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Capital Requirements, Market Structure, and Heterogeneous Banks

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Abstract

Bank regulators interfere with the efficient allocation of resources for the sake of financial stability. Based on this trade-off, I compare how different capital requirements affect default probabilities and the allocation of market shares across heterogeneous banks. In the model, banks' productivity determines their optimal strategy in oligopolistic markets. Higher productivity gives banks higher profit margins that lower their default risk. Hence, capital requirements indirectly aiming at high-productivity banks are less effective. They also bear a distortionary cost: Because incumbents increase interest rates, new entrants with low productivity are attracted and thus average productivity in the banking market decreases.

Keywords: bank competition, bank regulation, Basel III, capital requirements, heterogeneous banks, leverage ratio

JEL classification: G11, G21, G28

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1 Introduction

In many industries, dynamics that induce a reallocation of market shares towards more productive firms are shown to be main drivers of aggregate productivity growth (Bartelsman and Doms, 2000) and economic growth (Prescott, 1998), respectively. Due to frictions, e.g. asymmetric information and entry barriers, the banking industry though is prone to allocative inefficiency causing welfare losses (Vives, 2001a; Berger et al., 1993). Deregulation and increased competition can improve allocative efficiency (Stiroh, 2000; Stiroh and Strahan, 2003). Cross-country evidence suggests that more efficient banking systems are associated with higher development (Demirgüç-Kunt and Maksimovic, 1998; Rajan and Zingales, 1998; Beck et al., 2000). Better banks provide higher quality of intermediation which can directly imply a higher quality of real investment (Jayaratne and Strahan, 1996) and more financing for new entrepreneurs (Black and Strahan, 2002). Further, more efficient banks are less likely to be in distress (Schaeck et al., 2009; Kick and Prieto, 2014). This indicates that the allocation of market shares across heterogeneous banks matters not only for real outcomes and economic efficiency but also for financial stability.

Since the introduction of Basel III, banks are constrained by a set of multiple competing minimum capital requirements. The simultaneity of multiple rules implies that their effect on the allocation of market shares cannot be neutral since some banks are more affected by any one specific rule than others (BCBS, 2016; Cecchetti and Kashyap, 2018). The rationale of capital requirements is to favor safe behavior and charge risky choices. Different capital requirements are directed at different threats to financial stability: high asset-specific credit risk, procyclicality, high leverage, systemic risk, and more. If banks with high productivity act prudently in every respect, normative implications are clear. But what if banks with high productivity take risky choices? A social planner will face a trade-off between an efficient allocation of resources and financial stability when setting new regulatory guidelines (Allen and Gale, 2004).

These issues motivate the following research questions. How effective are capital requirements in enhancing financial stability when we take heterogeneity into account? What kind of reallocation do they induce? And what are the implications of such reallocation for financial stability and credit supply? I develop a model with an industrial organization view on banking to address these questions. My analysis assesses the impact of the interaction of minimum capital requirements on the allocation of market shares across banks with different productivity. It focuses on the interaction between two capital requirements: the revised risk-based capital framework of Basel II and the non-risk-based leverage ratio of the Basel III accord.

Building on Repullo and Suarez (2004), banks are funded with a limited amount of equity and an unlimited amount of deposits while choosing their portfolio strategy between two types of loans: high-risk and low-risk. I extend this model by adding heterogeneity in productivity among banks in the form of differences in marginal costs of intermediation and an oligopolistic market structure.[1] Banks choose their strategy in both credit markets with Cournot competition and are subject to the aforementioned capital constraints.

In this model, productivity creates positive charter value and market power. In an unregulated equilibrium without any capital requirements, market shares are allocated proportional to productivity. The bank with the highest productivity is the market leader in the market for high-risk loans as well as in the market for low-risk loans. The presence of capital requirements, however, introduces interdependence between both types of loans. In a Basel II equilibrium where banks are constrained by a risk-weighted capital ratio, banks find it optimal to specialize in one type of loan, as shown by Repullo and Suarez (2004). My model shows that banks with high productivity specialize in the low-risk market while banks with low productivity specialize in the high-risk market. Banks trade off higher returns with higher capital requirements and lower scale. Heterogeneity in costs introduces a unique cut-off value. Intuitively, banks with costs above this cutoff have more incentives to compensate their competitive disadvantage by choosing the higher yielding, riskier loans.

My main results derive from the comparison of this equilibrium to a Basel III equilibrium with multiple constraints. In particular, banks are then subject to the risk-weighted ratio and a leverage ratio. Because banks with high productivity choose the low-risk strategy which can be operated at a higher scale under the Basel II requirement, these banks are directly affected by the introduction of a leverage ratio. They rebalance their portfolio into a mix of low-risk and high-risk loans.In order to compensate the higher costs due to the additional requirement banks reduce their loan supply in both markets so that interest rates rise. This attracts new entrants with lower productivity than the incumbent banks in both markets: In the low-risk market incumbent high-productivity banks lose market shares to competitors with lower productivity who specialize in high-risk loans

[1] In perfect competition with productivity differences the most productive bank which has the lowest marginal costs would effectively be a monopolist.

under Basel II regulation. In the high-risk market incumbent low-productivity banks lose market shares to high-productivity banks on the one hand, and to new entrants with even lower productivity on the other. Hence the average productivity in the banking market decreases.

The model follows the efficient structure hypothesis (Demsetz, 1973). That is, it predicts that banks with higher productivity gain market shares so that less concentrated banking markets are associated with higher productivity and lower interest rates. As such it takes one side in a highly debated controversy about banking market structure. Proponents of the structure-conduct-performance paradigm would argue for the opposite. Ample empirical studies exist on both sides (see Berger et al. (2004) for a review) among which we find supporting evidence consistent with this model's predictions in the U.S. (Berger, 1995; Jayaratne and Strahan, 1998), Europe (Goldberg and Rai, 1996), and a group of advanced economies (Mirzaei et al., 2013).

