}$$

and

$$\widetilde{MC^h}^{BaselII} < \widetilde{MC^h}^{BaselIII} \tag{40}$$

Proof. Proof is in the appendix. □

Proposition 4 (Market share reallocation and average productivity).
By tightening capital requirements through the introduction of a leverage ratio, market shares in the low-risk market are reallocated towards less productive banks while market shares in the high-risk market are reallocated towards more productive banks and less productive new entrants. Because of these entrants, the banking market has a lower average productivity.

Proof. Proof follows directly from Proposition 3 and Corollary 1. □

The results of Corollary 1 are illustrated in Figure 3. Taking the order of N banks according to their marginal costs, I distinguish six groups of banks according to whether they are affected or unaffected by the leverage ratio (i.e. whether they change their strategies between the Basel II and Basel III equilibrium) and whether they are constrained or unconstrained: (i - solid line segment) low-risk market incumbents, (ii - dashed) affected constrained high-risk market incumbents, (iii - solid) unaffected constrained high-risk market incumbents, (iv - dashdotted) affected unconstrained high-risk market incumbents, (v - solid) unaffected unconstrained high-risk market incumbents, and (vi - dotted) new entrants.

The most productive banks are the low-risk market incumbents (i). Their business model is affected directly by the leverage ratio. They react by shifting their portfolio and choosing the mixed strategy v. Thereby they reduce their supply of low-risk loans in order to compensate the additional cost of being constrained with higher loan rates which are available in the high-risk market. This in turn makes the low-risk market attractive for less productive banks that shift from a specialized high-risk into a mixed portfolio strategy (ii). The high-risk market gets more competitive as more productive banks enter it. In a Cournot-equilibrium with asymmetric costs, an increase in the number of banks in a market implies that supply is reduced and prices increase. This phenomenon is termed "anti-competitive" behavior by Amir and Lambson (2000).[12] Some specialized banks in the high-risk market are unaffected by the leverage ratio and do not change their strategy (iii), although they profit from the increase in the high-risk interest rate. Formerly unconstrained banks are able to increase their supply of loans so that some of them grow to point where they are constrained by the risk-weighted ratio (iv) and others grow as well but less (v). Finally, since expected revenue in the high-risk market is higher in the new equilibrium, new banks enter the high-risk market (vi). As a result, market shares are reallocated between heterogeneous banks. More productive banks lose market shares in the market for low-risk loans but gain shares in the other market. Less productive high-risk markets incumbents lose market shares.

The reallocation of market shares in the low-risk market implies that the average productivity of banks participating in that market decreases. On the other hand average productivity in the high-risk market might increase, i.e. if the number of new entrants is relatively small. In the unregulated equilibrium, the most productive

[12]To rationalize this, consider that the competitive outcome is achievable in this model if the most productive bank 1 chooses to push every other bank out of the market by producing very high quantities at its marginal costs. Therefore, the more banks are active in equilibrium, the closer market outcomes are to monopoly outcomes. See sec. 5 for a discussion on how crucial the Cournot market is for the results.

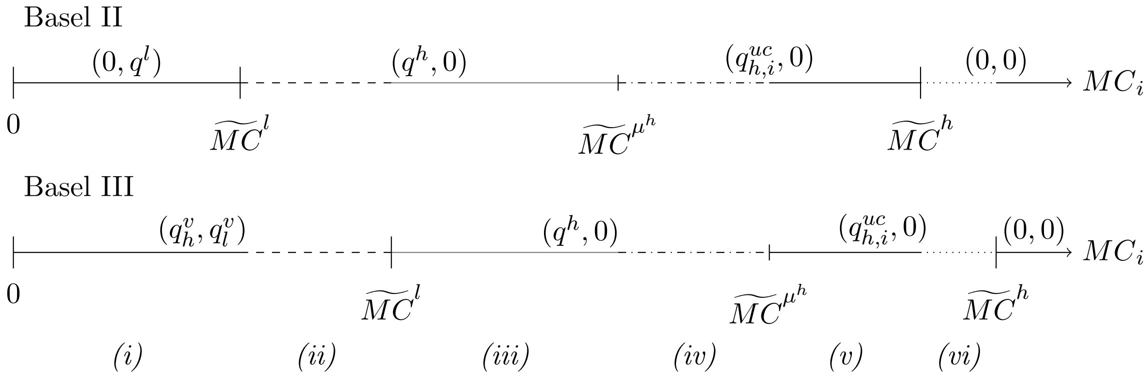

Figure 3: Optimal strategies and cutoff marginal costs in the Basel II equilibrium (upper line) and the Basel III equilibrium (lower line). Roman numbers on the bottom indicate groups of banks according to their change in strategy from the Basel II to Basel III equilibrium.

banks dominate both markets. Hence, any capital requirement indirectly protects market shares of less productive banks in the affected market.

Another implication of the model is that the regulator faces a trade-off between ample credit supply and higher equity ratios. As mentioned above, in order to cope with the additional capital requirement banks reduce aggregate credit supply in both markets so that they are able to maintain profitability.

Lemma 4 (Effect on interest rates)**.** *By tightening capital requirements through the introduction of the leverage ratio, aggregate loan supply decreases and interest rates increase in both markets.*

$$r_\eta(Q_\eta^{*BaselII}) < r_\eta(Q_v^{*BaselIII}) \quad \eta \in \{h, l\} \, . \tag{41}$$

Proof. Follows directly from Corollary 1. □

In terms of solvency, one might ask whether the risk-shifting of banks with high productivity increases their default probabilities as in Koehn and Santomero (1980) or if this effect is compensated by the increase of loss absorbing capital as in Smith et al. (2017). Besides the most productive banks that are directly affected by the leverage ratio and shift into the riskier loan class and reduce their leverage (group i), a subgroup of banks reacts in the opposite way (group ii). Because of the heterogeneity of banks, the effect of the leverage ratio differs between the six categories defined above. I focus on the first two groups (i) and (ii) because these change the portfolio composition of loans between the equilibria.

Lemma 5 (Risk shifting in the Basel III equilibrium)**.** *The risk-shifting channel introduced through the additional regulation by a risk insensitive capital requirement* *does not increase default probabilities as long as systematic risk realizes below a threshold $\tilde{z}$. Formally,*

$$\exists\ \tilde{z}\ : 0 < \tilde{z} < \hat{z} \quad s.t. \quad \begin{cases} \dfrac{\partial \Pi_i}{\partial \gamma_i} \geq 0 & if \quad z \leq \tilde{z} \\ \dfrac{\partial \Pi_i}{\partial \gamma_i} < 0 & if \quad z > \tilde{z} \end{cases}$$
$$where \quad \hat{z} = \frac{-\zeta_h^2 + \zeta_l^2 + 2\ln\left(\frac{r_h(Q_h)}{r_l(Q_l)}\right)(1-\rho)}{2\sqrt{\rho}\,(\zeta_h - \zeta_l)}. \tag{42}$$

Proof. Formal proof is in the appendix. □

Additional to the risk-shifting channel, the total effect on default probabilities depends on an interest rate channel and a leverage channel. As interest rates increase, profits of all banks c.p. increase rendering them more resilient.[13] Since the most productive banks of group (i) increase the share of high-risk loans which offer higher yields, the interest rate channel has a positive effect in reducing their likeliness to default. According to the leverage channel, banks are c.p. less likely to default if they finance their assets with a higher equity share. Since equity is normalized among banks, banks of group (i) increase their individual leverage ratio by reducing debt which makes them more resilient to default. To sum up, as long as systematic risk realizes below the threshold $\tilde{z}$, default probabilities of the most productive banks decrease even though they shift their portfolio into the riskier loan class. The opposite holds for banks of group (ii) for the leverage as well as interest rate channel.[14]

4.3 Comparative Statics

Next I consider how the aforementioned effects change if a stricter leverage ratio is implemented. The level of the leverage ratio directly determines the optimal portfolio structure and quantities of the most productive banks, i.e. strategy v, and indirectly interest rates and cutoffs in equilibrium.