Based on the grounds of efficient structure, the model yields several testable implications regarding the introduction of a leverage ratio. In Europe, for example, a leverage ratio has to be reported by banks since 2014 (European Parliament, 2013) and is expected to act as a binding requirement in 2021 (European Commission, 2019). According to my model, the leverage ratio should have affected banks with high productivity, high interest margins, or business models with low credit risk profiles or simply large banks. Affected banks should have lost market shares or demonstrated other signs of loss of competitiveness such as higher refinancing costs. Finally, the leverage ratio should have led to a reduction of credit supply.[2]

The comparative statics of the model reveal that the higher the leverage ratio, the smaller is the distortive effect on average productivity. A higher ratio leads to more shifting within the portfolio of affected banks towards the high-risk loan market and hence to stronger reactions of incumbent banks in that market. This leaves less room for new entrants with very low productivity. The model therefore advocates for a rather strict leverage ratio albeit it cannot be stricter than the risk-weight on high-risk loans. This sets an upper limit to the leverage ratio. If leverage ratios were above this limit, the market for low-risk loans becomes so unattractive that there would no longer be an equilibrium.

My analysis points out the possibility of unintended consequences in the actual regu-

[2]The model also predict a rise in interest rates. In reality, however, these rates are influenced by monetary policy that is absent in the model.

lation of many countries worldwide[3] (BCBS, 2019). In a recent survey, banks report "difficulty in achieving multiple constraints simultaneously" as one of the "most important challenges associated with meeting regulatory requirements" (BCBS Survey, 2019). A further tightening of the leverage ratio for the group of systemically important banks is scheduled for 2022 within the Basel framework (G-SIBs leverage ratio buffer requirement) and was already implemented in the U.S. (supplementary leverage capital ratio). The main channel demonstrated in this model is applicable to other requirements as well. For example, operational risk could be associated with complexity which in turn could be an outcome of high productivity.

The model gives cautious normative implications. Capital requirements are not tailored to individual banks. On the contrary, they intend to provide a level playing field.[4] In the model, banks with lower productivity do not have to provide more equity for taking the same risks, yet their default probabilities are higher due to lower charter values. A less distortive requirement would have to take this into account. Yet, any normative implication is based on decisive standpoints on open empirical questions. Additionally, setting capital ratios dependent on banks' productivity seems a daunting task in practice. As long as no consensus on how to measure bank productivity can be reached, determining a productivity-adequate level of capital can only be done within the realm of supervisory discretion. Indeed, policy makers stress the importance of proportionality in regulation (Restoy, 2019). Hence, the most practical implication of this work is that more research could be directed to finding the right methods to identify the most productive banks in the market and ascertain their behavior.

This paper contributes to the literature studying the role of productivity in banking. It leans on the idea that productivity differences play an important role in shaping firms' optimal strategies as proposed in trade theory by Melitz (2003). Theoretical work on capital requirements so far mostly neglected the impact of productivity on banks' decisions because models were based on the conduct of representative banks (VanHoose, 2007). My model is among the first to deviate from the view of a representative bank to emphasize how differences matter for aggregate outcomes. Apart from macroeconomic models with heterogeneous agents, e.g. Choi et al. (2015), only few microeconomic banking models

[3] Except Australia all other 17 member countries of the Basel Committee and the European Union have implemented the leverage ratio according to the definition of 2014 in combination with risk-based capital standards (BCBS, 2019).

[4] At least Pillar 1 capital requirements are not tailored to individual banks depending on their productivity. Under the Supervisory Review and Evaluation Process in Pillar 2, regulators can levy additional bank specific requirements.

consider heterogeneity. Closest to my results, Barth and Seckinger (2018) demonstrate that stricter capital requirements in the form of a leverage ratio induce agents with lower monitoring ability to become banks and thus that the average ability in the banking market deteriorates. Hakenes and Schnabel (2011) consider two distinct types of banks and find that smaller banks take more risks if big banks have a competitive advantage by choosing the internal ratings-based over the standardized approach in the Basel II framework. Opposed to my approach neither of these works uses an industrial organization approach with a focus on competitive interactions between banks of different productivity.

Nevertheless, empirical work has given much attention to the relationship between productivity and risk taking underpinning the idea that there is a connection although the evidence is yet inconclusive about its direction. Some findings support the efficiency-risk hypothesis[5] (Berger and di Patti, 2006; Altunbas et al., 2007) claiming that more productive banks expect higher future profits and thus need smaller capital buffers. Others support a negative relation (Fiordelisi et al., 2011) claiming that more productive banks protect these higher profits by choosing less risky strategies. My model provides a first attempt for a formal framework for the latter, i.e. a productivity-driven charter-value hypothesis. The mechanisms at work are akin to Keeley's (1990) charter value argument with the addition that market power is derived from productivity. Further, Delis et al. (2012) provide evidence for a panel of banks in 14 developed countries over the period from 1998 to 2008 that capital regulation affects banks' risk taking indeed in a heterogeneous way and that productivity, bank size, and market shares are among the factors explaining this heterogeneity.

Further, my work contributes to the literature on capital requirements and risk, in particular to the recent literature on the interaction of the leverage ratio and the risk-based capital ratio.[6] Closest to my work, Kiema and Jokivuolle (2014) study banks' optimal portfolio choice with both requirements building as well on Repullo and Suarez (2004). They show that bank portfolios get more alike and conclude that the role of the leverage ratio as a backstop to model risk is impeded by less diverse portfolio choices. However, they do not consider the implications of heterogeneity or imperfect competition in their

[5] Empirical studies use the term efficiency for what throughout the text I call productivity. In both cases, the terms refer to a bank-specific relation between input and output or cost and revenue.

[6] Several other papers study the interaction between the two requirements not based on banks' portfolio allocation (Blum, 2008; Wu and Zhao, 2016; Brei and Gambacorta, 2016; Gambacorta and Karmakar, 2018).

work. Acosta Smith et al. (2017) also examines banks' risk choices under the competing rules and evaluates whether the leverage ratio effectively reduces the probability of insolvency. They contrast the risk-taking incentives of the leverage ratio with the increase of loss absorbing capital due to the ratio. They show theoretically that the positive effect of higher capital outweighs the negative effect of increased risk-taking. I find a similar results with respect to the changes in default probabilities[7] but my analysis identifies additional equilibrium channels on default probabilities through the changing market outcomes and productivity differences.

Lastly, this model is related to the recent literature that stresses the importance of industrial organization set-ups for questions in banking. Mahoney and Weyl (2017) and Crawford et al. (2018) show how integrating imperfect competition in selection markets can change the welfare effects of adverse selection. My model in contrast shows how integrating imperfect competition in regulated markets can change welfare effects but also the effects of the regulation itself. Similar to my work, Corbae and D'Erasmo (2019) study the interaction of regulatory policies and banking market structure. They develop a rich dynamic model that quantifies the effects of capital requirements on allocative efficiency which produces compatible quantitative results with the qualitative ones here.[8]

The remainder of this paper is organized as follows. Section 2 introduces the main assumptions and setting of the model. Section 3 gives the baseline equilibrium without regulation. In Section 4 banking regulation is introduced and the equilibria with risk-weighted and competing capital requirements as well as comparative statics are derived. Section 5 discusses the results and possible limitations. Section 6 concludes.

2 The model

Consider a Cournot-Nash game with N banks competing in two markets. There is a market for low-risk loans and a market for high-risk loans. Banks have different unit costs and no fixed costs. Unit costs of bank i are denoted as c_i. In what follows, we rank banks according to their costs such that the bank with the lowest unit costs is

[7] These results are that indeed for most banks default probabilities decline when a leverage ratio is introduced at least as long as realizations of a common systematic risk-factor not exceed a threshold. Beyond this threshold, default rates in the high-risk market are so high that even the most productive banks are closer to default.

[8] Compare the section on size dependent capital requirements which mirrors best the setting of this paper.

denominated as bank 1 whereas bank N has the highest unit costs.

$$c_1 < c_2 < \cdots < c_N \tag{1}$$

Each market represents one of two types of entrepreneurs, risky and less risky entrepreneurs. Once in the game, there is perfect information about types. Unit costs can be interpreted as screening costs that banks have to incur in order to discern high-risk and low-risk entrepreneurs. Further, these costs can reflect monitoring and administrative costs, such as employment of loan officers, back-office administration of the loan portfolio, or maintenance of monitoring processes. Therefore, low costs represent a more efficient production technology. Banks that are able to operate their loan portfolio at lower costs are more productive. The model introduces productivity differences of banks in the simplest form of differing cost functions.[9] This leads to asymmetric Nash-equilibria where optimal strategies depend on marginal costs.[10]

Let the strategy of bank i be $q_i = (q_{h,i}, q_{l,i})$. Let $Q_{-i} = (Q_{h,-i}, Q_{l,-i})$ denote aggregate quantities of all banks except bank i and $Q = (Q_h, Q_l)$ the total aggregate supply of loans in the respective markets. Aggregate supply determines inverse demand $r_\eta(Q_\eta)$[11] from entrepreneurs of type $\eta = \{h, l\}$. Inverse demand functions are continuous, monotone, and concave:

$$r_\eta(Q_\eta) = r_\eta\left(\sum_{i=1}^{N} q_{\eta,i}\right) , \qquad r'_\eta(Q_\eta) < 0 , \qquad r''_\eta(Q_\eta) \le 0 . \tag{2}$$

Entrepreneurs demand a loan of size 1 if the interest rate is lower than their expected payoff. I assume expected payoffs are distributed such that it entails inverse demand functions of the described kind. Entrepreneurs have limited liability. They repay the interest rate only if their projects are successful. If their project defaults, entrepreneurs pay nothing to the bank, i.e. loss given default is 1. Banks consider the probability of success for each type of loan to take this into account in their loan supply decisions.

[9] Heterogeneous productivity is exogenous in the model. This is inspired by trade models with heterogeneous firms (Melitz, 2003). It is applicable since I do not want to study what constitutes productivity differences among banks but rather how they influence the portfolio decision and distribution of market shares. Caveats concerning this assumption are discussed in Section 5.

[10] I assume that banks are perfectly informed about their own as well as their rivals' marginal cost. It was shown at least for the case of linear demand functions that full disclosure of costs is optimal in Cournot games with initial uncertainty about rivals' costs (Shapiro, 1986).

[11] All interest rates are absolute returns. Therefore, think of r_h as $1 + interest_h$, etc..