Proposition 5 (Comparative statics)**.**
A tightening of the leverage ratio increases the cutoff marginal costs of the low-risk

[13]Default probabilities of banks of group (iii) therefore decrease. These banks profit from increasing rates but do neither change their portfolio composition nor size nor leverage.

[14]Unconstrained banks of group (iv) and (v) encounter ambiguous effects: Their default probabilities are reduced by increasing interest rates but increased through higher leverage since these banks are able to expand.

market, and decreases the cutoff marginal costs of the high-risk market, i.e.

$$\frac{\partial \widetilde{MC}^{l}}{\partial \beta} > 0 \; , \quad \frac{\partial \widetilde{MC}^{\mu_h}}{\partial \beta} < 0 \; , \quad \frac{\partial \widetilde{MC}^{h}}{\partial \beta} < 0 \tag{43}$$

Therefore, average productivity in the low-risk market decreases while average productivity in the high-risk market as well as the overall banking market increases.

Proof. Proof is in the appendix. □

A stricter leverage ratio ensures that less new entrants with low productivity enter the high-risk market so that average productivity in the banking market decreases less compared to the introduction of a more lenient leverage ratio. The reallocation of market shares within each market gets stronger. Market shares on the high-risk market are reallocated more strongly toward banks with high productivity and market shares on the low-risk market are reallocated more strongly toward banks with low productivity, respectively.

Lemma 6 (Comparative effect on interest rates)**.** *A tightening of the leverage ratio increases supply of high-risk loans but decreases supply of low-risk loans, i.e.*

$$\frac{\partial r_h(Q_h)}{\partial \beta} < 0 \; , \quad \frac{\partial r_l(Q_l)}{\partial \beta} > 0 \; . \tag{44}$$

Proof. Proof is in the appendix. □

Since banks with high productivity have to reduce a higher amount of debt with stricter leverage ratio, the supply of low-risk loans decreases even more than compared with a more lenient ratio while the supply of high-risk loans decreases less. On the other hand, this implies a stricter leverage ratio checks the increase of the interest rate for high-risk loans. Regarding the effect on banks' solvency, the positive effect of the interest rate channel is weakened while clearly the positive effect of the leverage channel is strengthened. Consequently, a tightening of the leverage ratio does not necessarily decrease default probabilities.

5 Discussion

The model highlights how regulation naturally interferes with regular market forces and thus creates side effects on financial stability. Productivity, irregardless of whether it stems from advantages in technology or information, influences banks' strategies and price setting. And ultimately, it influences market structure.

Regulators face a trade-off between assuring safety in the banking system and distorting competition. Banks should internalize risk-taking which is defined in various dimensions. Banks have different exposures to these dimensions. The model shows that these differences arise systematically due to the heterogeneity between banks. Therefore, as the regulator aims at confining risky banks it might as well narrow profitability of productive banks. Although unpleasant for a bank on its own, it can be seen as an exchange of intangible charter value into observable regulatory capital, both of which have a loss absorbing function.

A limitation to the model surely is the assumption that equity is fixed and the same amount for all banks. This serves to make banks comparable at some level. When in fact, productivity advantages and intangible charter value should be priced on the equity market in a way that more productive banks find it easier to refinance themselves. Increasing equity is an alternative strategy to risk-shifting as a reaction to the leverage ratio. Indeed, banks raised equity ever since the ratio was announced and monitored (Basel Committee on Banking Supervision, 2016; Smith et al., 2017) but investors should have been aware that the capital was needed to comply to tightened regulatory guidelines. However, for this model it would mean that the problem for more productive banks is moving from the product to the equity market. Loosening constraints by raising equity allows banks to move closer to an unregulated equilibrium where productivity sponsors market shares and size. Consequently, if a leverage ratio were to be binding for any bank at all, it still were binding for the more productive banks even if they do not change their portfolio composition as a response.

Another critical assumption is Cournot competition. While it plausibly implies that productivity produces market power in the form of market shares and profits, it implies that lower concentration comes along with less competitive outcomes (Amir and Lambson, 2000). Therefore, the set-up of the model is related to Efficient Structure theories. Such a relation between concentration and loan rates is confirmed by some empirical work, e.g., Jayaratne and Strahan (1998), yet it is challenged by as many (cf. VanHoose (2007) for a comprehensive literature review).

The focus of my work lies on the evaluation of capital requirements. In this light, you may note that the positive effect on less productive high-risk market incumbents' default probabilities hinges on exactly this anti-competitive behavior. In other settings, if banks had some price setting power –irrespective of the question of entry and exit– it is reasonable if they reacted by passing on costs to costumers by increasing loan rates. As long as excessive risk-taking is associated with high quantities, the regulator cannot avoid increasing financial stability at the expense of credit rationing.

In a competitive setting where banks cannot influence market loan rates, less productive banks would exit the market if new regulation causes additional costs. In fact, this is what happens when moving from the unregulated equilibrium to the Basel II equilibrium. But since banks are already constrained when the leverage ratio is introduced, they can circumvent incurring the costs of being regulated by adapting their business model and entering the high-risk market.

6 Conclusion

I study the optimal portfolio choice under competing capital requirements for heterogeneous banks. It points to the fact that productivity influences banks' exposures to risk systematically so that regulation indirectly affects certain types of banks. As a result, capital requirements shape the market structure in banking.

The model shows that the introduction of the leverage ratio in combination with the existing risk-weighted ratio directly affects banks with high productivity. This is because their productivity advantage induces them to chose a less risky strategy under risk-weighted regulation which can be operated at a higher scale. They react to the leverage ratio with risk-shifting. However, this higher share of high-risk loans does not increase their default probabilities, at least not as long as systematic risk is moderate. It induces a reallocation of market shares from more to less productive banks in the low-risk market. Average productivity in the low-risk market falls. These could be viewed as possible side effects of the current regulation. On the other hand, market shares in the high-risk market are distributed among a larger number of banks, including banks with high productivity. Compared to the Basel II equilibrium where high-risk loans are concentrated on low-productivity banks, this dispersion identified in my analysis highlights an unintended benefit of the new capital regulation regime.

Under the revision of the regulatory framework caused by the financial crisis numerous new instruments were implemented and discussed. It is important to consider the differential treatment caused by the interplay of different measures. The results could apply to other measures. For example, capital requirements on operational risk charge banks based on their gross income. While gross income is used as a proxy of risk caused by complexity, it is reasonable to assume that gross income depends on productivity as well. Productivity is hard to measure. Yet it can create positive charter value in an imperfect competitive environment. Since it might be a difficult to impossible task to formulate any requirements contingent on productivity in order to regulate heterogeneous banks, capital regulation should at least contemplate possible channels between productivity and risk. If risk measures are positively correlated to productivity measures, regulating these risks turns intangible charter value into observable capital. Generally, the banking market would be more transparent but not necessarily safer and market shares might be reshuffled. If on the other hand risk measures are negatively correlated to productivity, regulating these risks is more than called for. By using approaches with heterogeneous instead of representative banks, further theoretical work could systematically address the complex relationship between risk, capital, and productivity.

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Appendix

A.1 Proof of Lemma 1

Proof. This proof applies the results of Vives (2001b) and checks whether the conditions formulated therein are met in all games. According to Vives (2001b) Theorem 2.1, a Nash equilibrium for a game with strategy set Ω_i, payoffs Π_i, and players $i \in \{1, \dots, N\}$ exists, if

a) strategy sets Ω_i are non-empty, convex, and compact subsets of Euclidean space, and

b) payoff Π_i is continuous in the actions of all firms and

c) quasi-concave in its own action.

a) The strategy set of bank i consists of all possible quantities of loans. The model facilitates the view of a bank to a simple loan generating and deposit taking intermediary and therefore abstracts from other financial products where negative positions would be attainable. A potential strategy is therefore non-negative and the strategy set focuses on the upper right quadrant of $\mathbb{R}^2$ which is a non-empty convex set and subset of Euclidean space. Since zero is included in the strategy set, it is closed. Given a capacity limit $0 \leq q_i \leq W_i$, the set is bounded. The Heine-Borel theorem states that any bounded and closed subset of Euclidean space is also compact. Consequently, the first condition is met by an unregulated market.

The capital requirements essentially lower the upper bound on the strategy set. Both constraints are linear and define a triangle in $\mathbb{R}^2$, which is convex. Figure 2 illustrates both constraints. In the case of joint regulation with both constraints, the strategy set is an intersection of the two strategy sets of the preceding games which are both convex. Hence, their intersection is convex as well. In all constrained cases, they include the upper bound and zero as the lower bound. Consequently, strategy sets of the constrained games are non-empty, convex, and compact subsets of Euclidean space. Let the strategy set Ω_i be defined as

$$
\begin{aligned}
&\textit{(without constraints)} \quad && \Omega_i = \{q_i \mid 0 \leq q_i \leq W_i\} \\
&\textit{(risk-weighted)} && \Omega_i = \{q_i \mid 0 \leq \beta_h q_{h,i} + \beta_l q_{l,i} \leq e\} \\
&\textit{(both constraints)} && \Omega_i = \{q_i \mid 0 \leq \max\left[\beta_h q_{h,i} + \beta_l q_{l,i}, \beta(q_{h,i} + q_{l,i})\right] \leq e\}
\end{aligned}
$$

b) The payoff function of bank i is given as

$$\Pi_i(q_i) = (p_h r_h(Q_h) - MC_i)q_{h,i} + (p_l r_l(Q_l) - MC_i)q_{l,i}$$

where continuity follows from the continuity of its components. The inverse demand functions in $r(Q)$ are continuous by definition and q_i itself is continuous. Hence their product and difference is. Adding constraints was shown to alter the strategy space but not the payoff function. Therefore, the second condition for the existence of an equilibrium is fulfilled in all scenarios.

c) Profits are quasi-concave with respect to banks' own strategy choices, if all principal minors of the bordered Hessian matrix of $\Pi_i(q_i)$ are of alternating signs. Bordered Hessian of $\Pi(q_i)$ holding Q_{-i} constant is

$$H = \begin{pmatrix} 0 & \frac{\partial \Pi_i}{\partial q_{l,i}} & \frac{\partial \Pi_i}{\partial q_{h,i}} \\ \frac{\partial \Pi_i}{\partial q_{l,i}} & \frac{\partial^2 \Pi_i}{\partial q_{l,i}^2} & 0 \\ \frac{\partial \Pi_i}{\partial q_{h,i}} & 0 & \frac{\partial^2 \Pi_i}{\partial q_{h,i}^2} \end{pmatrix} .$$

The first principal minor is

$$-\left(\frac{\partial \Pi_i}{\partial q_{l,i}}\right)^2 \leq 0 \,,$$

which is non-positive by construction of H. The second principal minor is equal to the determinant of H which is

$$-\frac{\partial^2 \Pi_i}{\partial q_{l,i}^2}\left(\frac{\partial \Pi_i}{\partial q_{h,i}}\right)^2 - \frac{\partial^2 \Pi_i}{\partial q_{h,i}^2}\left(\frac{\partial \Pi_i}{\partial q_{l,i}}\right)^2 \geq 0 \,.$$

This is non-negative since

$$\frac{\partial^2 \Pi_i}{\partial q_{\eta,i}^2} = 2p_\eta \frac{\partial r_\eta}{\partial q_{\eta,i}} + p_\eta q_{\eta,i} \frac{\partial^2 r_\eta}{\partial q_{\eta,i}^2}$$

and inverse demand is concave so that $\frac{\partial r_\eta}{\partial q_{\eta,i}} < 0$ and $\frac{\partial^2 r_\eta}{\partial q_{\eta,i}^2} < 0$ (See assumption in eq. (2)). Therefore, Π_i is quasi-concave with respect to q_i. Constraints on the strategy set in form of capital requirements do not alter the profit function, hence the third condition for existence is fulfilled in all scenarios. We conclude that at least one Nash-equilibrium must exist in each game. □

A.2 Proof of Proposition 1

Proof. The Karush-Kuhn-Tucker conditions to the optimization problem given in eq. (8) for bank i in each market η are

$$\frac{\partial \Pi_i}{\partial q_{\eta,i}} \leq 0 \ , \tag{45}$$

$$q_{\eta,i} \frac{\partial \Pi_i}{\partial q_{\eta,i}} = 0 \ , \tag{46}$$

$$\Pi_i(q_i) - (r_e - r_d)e \geq 0 \tag{47}$$

$$W_i - q_i \geq 0 \tag{48}$$

$$q_i \geq 0 \tag{49}$$

where $\frac{\partial \Pi_i}{\partial q_{\eta,i}} = p_\eta(r_\eta(Q_\eta) + r'_\eta(Q_\eta)q_{\eta,i}) - MC_i$. From eq. (46) and (49), we know that banks either produce nothing or, if they supply a positive amount of loans, marginal profits must be zero. There are two markets to cater to, so banks decide on their participation and the extend of it in both markets. They do this separately, since the extend to which they choose to produce in one market does not affect their actions or the actions of other banks in the other market. Hence by solving eq. (45) for $q_{\eta,i}$ in equality, we get bank i's best reply function in market η

$$q_{\eta,i} = \max\left[0, \frac{p_\eta r_\eta(q_{\eta,i}, Q_{\eta,-i}) - MC_i}{-p_\eta r'_\eta(q_{\eta,i}, Q_{\eta,-i})}\right] \ . \tag{50}$$

Summing the FOCs for marginal profits in eq. (45) over all banks gives

$$p_\eta(\,N r_\eta(Q_\eta) + r'_\eta(Q_\eta)Q_\eta\,) - \sum_{i=1}^{N} MC_i \leq 0 \tag{51}$$

Let ν_η denote the bank with the highest marginal costs that is still able to supply loans at a profit in market η. We can rewrite eq. (51) as

$$p_\eta(\,\nu_\eta r_\eta(Q_\eta) + r'_\eta(Q_\eta)Q_\eta\,) - \sum_{i=1}^{\nu_\eta} MC_i = 0 \tag{52}$$

Solving eq. (52) for $r'_\eta(Q_\eta)$ and inserting into eq. (50), we get

$$q_{\eta,i}(Q_\eta) = \max\left[0, \frac{p_\eta r_\eta(Q_\eta) - MC_i}{\nu_\eta p_\eta r_\eta(Q_\eta) - \sum_{i=1}^{\nu_\eta} MC_i} Q_\eta\right] \ . \tag{53}$$