To determine success probabilities of entrepreneurs, I use the representation by Repullo and Suarez (2004) and Kiema and Jokivuolle (2014) of the Vasicek model (Vasicek (1987),Vasicek (2002)). This risk model underpins the framework of risk-sensitive capital requirements of the Basel II accord. There is a common risk factor captured in z as well as idiosyncratic risk ϵ_j that are both standard normally distributed. Successes of high-risk and low-risk projects are correlated and ρ is the correlation parameter. The project of entrepreneur j is successful if a latent random variable $x_j \leq 0$, where

$$\begin{aligned} & x_j = \zeta_\eta + \sqrt{\rho}\, z + \sqrt{1-\rho}\, \epsilon_j \qquad \eta = \{h, l\} \\ & z \sim N(0,1), \quad \epsilon_j \sim N(0,1)\ . \end{aligned} \tag{3}$$

The two types differ in ζ_η which represents the financial vulnerability of entrepreneurs of type η and $0 < \zeta_l < \zeta_h$. If banks know the types of entrepreneurs, they know ζ_l and ζ_h. The probability to default for loans of type η conditional on the realization of systematic risk factor z is hence given by

$$PD_\eta(z) = Pr(\zeta_\eta + \sqrt{\rho} z + \sqrt{1-\rho}\epsilon_j > 0) = \Phi\left(\frac{\zeta_\eta + \sqrt{\rho}\, z}{\sqrt{1-\rho}}\right)\ , \tag{4}$$

where Φ is the cumulative distribution function of the standard normal distribution. Consequently, the unconditionally expected probability to default of loans of type η, i.e. the average default probability, is $\overline{PD}_\eta = \Phi(\zeta_\eta)$.

To abbreviate notation, let the conditional probability of success be $p_\eta(z) = 1 - PD_\eta(z)$, and the expected probability of success be $p_\eta = 1 - \overline{PD}_\eta$, respectively. Note that $p_h(z) < p_l(z)$ for all z since low-risk entrepreneurs are less likely to default. Assume that investing in the riskier project has a higher expected yield so that

$$p_l(z) r_l(Q_l) < p_h(z) r_h(Q_h)\ . \tag{5}$$

I assume depositors are insured and consequently ignorant of bank risk. They supply an inexhaustible amount of savings at an interest rate r_d. The deposit rate could be the value of an outside option of depositors, e.g. holding cash or a safe asset instead of investing their endowment in a bank. Depositors will then invest in banks whenever these offer a deposit rate at least as high as their outside option. Bank i's profit from

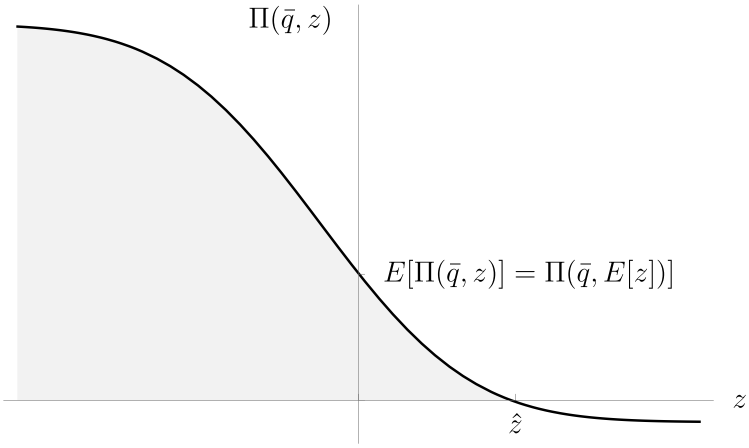

Figure 1: Profit depending on systematic risk.

intermediation are given as

$$\Pi_i(q_i, z) = p_h(z)r_h(Q_h)q_{h,i} + p_l(z)r_l(Q_l)q_{l,i} - c_i(q_{h,i} + q_{l,i}) - r_d d_i \ . \tag{6}$$

Banks optimize expected profits and have limited liability. If their loan portfolio generates losses, they do not pay back depositors. Their objective function $V(q_i, z)$ is therefore given as

$$V(q_i, z) = E[\max\{\Pi_i(q_i, z), 0\}] = \int_{-\infty}^{\hat{z}(q_i)} \Pi_i(q_i, z) \ \ d\Phi(z) \tag{7}$$

where $\hat{z}$ is defined as the critical value of systematic risk for which $\Pi_i(q_i, \hat{z}_i) = 0$. Figure 1 illustrates how $\hat{z}$ is defined.

This objective however is subject to several constraints. Each banker is equally endowed with an amount of equity e. Let r_e denote the opportunity costs of equity capital and let it be higher than the opportunity costs of depositors, s.t. $r_d < r_e$.[12] Banks are only operated if expected profits from intermediation are higher than the outside option of bankers. Therefore, I assume that bankers have to invest their equity in the bank in order to employ the banking technology. Their participation constraint is given by

$$V(q_i, z) \geq r_e e \ . \tag{8}$$

[12]This assumes that equity is costly contrary to the discussion in Admati et al. (2010).

Banks' balance sheet constraint is given by

$$e + d_i = q_{h,i} + q_{l,i} \; . \tag{9}$$

For simpler notation, I define marginal costs of intermediation as

$$MC_i = c_i + r_d \; . \tag{10}$$

and insert eq. (9) and eq. (10) into eq. (6) so bank i's profit function becomes

$$\Pi_i(q_i, z) = p_h(z) r_h(Q_h) q_{h,i} + p_l(z) r_l(Q_l) q_{l,i} - MC_i(q_{h,i} + q_{l,i}) \tag{11}$$

In addition, each bank has a capacity limit W_i which is finite but arbitrarily high so it cannot produce more than W_i in any market. This assumption ensures that banks' strategy sets are bounded in the unregulated case. Furthermore, banks are not allowed to take short positions in neither loans nor deposits, so that $q_i \geq 0$ and $d_i \geq 0$.

3 Unregulated equilibrium

Consider the optimization problem of bank i without capital requirements given as

$$\underset{q_i}{\text{Max}} \; V_i(q_i, z) \qquad \text{s.t.} \quad V_i(q_i, z) \geq r_e e \quad \text{and} \quad 0 \leq q_i \leq W_i \; . \tag{12}$$

Definition 1. A Cournot-Nash equilibrium in pure strategies is characterized by optimal strategy vector Q^* resulting in equilibrium interest rate vector $r^*(Q^*)$ with best-response correspondence such that

$$q_i^* = \arg\max V_i(q_i, Q_{-i}^*) \quad \forall i \in \{1, \ldots, N\} \, . \tag{13}$$

Given the assumption of concavity of inverse loan demand, the unregulated equilibrium exists. Similarly, the constrained equilibria, which are introduced in the next sections, must exist as well.

Lemma 1 (Existence of equilibria)**.** *The unregulated game, the game with a risk-weighted capital requirement, and the game with a leverage ratio and a risk-weighted capital requirement have at least one Nash-equilibrium in pure strategies.*

Proof. Proof is in the appendix. □

Because of the flat deposit rate due to the deposit insurance and the fact that debt financing is cheaper than equity financing, banks have strong incentives to increase their balance sheet size through levering. In a Cournot game though, competition ensures that bank size stays limited. If any bank expands its loan business the interest rates decrease for all banks so that competitors reduce their supply. Furthermore, the lower interest rates are, the fewer banks are able to participate in the loan market because some banks' marginal costs would be too high to make a profit. Consequently, the least productive banks do not provide loans in equilibrium and some less productive banks only provide loans in the high-risk market where expected revenues are higher.

By taking the first derivative of the objective function eq. (7) with respect to quantities $q_{h,i}$ and $q_{l,i}$, summing first-order conditions over all banks, and rearranging, the best response function of a bank i is[13]

$$q^*_{\eta,i} = \max\left[0, \frac{r_\eta(Q^*_\eta) - G_{\eta,i}(\hat{z}_i)MC_i}{\nu_h r_\eta(Q^*_\eta) - \sum_{i=1}^{\nu_h} G_{\eta,i}(\hat{z}_i)MC_i} Q^*_\eta\right] , \tag{14}$$

where $\nu_\eta \in \{1, \dots, N\}$ denotes the least productive bank still active in market η, s.t. $q_{\eta,\nu_\eta} > 0$ while $q_{\eta,\nu_\eta+1} = 0$, and for a simpler notation I introduced a function $G_{\eta,i}(\hat{z}_i)$ defined as

$$G_\eta(\hat{z}_i) = \frac{\Phi(\hat{z}_i)}{\int_{-\infty}^{\hat{z}_i} p_\eta(z)\, d\Phi(z)} = \frac{Pr(z \leq \hat{z}_i)}{Pr(x_j \leq 0 | z \leq \hat{z}_i)} . \tag{15}$$

Let $\kappa_{\eta,i}$ denote the market share of bank i in market η so eq. (14) can be written as $q^*_{\eta,i} = max[0, \kappa_{\eta,i}(MC_i, \hat{z}_i)Q^*_\eta]$. On the one hand market shares are determined by banks' productivity. This is reflected in the dependence of κ on banks' marginal costs of intermediation MC. It is a positive relationship: Productivity generates market power. Banks with lower marginal costs *ceteris paribus* should have higher market shares.

On the other hand, market shares are also determined by banks' distance to default captured in the dependence of κ on $G(\hat{z})$. This function describes the probability of bank success relative to the probability of projects success given that the bank does not default. It reflects the limited liability of banks. They only take positive earnings into account. The function G increases in $\hat{z}$ since the unconditional probability of success grows at a higher pace than the conditional probability. Hence, any given bank faces a trade-off

[13] More details on the derivation of equilibrium are provided in the proof of Proposition 1.

between higher market shares and higher safety. However, it is not straightforward to see how $\hat{z}$ varies between banks. But using eq. (14), it must be that banks with higher productivity have a higher distance to default. I call this a productivity-driven charter value. That is banks with higher productivity generate higher positive profits due to their relative low costs. They protect these profits by choosing relatively safer strategies than their peers. This effect even occurs despite potential losses of market power due to the trade-off between safety and market shares. In this respect, this notion of charter value differs from the traditional view coined by Keeley (1990).

Lemma 2 (Productivity-driven charter value)**.** *In the unregulated equilibrium, banks with higher productivity have higher distances to default. Formally,*

$$\hat{z}_i > \hat{z}_{i+1} \quad \forall i \in \{1, ..., \nu_\eta\},\ \eta \in \{h, l\} \ . \tag{16}$$

Proof. Proof is in the appendix. □

Consequently, high-productivity banks should have on the one hand lower market shares due to the indirect effect of lower costs resulting in safer strategies, on the other hand higher market shares due to the direct effect of lower costs. All in all, the relation between market power and productivity therefore depends on the relative importance between these two competing channels which can be expressed as the relative rate of change between any two banks i and $i+1$. Given

$$\frac{G_h(\hat{z}_i) - G_h(\hat{z}_{i+1})}{G_h(\hat{z}_{i+1})} < \frac{MC_{i+1} - MC_i}{MC_i} \ , \tag{17}$$

the positive relation between productivity and market shares dominates and we can state the following.

Proposition 1 (Unregulated equilibrium)**.**
In an unregulated equilibrium, if eq. (17) holds, more productive banks gain higher market shares in both markets and are therefore larger than less productive banks.

$$\kappa_{\eta,i}(MC_i, \hat{z}_i) > \kappa_{\eta,i+1}(MC_{i+1}, \hat{z}_{i+1}) \quad \forall i \in \{1, \ldots, \nu_\eta\},\ \eta \in \{h, l\} \ . \tag{18}$$

Proof. Proof is in the appendix. □

Here, bank 1 with the lowest marginal costs MC_1 has the highest market share in the market for low-risk loans and the market for high-risk loans, whereas bank ν_h has the lowest market share in the market for high-risk loans and its marginal costs MC_{ν_h} are only slightly smaller than or equal to the market interest rate $r_h(Q_h)$. Consequently, bank 1 has the biggest balance sheet and the highest debt-to-equity ratio. Therefore, in the unregulated equilibrium with Cournot competition and heterogeneous cost functions productivity advantages translate into scale and market power.