Because $p_h r_h(Q_h) > p_l r_l(Q_l)$ by assumption, if $q_{l,i} > 0$, then $q_{h,i} > 0$. Banks have three strategies: First, any bank that is able to offer low-risk loans at a profit supplies high-risk loans as well. Second, not every bank that is able to provide high-risk loans profitably incurs costs that are low enough to participate in the low-risk market. And third, some banks cannot participate in neither of the markets. Consider the bank ν_h with the lowest productivity which is still able and willing to participate. Assuming that $p_h r_h(Q_h) > MC_{\nu_h} > p_l r_l(Q_l)$, bank ν_h specializes on high risk loans. According to eq. (47) it must hold that

$$(p_h r_h(Q_h) - MC_{\nu_h})q_{h,\nu_h} - (r_e - r_d)e = 0\,. \tag{54}$$

Inserting eq. (50) given that $q_{h,\nu_h} > 0$ and solving for MC_{ν_h}, we get the cutoff marginal costs for the last bank producing in the high-risk market denoted as $\widetilde{MC}^h$ as

$$\widetilde{MC}^h = MC_{\nu_h} = p_h r_h(Q_h) - ((-p_h r_h'(Q_h))(r_e - r_d)e)^{\frac{1}{2}} \tag{55}$$

From Lemma 1 we know an equilibrium must exist. An equilibrium is characterized by best-response correspondence such that

$$q_i^* = \arg\max \Pi_i(q_i, Q_{-i}^*) \quad \forall i \in \{1, \dots, N\}\,. \tag{56}$$

Hence, if there is an equilibrium, optimal strategies of banks must be defined as

$$q_i^*(Q_{-i}^*) = \begin{cases} \left(\frac{p_h r_h^*(Q_h^*) - MC_i}{\nu_h p_h r_h^*(Q_h^*) - \sum_{i=1}^{\nu_h} MC_i} Q_h^*, \frac{p_l r_l^*(Q_l^*) - MC_i}{\nu_l p_l r_l^*(Q_l^*) - \sum_{i=1}^{\nu_l} MC_i} Q_l^*\right) & \text{if} \quad MC_i \le p_l r_l^*(Q_l^*) \\ \left(\frac{p_h r_h^*(Q_h^*) - MC_i}{\nu_h p_h r_h^*(Q_h^*) - \sum_{i=1}^{\nu_h} MC_i} Q_h^*, 0\right) & \text{if} \quad p_l r_l^*(Q_l^*) < MC_i \le \widetilde{MC}^h \\ (0,0) & \text{if} \quad \widetilde{MC}^h < MC_i. \end{cases} \tag{57}$$

Let $\kappa_{\eta,i} = \left(\frac{q_{\eta,i}}{Q_\eta}\right)$ denote the market share of bank i in market η. Then

$$\begin{gathered} \kappa_{\eta,i} > \kappa_{\eta,i+1} \\ \frac{p_\eta r_\eta(Q_\eta) - MC_i}{-p_\eta r_\eta'(Q_\eta)} > \frac{p_\eta r_\eta(Q_\eta) - MC_{i+1}}{-p_\eta r_\eta'(Q_\eta)} \\ MC_i < MC_{i+1} \end{gathered} \tag{58}$$

holds in both markets. Therefore, banks with higher marginal costs have lower market shares. Consequently,

$$\begin{aligned} q^*_{h,i} + q^*_{l,i} &> q^*_{h,i+1} + q^*_{l,i+1} \\ \kappa_{h,i} Q^*_h + \kappa_{l,i} Q^*_l &> \kappa_{h,i+1} Q^*_h + \kappa_{l,i+1} Q^*_l \\ (\kappa_{h,i} - \kappa_{h,i+1}) Q^*_h &> (\kappa_{l,i+1} - \kappa_{l,i}) Q^*_l \end{aligned} \tag{59}$$

is always true, because the left-hand side of the last inequality is positive while the right-hand side is always negative due to Equation (58). Hence, the higher the marginal costs of a bank, the smaller is its balance sheet.

□

A.3 Proof of Lemma 2

Proof. I show that $\Pi_i(c_i, q_i, r(Q), z)$ defined in eq. (12) has a unique root for mixed strategies. Note that in case of mixed strategies, i.e. if $q_{h,i} > 0$ and $q_{l,i} > 0$, eq. (12) cannot be solved for $z_{i,crit}$. But

$$\frac{\partial \Pi_i}{\partial z} = -r_h(Q_h) q^v_h \frac{\partial PD_h}{\partial z} - r_l(Q_l) q^v_l \frac{\partial PD_l}{\partial z} < 0 \tag{60}$$

with

$$\frac{\partial PD_\eta}{\partial z} = \sqrt{\frac{\rho}{1-\rho}} \phi \left(\frac{\zeta_\eta + \sqrt{\rho} z}{\sqrt{1-\rho}} \right) > 0 \tag{61}$$

so that $\Pi_i(c_i, q_i, r(Q), z)$ is a decreasing function. Furthermore, it is monotone due to the monotonicity of the CDF in $PD_\eta(z)$. We know from optimality conditions of an equilibrium solution that $\Pi_i(c_i, q_i, r(Q), 0) \geq 0$ (note that $1 - PD_\eta(0) = p_\eta$). Therefore, $\Pi_i(c_i, q_i, r(Q), z) - r_d d_i$ has a unique root at $z_{i,crit} \geq 0$. Hence $\hat{z}_i > \hat{z}_{i+1}$ if

$$\Pi_i(c_i, q_i, r(Q), z) > \Pi_{i+1}(c_i, q_i, r(Q), z) \quad \forall\ z\ \wedge\ i \in \{1, ..., \max[\nu_l, \nu_h]\} \tag{62}$$

Using eq. (12) and defining $p_\eta(z) = 1 - PD_\eta(z)$ we can write

$$\begin{aligned} &(p_l(z) r_l(Q^*_l) - c_i)\, q^*_{l,i} + (p_h(z)\, r_h(Q^*_h) - c_i)\, q^*_{h,i} > \\ &(p_l(z) r_l(Q^*_l) - c_{i+1})\, q^*_{l,i+1} + (p_h(z)\, r_h(Q^*_h) - c_{i+1})\, q^*_{h,i+1} \end{aligned} \tag{63}$$

which is always true given equilibrium strategies in eq. (57) because banks with lower marginal costs earn higher mark-ups and offer higher quantities.

□

A.4 Proof of Proposition 2

Proof. The proof is structured as follows. First, I compare all possible strategies to eliminate dominated strategies. Then, I derive the conditions for feasibility of the dominating strategies.

The Karush-Kuhn-Tucker conditions of eq. (18) are

$$\frac{\partial \Pi_i}{\partial q_{\eta,i}} \leq 0 \ , \tag{64}$$

$$\Pi_i(q_i) - (r_e - r_d)e \geq 0 \tag{65}$$

$$e - \beta_h q_{h,i} - \beta_l q_{l,i} \geq 0 \tag{66}$$

$$q_{\eta,i} \frac{\partial \Pi_i}{\partial q_{\eta,i}} = 0 \ , \ \mu_i \left(e - \beta_h q_{h,i} - \beta_l q_{l,i}\right) = 0 \tag{67}$$

$$q_i \geq 0 \ , \ \mu_i \geq 0 \tag{68}$$

I derive five possible strategies $s \in \{l, rw, h, uc, 0\}$ which are defined in eq. (19). If feasible, constrained strategies (l, rw, h) dominate the unconstrained strategy (uc) and clearly the non-participating strategy (0), since in the unconstrained strategy banks are left with unused equity. If no constrained strategy is feasible for a bank, but still $MC_i \leq \widetilde{MC}^h$, see eq. (27)), banks participate (see eq. (65)) with an unconstrained strategy.