Under any continuous distribution of risk, here it is the standard normal distribution, extreme realizations of systematic or idiosyncratic risk are possible, so that default cannot be prevented with absolute certainty no matter how much loss absorbing capital is available to a bank. A regulator would try to avoid bank failures and banking crisis because these are associated with costs and loss of economic output (Laeven and Valencia, 2013). The micro-prudential approach of the Basel Committee is to set a maximal admissible default probability.

Assumption 1 (Necessity of regulation)**.** Given that the regulator sets maximal admissible default probability at α, I assume

$$Pr(z \leq \hat{z}_1) < 1 - \alpha \,. \tag{19}$$

Therefore, in the unregulated equilibrium the probability that all banks default is unacceptably high. Given that critical values are ordered according to Lemma 2, if systematic risk z realizes higher than critical value of some bank i, then bank i is expected to default and all banks with marginal costs higher than MC_i are expected to default as well. Hence, for any realization of z above $\hat{z}_1$, the whole banking system is expected to default. According to eq. (19), the probability of this event happening is higher than α.

4 Regulating heterogeneous banks

4.1 Basel II equilibrium

The Basel II accord introduced risk-sensitive capital requirements to avoid the risk-shifting phenomenon described by Koehn and Santomero (1980), Kim and Santomero (1988) and others. They show that if capital requirements are not risk-sensitive, banks have incentives to shift their portfolio towards riskier assets. Following the Basel II approach for credit risk, banks must categorize their assets with respect to their riskiness into different buckets for which different risk-weights are applied. In the Standard Approach these weights are set by the regulator. In the Internal Ratings-based Approach (IRB) banks are allowed to use internal risk models to provide expected default probabilities or more inputs, e.g. loss given default, for the calibration of the weights.

This model describes the IRB approach where default probabilities of loans of a certain type are used to calculate capital requirements. The model is static so that the maturity of all loans is one period. The risk-weighted requirement is constructed such that the probability that unexpected losses of the asset portfolio exceed available equity is lower than a threshold α, i.e. the admissible probability of default set by the regulator.[14] Let us assume the regulator sets α for some representative bank. As a results, equity is insufficient to cover unexpected losses only with probability α for that bank.

The regulator infers the critical value of systematic risk $z_\alpha = \Phi^{-1}(1-\alpha)$ such that $Pr(z \leq z_\alpha) = 1-\alpha$. Consequently, if the representative bank holds at least $PD_\eta(z_\alpha)$ equity for each loan of type η, it is able to cover losses with probability $1-\alpha$. In detail, the capital requirement has two components: loan loss provisions for expected losses ($\overline{PD}_\eta$) and equity capital for unexpected losses ($PD(z) - \overline{PD}_\eta$). In this model the risk-adequate capital requirement for a loan of type η simplifies to

$$\beta_\eta = PD_\eta(z_\alpha) = \Phi\left(\frac{\zeta_\eta + \sqrt{\rho}\,\Phi^{-1}(1-\alpha)}{\sqrt{1-\rho}}\right) . \tag{20}$$

The requirement is additive for both types of loans given that banks hold a well-diversified portfolio within each class of loans (Vasicek, 2002). Since high-risk firms have a higher financial vulnerability ($\zeta_h > \zeta_l$), the capital requirement for high-risk loans is higher than for low-risk loans. The risk-weighted capital constraint of Basel II is given by

$$e \geq \beta_h q_{h,i} + \beta_l q_{l,i} \quad \textit{where} \quad 0 < \beta_l < \beta_h < 1 . \tag{21}$$

Adding the risk-weighted capital constraint to bank i's optimization problem gives

$$\begin{aligned} \underset{q_i}{\text{Max}} \quad & V(q_i, z) = E[\max\{\Pi_i(q_i, z), 0\}] \\ \text{s.t.} \quad & V(q_i, z) \geq r_e e \,, \quad 0 \leq q_i \leq W_i \,, \quad e \geq \beta_h q_{h,i} + \beta_l q_{l,i} \,. \end{aligned} \tag{22}$$

For a fixed amount of equity e, the highest total quantity a bank could possibly produce

[14] Confer Kiema and Jokivuolle (2014) for a detailed account of how default probabilities are effectively restricted by Basel II capital requirements in a representative bank model.

is $\frac{e}{\beta_l}$ when equity is fully invested in the low-risk market with a lower capital requirement. Let us therefore assume that $W_i > \frac{e}{\beta_l}$, so that the capital requirement is strictly more binding than the capacity constraint. Hence, expected profits are maximized under the participation constraint, the short-selling restriction, and the capital requirement.

Let $\Pi_i(q_i^s, z)$ denote the profit of bank i implementing strategy s. Banks can potentially implement one of five strategies: They can stay out of both markets and not offer any loans ($s = 0$), they can chose an unconstrained strategy ($s = uc$) according to eq. (14) but only if $e > \beta_h q_{h,i}^{uc} + \beta_l q_{l,i}^{uc}$, or they can make full use of their equity and specialize in high-risk loans ($s = h$), specialize in low-risk loans ($s = l$), or chose a mixed portfolio structure ($s = rw$). Figure 3 illustrates the notation and feasible strategies are

$$q_i^l = \left(0, \frac{e}{\beta_l}\right) \;,\; q_i^h = \left(\frac{e}{\beta_h}, 0\right) \;,\; q_i^{rw} = \left(q_{h,i}^{rw}, q_{l,i}^{rw}\right) \;,\; q_i^{uc} = \left(q_{h,i}^{uc}, q_{l,i}^{uc}\right) \;,\; q_i^0 = (0, 0) \tag{23}$$

where $q_{\eta,i}^{uc}$ is defined in eq. (14) and $q_{\eta,i}^{rw}$ is defined in eq. (23) in the Appendix. Of course, banks' expected payoff with constrained strategies (l, h, rw) is strictly higher than expected payoff with an unconstrained strategy or in case of non-participation. Therefore, I first discuss how banks choose between these three strategies.

Due to the limited availability of equity, the capital requirements introduce an interdependence between both types of loans. Through the capital constraint the any loan offered in one market reduces the capacity to offer loans in the other market. Because the requirement in eq. (21) is additive, banks enjoy no immediate advantage by diversifying their portfolio between different loan types. To illustrate this, consider the case where expected returns of both types of loans were equal, then these form of requirements incentivize banks to fully specialize in the type of loan which requires less capital. Furthermore, in a mixed portfolio, losses in one loan class are cross-subsidized by returns stemming from loans of the other type. To make use of limited liability, bankers would have incentives to operate both loan portfolios as separate entities (Repullo and Suarez, 2004; Kiema and Jokivuolle, 2014). Therefore, a specialized portfolio is always preferred over a mixed portfolio strategy if it is feasible. Moreover, by comparing $V_i(q_i^h, z)$ and $V_i(q_i^l, z)$, whenever

$$p_l(z) r_l(Q_l) - MC_i > \frac{\beta_l}{\beta_h} \left(p_h(z) r_h(Q_h) - MC_i\right) \tag{24}$$

bank i has incentives to fully specialize in low-risk loans. Equation (24) shows that banks trade off expected marginal return of offering a loan between the two markets discounted by the amount of capital required for offering it. It further shows that this trade-off depends on banks' marginal costs. Rearranging eq. (24) for MC_i gives the cutoff marginal costs of the bank with the lowest productivity which specializes on low-risk loans. It is therefore the cutoff of the low-risk market, meaning that only banks which marginal costs below this cutoff offer loans in the low-risk market.[15] It is denoted as $\widetilde{MC}^l$, and defined s.t.

$$\begin{aligned} &V_i(q_i^l, z) \geq V_i(q_i^h, z) \qquad \forall\, z \;\wedge\; \forall\, i \in \{1, ..., N\} : MC_i \leq \widetilde{MC^l} \\ &\text{where} \quad \widetilde{MC^l} = \frac{\beta_h p_l(z) r_l(Q_l) - \beta_l p_h(z) r_h(Q_h)}{\beta_h - \beta_l} \,. \end{aligned} \tag{25}$$

An equilibrium can only exist if this cutoff is positive and there are banks that specialize in low-risk loans as well as banks that specialize in high-risk loans. It follows that in equilibrium capital requirements pose an upper bound on the interest rate on high-risk loans relative to the interest rate of low-risk loans, i.e.

$$\frac{p_h(z) r_h(Q_h^*)}{p_l(z) r_l(Q_l^*)} < \frac{\beta_h}{\beta_l} \,. \tag{26}$$

Whereas in the unregulated equilibrium competitive pressures are the main force limiting bank size and determining the bank portfolio composition, under assumption 1 capital requirements pose much stricter limits on size and composition. In an unregulated equilibrium, the most productive bank gains the highest market share in both markets and is the largest bank. Consequently, it has the highest shadow price of being now constrained by the capital requirement. Hence, banks with higher marginal costs have lower shadow prices. Let the shadow price of being constrained by the risk-weighted capital requirement of bank i that chooses strategy s be denoted as μ_i^s[16] This negative relation between shadow prices and marginal costs implies that there exists another marginal cost cutoff for each specialization strategy ($s \in \{h, l\}$), denoted $\widetilde{MC}^{\mu_s}$, below which shadow prices would turn negative. This is not possible since shadow prices must be non-negative, i.e. $\mu_i^s \geq 0$, so that these cutoffs can be seen as feasibility constraints for choosing one of the specialization strategies. The following condition therefore ensures that strategy l is

[15] I show in the following that none of the banks with marginal costs above this cutoff chooses a mixed portfolio strategy so that this is indeed the cutoff in the low-risk market.

[16] See Appendix Proof of Proposition 2 for a full account of the optimization problem with a Lagrange objective function where μ_i is the Lagrange parameter of the capital constraint.

feasible for all banks with marginal costs below $\widetilde{MC}^l$, i.e. $\mu_i^l \geq 0$ and $\widetilde{MC}^l < \widetilde{MC}^{\mu_l}$ if[17]

$$-\frac{\beta_l}{\beta_h - \beta_l}(p_h(z)r_h(Q_h^*) - p_l(z)r_l(Q_l^*)) < \frac{e}{\beta_l}p_l(z)r_l'(Q_l^*) \,. \tag{27}$$

Similarly, strategy h is only feasible for banks with marginal costs above $\widetilde{MC}^l$ if $\mu_i^h \geq 0$. Let the cutoff marginal costs of the least productive bank that specializes on high-risk loans and is fully constrained by the capital requirement be denoted as $\widetilde{MC}^{\mu_h}$ and defined as

$$\begin{aligned} &V_i\left(q_i^l, z\right) < V_i\left(q_i^h, z\right) \;\wedge\; \mu_i^h \geq 0 \qquad \forall\, z \;\wedge\; \forall\, i \in \{1, ..., N\} : \widetilde{MC}^l < MC_i \leq \widetilde{MC}^{\mu_h} \\ &\text{where} \quad \widetilde{MC}^{\mu_h} = p_h(z)r_h(Q_h) + \frac{e}{\beta_h}p_h(z)r_h'(Q_h) \,. \end{aligned} \tag{28}$$