From eq. (22) we know that banks with marginal costs below the cutoff prefer strategy l over h. Comparing l and rw gives

$$\begin{aligned}
\Pi_i^{rw}(q_i^{rw}) &< \Pi_i^l(q_l^l) \\
(p_h r_h - MC_i)q_{h,i}^{rw} + (p_l r_l - MC_i)q_{l,i}^{rw} &< (p_l r_l - MC_i)q_l^l \\
\frac{(p_h r_h - MC_i)}{(p_l r_l - MC_i)} &< \frac{q_l^l - q_{l,i}^{rw}}{q_{h,i}^{rw}} \\
\frac{(p_h r_h - MC_i)}{(p_l r_l - MC_i)} &< \frac{\beta_h}{\beta_l}
\end{aligned} \tag{69}$$

which is the same condition as in eq. (21). For the last step, I used the fact that eq. (66) holds with equality for strategy l and rw. Consequently, whenever strategy l dominates h, l dominates rw as well. It can be shown in a similar way, that whenever strategy h dominates l, it dominates rw as well. Hence, banks would never choose a mixed portfolio strategy if a specialization strategy is available.

Now, I derive conditions for feasibility of all strategies. First, the cutoff $\widetilde{MC}^l$ has to be positive. Otherwise all banks find it optimal to specialize on high-risk loans which can never be a Nash-equilibrium. Then supply of high-risk loans would be very high and the loan rate falls whereas there is no supply of low-risk loans so that the interest rate on low-risk loans rises and ultimately $p_h r_h > p_l r_l$ is violated or $\widetilde{MC}^l > 0$. The first condition is therefore

$$p_l r_l(Q_l^*) < p_h r_h(Q_h^*) < \frac{\beta_h}{\beta_l} p_l r_l(Q_l^*) \,. \tag{70}$$

Secondly, a strategy s is only feasible if $\mu_i^s \geq 0$, see eq. (68). The shadow prices are functions of marginal costs and market prices ($\mu_i^s[MC_i, r(Q)]$) which imply cutoffs $\widetilde{MC}^{\mu^s}$ which themselves have to be positive to be meaningful, s.t.

$$\mu_i^s \geq 0 \quad \forall i \in \{1, ..., N\} : MC_i \leq \widetilde{MC}^{\mu^s} \quad \text{where} \quad \widetilde{MC}^{\mu^s} > 0 \tag{71}$$

For strategies l and h this means that

$$\mu_i^l \geq 0 \quad \forall i \in \{1, ..., N\} : MC_i \leq \widetilde{MC}^{\mu^l} \text{ where } \widetilde{MC}^{\mu^l} = p_l r_l + \frac{e}{\beta_l} p_l r_l' > 0 \tag{72}$$

$$\mu_i^h \geq 0 \quad \forall i \in \{1, ..., N\} : MC_i \leq \widetilde{MC}^{\mu^h} \text{ where } \widetilde{MC}^{\mu^h} = p_h r_h + \frac{e}{\beta_h} p_h r_h' > 0 \tag{73}$$

Thirdly, the following conditions ensure that there is a certain order between feasibility cutoffs $\widetilde{MC}^{\mu^s}$ and dominance cutoffs $\widetilde{MC}^l$ and $\widetilde{MC}^h$. All banks with $MC_i \leq \widetilde{MC}^l$ can choose l only if

$$\begin{gathered} \widetilde{MC}^l < \widetilde{MC}^{\mu^l} \\ -\frac{\beta_l}{\beta_h - \beta_l}(p_h r_h(Q_h^*) - p_l r_l(Q_l^*)) < \frac{e}{\beta_l} p_l r_l'(Q_l^*) < 0 \,. \end{gathered} \tag{74}$$

All banks with $MC_i \leq \widetilde{MC}^{\mu^h}$ can choose h only if

$$\begin{gathered} \widetilde{MC}^{\mu^{rw}} < \widetilde{MC}^{\mu^h} \\ (p_l r_l(Q_l^*) - p_h r_h(Q_h^*)) < \frac{e}{\beta_h} p_h r_h'(Q_h^*) < 0 \,. \end{gathered} \tag{75}$$

Eq. (75) usefully implies that

$$\begin{aligned} p_l r_l(Q_l^*) &< p_h r_h(Q_h^*) + \frac{e}{\beta_h} p_h r_h'(Q_h^*) \\ p_l r_l(Q_l^*) &< \widetilde{MC}^{\mu^h} \end{aligned} \tag{76}$$

so that if banks choose the unconstrained strategy, they specialize on high-risk loans and are not able to supply low-risk loans profitably.

Furthermore, condition (74) is always stricter than condition (72), and condition (75) is always stricter than condition (73). Hence, given eq. (23),(74), and (75) optimal strategies in equilibrium are

$$(q^*_{h,i}, q^*_{l,i}, \mu^*_i) = \begin{cases} \left(0, q^l_l, \mu^l_i(MC_i)\right) & \text{if} \quad MC_i \leq \widetilde{MC}^l \\ \left(q^h_h, 0, \mu^h_i(MC_i)\right) & \text{if} \quad \widetilde{MC}^l < MC_i \leq \widetilde{MC}^{\mu^h} \\ \left(q^{uc}_{h,i}(MC_i), 0, 0\right) & \text{if} \quad \widetilde{MC}^{\mu^h} < MC_i \leq \widetilde{MC}^h \,. \end{cases} \tag{77}$$

□

A.5 Proof of Lemma 3

Proof. First, I show that within each strategy, banks with lower marginal costs have higher critical values and therefore lower default probabilities. For the specialized strategies, we can solve eq. (12) for $z^\eta_{i,crit}$ which is the critical value of bank i if it specializes on strategy η. Given equilibrium strategies and outcomes we get

$$\begin{aligned} \left(1 - PD_\eta(z^\eta_{i,crit})\right) \, r_\eta(Q^*_\eta) q^{\eta*}_\eta \, - MC_i q^{\eta*}_\eta + r_d e \, &= 0 \\ \left(1 - \Phi\left(\frac{\zeta_\eta + \sqrt{\rho}\, z^\eta_{i,crit}}{\sqrt{1-\rho}}\right)\right) \, r_\eta(Q^*_\eta)\frac{e}{\beta_\eta} - MC_i \frac{e}{\beta_\eta} + r_d e &= 0\,. \end{aligned} \tag{78}$$

Rearranging gives

$$z^\eta_{i,crit} = \frac{\sqrt{1-\rho}}{\sqrt{\rho}}\, \Phi^{-1}\left(1 - \frac{MC_i - r_d \dfrac{e}{q^{\eta*}_\eta}}{r_\eta(Q^*_\eta)}\right) - \frac{\zeta_\eta}{\sqrt{\rho}}\,. \tag{79}$$

Except MC_i, all parameters in eq. (79) are equal for banks with the same constrained equilibrium strategy. Taking the derivative with respect to MC_i gives

$$\frac{\partial z^\eta_{i,crit}}{\partial MC_i} = (-1)\, \frac{\sqrt{1-\rho}}{\sqrt{\rho}\, \phi\left(\Phi^{-1}\left(1 - \dfrac{MC_i - r_d \frac{e}{q^{\eta*}_\eta}}{r_\eta(Q^*_\eta)}\right)\right)} < 0 \tag{80}$$

where $\phi(x)$ is the PDF of the standard normal distribution. Therefore, if $MC_i < MC_{i+1}$, then $z^\eta_i > z^\eta_{i+1}$ for $\eta = \{h, l\}$.

For high-risk specialists that are not constrained (strategy uc), the parameters MC_i and q^{uc*}_h change in eq. (79). Simplifying $z^{uc}_{i,crit} > z^{uc}_{i+1,crit}$ yields

$$(MC_{i+1} - MC_i) q^{uc*}_{h,i+1} q^{uc*}_{h,i} > r_d e (q^{uc*}_{h,i+1} - q^{uc*}_{h,i}) \tag{81}$$

which is always true since $q^{uc*}_{h,i+1} - q^{uc*}_{h,i} < 0$. Hence, when comparing different banks with the same strategy, we find that within each market banks with lower marginal costs have higher critical values and therefore lower default probabilities.

Next, I show that within the same bank and given $p_h r_h(Q^*_h) < \frac{\beta_h}{\beta_l} p_h r_l(Q^*_l)$, strategies with a higher share of high-risk loans have a higher default probability. Let us now compare default probabilities of different strategies for one bank i. If $p_h r_h(Q^*_h) < \frac{\beta_h}{\beta_l} p_h r_l(Q^*_l)$, then

$$1 - \frac{MC_i - r_d\beta_l}{r_l(Q^*_l)} > 1 - \frac{MC_i - r_d\beta_h}{r_h(Q^*_h)} \tag{82}$$

and hence

$$\Phi^{-1}\left(1 - \frac{MC_i - r_d\beta_l}{r_l(Q^*_l)}\right) > \Phi^{-1}\left(1 - \frac{MC_i - r_d\beta_h}{r_h(Q^*_h)}\right) \tag{83}$$

so that the right hand side in the following is negative which ensures that it is true that

$$\frac{\zeta_h - \zeta_l}{\sqrt{1-\rho}} > \Phi^{-1}\left(1 - \frac{MC_i - r_d\beta_h}{r_h(Q^*_h)}\right) - \Phi^{-1}\left(1 - \frac{MC_i - r_d\beta_l}{r_l(Q^*_l)}\right) \tag{84}$$

and thus

$$z^l_{i,crit} > z^h_{i,crit} \ . \tag{85}$$

Since we know that $z^h_{i,crit} > z^h_{i+1,crit}$, we can compare the default probabilities of the least productive bank in the low-risk market ν_l (which has marginal cost just below or at the cutoff: $MC_{\nu_l} \leq \widetilde{MC}^l$) with the next bank ν_{l+1} that is the most productive bank in the high-risk market with $MC_{\nu_l+1} > \widetilde{MC}^l$, and state that

$$z^l_{1,crit} > \cdots > z^l_{\nu_l,crit} > z^h_{\nu_l,crit} > z^h_{\nu_l+1,crit} > \cdots > z^{uc}_{\nu_h,crit} \ . \tag{86}$$

$\square$

A.6 Proof of Proposition 3

Proof. The proof is structured similarly as the proof of Proposition 2. First, I compare all possible strategies to eliminate dominated strategies. Then, I derive conditions for feasibility of dominating strategies.

The Karush-Kuhn-Tucker conditions of eq. (31) are

$$\frac{\partial \Pi_i}{\partial q_{\eta,i}} \leq 0 \ , \tag{87}$$

$$\Pi_i(q_i) - (r_e - r_d)e \geq 0 \ , \tag{88}$$

$$
\begin{aligned}
& & e-\beta_h q_{h,i}-\beta_l q_{l,i} \geq 0\ , & \qquad e-\beta(q_{h,i}+q_{l,i}) \geq 0\ , \quad (89)\\
& q_{\eta,i}\frac{\partial \Pi_i}{\partial q_{\eta,i}} = 0\ , & \mu_i\left(e-\beta_h q_{h,i}-\beta_l q_{l,i}\right) = 0\ , & \qquad \lambda_i\left(e-\beta(q_{h,i}+q_{l,i})\right) = 0\ , \quad (90)\\
& q_i \geq 0\ , & \mu_i \geq 0\ , & \qquad \lambda_i \geq 0\ . \quad (91)
\end{aligned}
$$

If feasible, banks choose constrained over the unconstrained or the non-participating strategy. Comparing the payoff of strategies v and l gives

$$
\begin{aligned}
\Pi_i^v(q^v) &> \Pi_i^l(q^l)\\
p_h r_h q_h^v + p_l r_l q_l^v - MC_i(q_h^v+q_l^v) &> p_l r_l q^l - MC_i q^l\\
p_h r_h q_h^v - p_l r_l(q_i^l - q_l^v) &> 0\\
(p_h r_h - p_l r_l)q_h^v &> 0\ .
\end{aligned} \tag{92}
$$

Note that for all strategies constrained by the leverage ratio eq. (30) holds with equality so that bank i's costs are equal for strategies l,lr, and v. Furthermore, since $q_l^l = \frac{e}{\beta}$, from eq. (30) follows that $q_l^l - q_l^v = q_h^v$. Comparing the payoff of strategies v and lr gives

$$
\begin{aligned}
\Pi_i^v(q^v) &> \Pi_i^{lr}(q^{lr})\\
p_h r_h q_h^v + p_l r_l q_l^v - MC_i(q_h^v+q_l^v) &> p_h r_h q_h^{lr} + p_l r_l q_l^{lr} - MC_i(q_h^{lr}+q_l^{lr})\\
p_h r_h(q_h^v - q_h^{lr}) - p_l r_l(q_l^{lr} - q_l^v) &> 0\\
(p_h r_h - p_l r_l)(q_h^v - q_h^{lr}) &> 0
\end{aligned} \tag{93}
$$

For the last step, reckon that the leverage ratio constraint in eq. (30) holds with equality for strategies v and lr. Equation (93) and eq. (92) are true for all banks irregardless of MC_i. Hence, strategy v dominates strategies l and lr.

$$
\begin{aligned}
\Pi_i^v(q^v) &> \Pi_i^l(q^l) \qquad \forall MC_i\\
\Pi_i^v(q^v) &> \Pi_i^{lr}(q^{lr}) \qquad \forall MC_i
\end{aligned} \tag{94}
$$

Comparing strategy v to h gives the cutoff defined in eq. (34), and comparing it to strategy rw gives

$$\begin{aligned}
\Pi_i^{rw}(q_i^{rw}) &< \Pi_i^v(q^v) \\
(p_h r_h - MC_i)q_{h,i}^{rw} + (p_l r_l - MC_i)q_{l,i}^{rw} &< (p_h r_h - MC_i)q_h^v + (p_l r_l - MC_i)q_l^v \\
\frac{(p_h r_h - MC_i)}{(p_l r_l - MC_i)} &< \frac{q_l^v - q_{l,i}^{rw}}{q_{h,i}^{rw} - q_h^v} \\
\frac{(p_h r_h - MC_i)}{(p_l r_l - MC_i)} &< \frac{\beta_h}{\beta_l}
\end{aligned} \tag{95}$$

which gives the same cutoff as in eq. (34). For the last step, note that eq. (17) holds with equality for both strategies. Hence, strategy v only dominates strategies h and rw if marginal costs are below the cutoff, i.e.

$$\begin{aligned}
\Pi_i^v(q^v) > \Pi_i^h(q^h) \qquad & \forall MC_i : MC_i \leq \widetilde{MC}^l \\
\Pi_i^v(q^v) > \Pi_i^{rw}(q^{rw}) \qquad & \forall MC_i : MC_i \leq \widetilde{MC}^l
\end{aligned} \tag{96}$$

Comparing strategies h and rw gives

$$\begin{aligned}
\Pi_i^{rw}(q_i^{rw}) &< \Pi_i^h(q^h) \\
(p_h r_h - MC_i)q_{h,i}^{rw} + (p_l r_l - MC_i)q_{l,i}^{rw} &< (p_h r_h - MC_i)q_h^h \\
\frac{(p_h r_h - MC_i)}{(p_l r_l - MC_i)} &> \frac{q_{l,i}^{rw}}{q_{h,i}^{rw} - q_h^h} \\
\frac{(p_h r_h - MC_i)}{(p_l r_l - MC_i)} &> \frac{\beta_h}{\beta_l}
\end{aligned} \tag{97}$$

which again gives the same cutoff as in eq. (34). Hence,

$$\begin{aligned}
\Pi_i^h(q^h) > \Pi_i^{rw}(q_i^{rw}) \qquad & \forall i \in \{1, ..., N\} : MC_i > \widetilde{MC}^l \\
\Pi_i^h(q^h) > \Pi_i^v(q_i^v) \qquad & \forall i \in \{1, ..., N\} : MC_i > \widetilde{MC}^l .
\end{aligned} \tag{98}$$

Now, I derive conditions for feasibility of all strategies. Firstly, we need condition (23) to ensure that the cutoff $\widetilde{MC}^l$ separating strategy v and h is positive. Secondly, a strategy s is only feasible if $\mu_i^s \geq 0$ and $\lambda_i^s \geq 0$. Some shadow prices are functions of marginal costs and market prices ($\mu_i^s[MC_i, r(Q)]$ or $\lambda_i^s[MC_i, r(Q)]$) which imply cutoffs $\widetilde{MC}^{\mu^s}$ or $\widetilde{MC}^{\lambda^s}$ which themselves have to be positive to be meaningful, s.t.

$$\mu_i^s \geq 0 \quad \forall i \in \{1, ..., N\} : MC_i \leq \widetilde{MC}^{\mu^s} \quad \text{where} \quad \widetilde{MC}^{\mu^s} > 0 \tag{99}$$

For strategies v and h this means

$$\mu_i^h \geq 0 \; \forall i \in \{1, ..., N\} : MC_i \leq \widetilde{MC}^{\mu^h} \text{ where } \widetilde{MC}^{\mu^h} > 0 \tag{100}$$

$$\lambda_i^v \geq 0 \; \forall i \in \{1, ..., N\} : MC_i \leq \widetilde{MC}^{\lambda^v} \text{ where } \widetilde{MC}^{\lambda^v} > 0 \tag{101}$$

$$\mu^v \geq 0 \; \forall i \in \{1, ..., N\} \tag{102}$$

where

$$\widetilde{MC}^{\mu^h} = p_h r_h + \frac{e}{\beta_h} p_h r_h' \tag{103}$$

$$\widetilde{MC}^{\lambda^v} = \frac{\beta_h p_l r_l - \beta_l p_h r_h}{(\beta_h - \beta_l)} + \frac{\beta_h(\beta_h - \beta)}{\beta(\beta_h - \beta_l)^2} p_l r_l' e - \frac{\beta_l(\beta - \beta_l)}{\beta(\beta_h - \beta_l)^2} p_h r_h' e \tag{104}$$

$$\mu^v = \frac{p_h r_h - p_l r_l}{(\beta_h - \beta_l)} - \frac{(\beta_h - \beta)}{\beta(\beta_h - \beta_l)^2} p_l r_l' e + \frac{(\beta - \beta_l)}{\beta(\beta_h - \beta_l)^2} p_h r_h' e \tag{105}$$

Thirdly, strategies v and h should be viable for all banks for whom these strategies are profit maximizing. That is the case if

$$\widetilde{MC}^{l} < \widetilde{MC}^{\lambda^v} < \widetilde{MC}^{\mu^h} \tag{106}$$

$$\widetilde{MC}^{\mu^h} > \max\left[\widetilde{MC}^{\mu^{rw}}, \widetilde{MC}^{\lambda^l}, \widetilde{MC}^{\lambda^{lr}}, p_l r_l\right] . \tag{107}$$

The conditions given in eq. (100), (101), (102), (106), and (107) simplify to eq. (35) and (37) in the following way: Given (100) and (101), $\widetilde{MC}^{l} < \widetilde{MC}^{\lambda^v}$ in (106) is true. Given $\widetilde{MC}^{\lambda^v} < \widetilde{MC}^{\mu^h}$ in (106), (100) is true. If (101) and

$$-\beta_h(p_h r_h - p_l r_l) < p_h r_h' e \, , \tag{108}$$

then $\widetilde{MC}^{\mu^h} > p_l r_l$ in (107) which itself implies $\widetilde{MC}^{\mu^h} > \widetilde{MC}^{\lambda^l}$, and $\widetilde{MC}^{\mu^h} > \widetilde{MC}^{\mu^{rw}}$ in (107). If (108) and

$$p_h r_h' e < -\beta(p_h r_h - p_l r_l) \, , \tag{109}$$

then $\widetilde{MC}^{\mu^h} > \widetilde{MC}^{\lambda^{lr}}$ in (107). To sum up, condition (35) is equal to eq. (101), and eq. (108) and (109) combine to condition (37) which is stricter than (102) and $\widetilde{MC}^{\lambda^v} < \widetilde{MC}^{\mu^h}$ in (106).

Hence, given eq. (101), (108), and (109) optimal strategies in equilibrium are

$$(q^*_{h,i}, q^*_{l,i}, \mu^*_i, \lambda^*_i) = \begin{cases} (q^v_h, q^v_l, \mu^v_i, \lambda^v_i(MC_i)) & \text{if} \quad MC_i \leq \widetilde{MC}^l \\ \big(q^h, 0, \mu^h_i(MC_i), 0\big) & \text{if} \quad \widetilde{MC}^l < MC_i \leq \widetilde{MC}^{\mu^h} \\ \big(q^{uc}_{h,i}(MC_i), 0, 0, 0\big) & \text{if} \quad \widetilde{MC}^{\mu^h} < MC_i \leq \widetilde{MC}^h \end{cases} \tag{110}$$

□

A.7 Proof of Corollary 1

Proof. I proof Corollary 1 by contradiction. Assume the cutoff $\widetilde{MC}^l$ decreases. It implies that the number of banks participating in low-risk market decreases. Then fewer banks produce a smaller quantity each so that the total supply of low-risk loans decreases. Note that these banks previously produced $q^l_l = \frac{e}{\beta_l}$ and now produce $q^v_l = \frac{(\beta_h - \beta)e}{\beta(\beta_h - \beta_l)} < q^l_l$. Hence, the interest rate on low-risk loans increases. From eq. (34) follows that the interest rate on high-risk loans must increase as well (and even more) otherwise the cutoff would not decrease as was assumed.

Due to eq. (2) the interest rate on high-risk loans only increases if total supply decreases. On the other hand an increase of r_h implies that the cutoffs $\widetilde{MC}^h$ and $\widetilde{MC}^{\mu_h}$ both increase while $\widetilde{MC}^l$ decreases. Thus, the number of specialized banks in the high-risk market increases and more productive banks with strategy v enter the high-risk market. All in all, this implies that the aggregate supply of high-risk loans must increase which contradicts the necessary decrease of aggregate supply such that the interest rate could rise. Hence, the cutoff $\widetilde{MC}^l$ cannot decrease but has to increase.

Assume further the cutoff $\widetilde{MC}^h$ decreases. Then the interest rate on high-risk loans necessarily decreases and aggregate supply increases. That is

$$\begin{gathered} Q^{*B2}_h < Q^{*B3}_h \\ \nu^{B3}_l(1 + \frac{q^v_h}{q^h_h}) - \nu^{B2}_l < (\nu^{B3}_h - \nu^{B2}_h) \end{gathered} \tag{111}$$

which cannot be true since the right hand side is negative if the cutoff decreases, as was assumed, while the left hand side is positive because the cutoff in the low-risk market increase as was shown earlier. Hence, the cutoff in the high-risk market must increase as well.

□

A.8 Proof of Lemma 5

Proof. We rewrite eq. (12) by defining the share of high-risk loans in bank i's portfolio as $\gamma_i = \frac{q_{h,i}}{d_i+e}$ as

$$\Pi_i(c_i, \gamma_i, q_i, r(Q), z) = (p_h r_h(Q_h)\gamma_i + p_l r_l(Q_l)(1-\gamma_i) - MC_i)\,(q_{h,i} + q_{h,i}) + r_d e\ . \tag{112}$$

The effect of a higher share of high-risk loans on default probabilities is implicitly defined by

$$\frac{\partial \Pi_i}{\partial \gamma_i} = (q_h^v + q_l^v)\,(r_h(Q_h)(1 - PD_h(z)) - r_l(Q_l)(1 - PD_l(z))) \tag{113}$$

which could be either negative or positive depending on z in the following way:

$$\begin{aligned} &lim_{z\to-\infty} \quad r_h(Q_h)(1 - PD_h(z)) - r_l(Q_l)(1 - PD_l(z)) = r_h(Q_h) - r_l(Q_l) \\ &lim_{z\to\infty} \quad r_h(Q_h)(1 - PD_h(z)) - r_l(Q_l)(1 - PD_l(z)) = 0 \\ &\qquad\qquad\ \ r_h(Q_h)(1 - PD_h(0)) - r_l(Q_l)(1 - PD_l(0)) = p_h r_h(Q_h) - p_l r_l(Q_l) \end{aligned} \tag{114}$$

This means that the effect is positive for non-positive z and vanishes for very high z. But the effect can be negative, because $\frac{\partial \Pi_i}{\partial \gamma_i}$ has a local minimum given at $\hat{z}$ defined by

$$\frac{\partial^2 \Pi_i}{\partial \gamma_i \partial z} = 0 \quad \Leftrightarrow \quad \hat{z} = \frac{-\zeta_h^2 + \zeta_l^2 + 2\ln(\frac{r_h}{r_l})(1-\rho)}{2\sqrt{\rho}(\zeta_h - \zeta_l)}\ . \tag{115}$$

Therefore, as $z \to \infty$, $\frac{\partial \Pi_i}{\partial \gamma_i}$ must approach the limit 0 from below implying

$$\exists\ \tilde{z}\ : 0 < \tilde{z} < \hat{z} \quad s.t. \quad \begin{cases} \frac{\partial \Pi_i}{\partial \gamma_i} \geq 0 & \text{if} \quad z \leq \tilde{z} \\ \frac{\partial \Pi_i}{\partial \gamma_i} < 0 & \text{if} \quad z > \tilde{z} \end{cases} \tag{116}$$

□

A.9 Proof of Proposition 5

Proof. First I show that the cutoff in the low-risk market $\widetilde{MC}^l$ increases by contradiction. Imagine the cutoff decreases. Because $\frac{\partial q_l^v}{\partial \beta} < 0$, each bank in the low-risk market provides smaller quantities and there are less banks active. Then clearly aggregate supply of low-risk loans (Q_l) decreases. This implies that the interest rate for low-risk loans increases. Consequently, the interest rate for high-risk loans must increase as well, otherwise the cutoff cannot decrease. But the interest rate for high-risk loans cannot increase because all these changes taken together imply

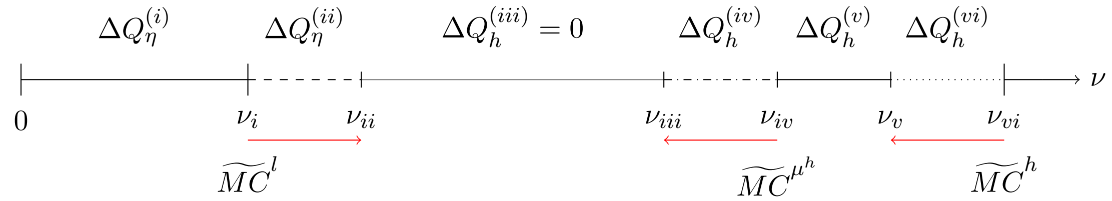

Figure 4: Illustration of notation for proof of Proposition 5. $\Delta Q_\eta^{(g)}$ is the change in aggregate supply of loans of type η for banks of group $g \in \{i, ii, iii, iv, v, vi\}$ induced by an increase of β. ν_g indicates the index of the bank with the highest marginal costs within group g. For example, $\Delta Q_h^{(ii)} = (\nu_{ii} - \nu_i)(q_h^v - q_h^h)$, i.e. when the leverage ratio is tightened, a number of $(\nu_{ii} - \nu_i)$ banks reduce their supply by $(q_h^v - q_h^h) < 0$ because they choose the vertex strategy instead of the specialization on loans of type h.

that the aggregate supply of high-risk loans (Q_h) must increase and hence the interest rate cannot increase: Banks with the vertex strategy produce a higher amount ($\frac{\partial q_h^v}{\partial \beta} > 0$), there are more banks that choose to specialize on high-risk loans and additional banks entering with small quantities in the high-risk market. Hence, $\widetilde{MC}^l$ cannot decrease.

Next I show that the cutoffs in the high-risk market $\widetilde{MC}^h$ and $\widetilde{MC}^{\mu_h}$ decrease by contradiction using the fact that $\widetilde{MC}^l$ increases. To ease notation, let $\Delta Q_\eta^{(g)}$ be defined as the change in aggregate supply of loans of type η for banks of group $g \in \{i, ii, iii, iv, v, vi\}$ induced by an increase of β. ν_g indicates the index of the bank with the highest marginal costs within group g. These groups are illustrated in Fig. 4. Imagine the cutoffs would increase. This necessitates that the interest rate for high-risk loans increases and aggregate supply decreases. Given the movement in the cutoffs we get the following condition for this scenario.

$$\begin{aligned} -\Delta Q_h^{(ii)} &> \Delta Q_h^{(i)} + \Delta Q_h^{(iv)} + \Delta Q_h^{(v)} + \Delta Q_h^{(vi)} \\ \nu_{ii} &> \nu_i \left(1 + \frac{\beta_h}{\beta(\beta_h - \beta)}\right) + \frac{\beta\beta_h(\beta_h - \beta_l)}{\beta_l(\beta_h - \beta)e} \left(\Delta Q_h^{(iv)} + \Delta Q_h^{(v)} + \Delta Q_h^{(vi)}\right) \end{aligned} \tag{117}$$

We get a second condition ensuring that the interest rate for low-risk loans increases as well otherwise $\widetilde{MC}^l$ could never increase.

$$\begin{aligned} -\Delta Q_l^{(i)} &> \Delta Q_l^{(ii)} \\ \nu_{ii} &< \nu_i \left(1 + \frac{\beta_h}{\beta(\beta_h - \beta)}\right) \end{aligned} \tag{118}$$

We see that both conditions can never be true at the same time because $\Delta Q_h^{(iv)} +$

$\Delta Q_h^{(v)} + \Delta Q_h^{(vi)} > 0$. Hence cutoffs in the high-risk market $\widetilde{MC}^h$ and $\widetilde{MC}^{\mu_h}$ cannot increase. □

A.10 Proof of Lemma 6

Proof. Given Proposition 5 we know that cutoffs in the high-risk market must decrease which implies that the high-risk loan rate decreases. We can rewrite Equation (117) for the opposite case of decreasing interest rate and increasing aggregate demand as

$$\Delta Q_h^{(i)} < -\Delta Q_h^{(ii)} - \Delta Q_h^{(iv)} - \Delta Q_h^{(v)} - \Delta Q_h^{(vi)}$$
$$\nu_i \left(1 + \frac{\beta_h}{\beta(\beta_h - \beta)}\right) > \nu_{ii} - \frac{\beta\beta_h(\beta_h - \beta_l)}{\beta_l(\beta_h - \beta)e} \left(\Delta Q_h^{(iv)} + \Delta Q_h^{(v)} + \Delta Q_h^{(vi)}\right) . \tag{119}$$

We know that Equation (119) must be true, so Equation (118) is true as well, because $-(\Delta Q_h^{(iv)} + \Delta Q_h^{(v)} + \Delta Q_h^{(vi)}) > 0$. Therefore, interest rates in the low-risk market increase and aggregate loan supply in the low-risk market decreases when the leverage ratio is tightened. □

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