This, however, must not be the least productive bank active in the banking market. Still, banks with marginal costs higher than $\widetilde{MC}^{\mu_h}$ could be active in equilibrium as long as their participation constraint (eq. 8) is non-negative. The participation constraint determines the cutoff for the least productive bank that offers loans in the banking market. Since r_e can be set arbitrarily, this cutoff could be set such as to coincide with $\widetilde{MC}^{\mu_h}$. Anyhow, if we allow banks with marginal costs higher than $\widetilde{MC}^{\mu_h}$ to be active, in principle, these banks could choose the unconstrained strategy (uc) or constrained mixed strategy (rw). But $\widetilde{MC}^{\mu_{rw}} < \widetilde{MC}^{\mu_h}$ and $p_l(z)r_l(Q_l) < \widetilde{MC}^{\mu_h}$ if

$$(p_h r_h(Q_h) - p_l r_l(Q_l)) < -\frac{e}{\beta_h}p_h r_h'(Q_h) \tag{29}$$

so that any bank with $MC_i > \widetilde{MC}^{\mu_h}$ chooses the unconstrained strategy (q_i^{uc}) with $q_{h,i}^{uc}$ defined in eq. (14) and $q_{l,i}^{uc} = 0$. Under eq. (29), the cutoff defined by the participation constraint determines the least productive bank offering loans in the market for high-risk loans since then all banks with $MC_i > \widetilde{MC}^l$ specialize in high-risk loans. Let this cutoff marginal costs be denoted as $\widetilde{MC}^h$ and defined as

$$V_i^{uc}\left(q_i^{uc}, z\right) - r_e e \geq 0 \qquad \forall z \;\wedge\; \forall\, i \in \{1, ..., N\} : \widetilde{MC}^{\mu_h} < MC_i \leq \widetilde{MC}^h \tag{30}$$

Hence all banks with marginal costs below $\widetilde{MC}^l$, i.e. the more productive banks, specialize in low-risk loans while banks with marginal costs above $\widetilde{MC}^l$, i.e. the less productive

[17] This condition is derived in more detail in the Appendix in the proof of Proposition 2.

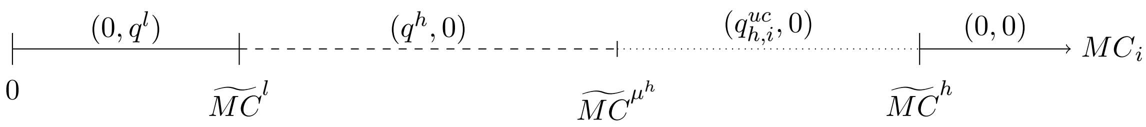

Figure 2: Optimal strategies and cutoff marginal costs in the Basel II equilibrium.

banks, specialize in high-risk loans. The equilibrium is illustrated in Figure 2 and is summarized in Proposition 2.

Proposition 2 (Basel II equilibrium)**.**
Consider the case with additive risk-weighted capital requirements. If eq. (26)*, (27), and (29) hold, then banks with marginal costs* $MC_i \leq \widetilde{MC}^l$ *specialize in low-risk loans while less productive banks with marginal costs* $\widetilde{MC}^l < MC_i \leq \widetilde{MC}^h$ *specialize in high-risk loans in equilibrium.*

Proof. It follows from eq. (25) and the arguments above. Detailed proof is in the appendix □

Based on these equilibrium strategies, it is possible to derive an order of all banks beginning with the bank with the lowest distance to default ($\hat{z}$) to the bank with the highest distance to default. The direct effect of productivity advantages on the critical value of systematic risk $\hat{z}_i$ is positive, i.e. banks with lower marginal costs *ceteris paribus* have higher profits. Positive profits constitute positive charter value and add to loss absorbing capacity. Therefore, when comparing banks that choose the same strategy, the relationship between productivity and default probability is straightforward. These banks offer the same quantities and earn the same interest rate. Hence, banks with lower marginal costs have lower default probabilities than banks with higher marginal costs that are active in the same loan market.

When comparing specialists on the high-risk and low-risk market, e.g. those two banks with marginal costs right below and above the cutoff $\widetilde{MC}^l$, the relationship between productivity and distance to default is not straightforward. On the one hand, high-risk specialists have a riskier investment strategy and higher costs. On the other hand, they are less levered and earn a higher return on their non-defaulting loans. If we impose a stricter limit on the upper bound of the high-risk market interest rate than eq. (26) and therewith limit the influence of the interest rate channel, a relationship can be clearly stated. In that case, the direct disadvantage of higher costs and the more risk-intensive investment outweigh the positive effect of lower leverage, so that banks with higher productivity are definitely less likely to default. Lemma 3 summarizes.

Lemma 3 (Risk taking in the Basel II equilibrium)**.** *In equilibrium, more productive banks have lower default probabilities than less productive banks in the same market, i.e.*

$$\begin{aligned} \hat{z}_i > \hat{z}_{i+1} \quad \forall\, i \in \{1, ..., n_l\} : \; q^*_{l,i} > 0 \\ \hat{z}_i > \hat{z}_{i+1} \quad \forall\, i \in \{n_l+1, ..., n_h\} : \; q^*_{h,i} > 0 \; . \end{aligned} \tag{31}$$

If $p_h(z) r_h(Q^*_h) < \frac{\beta_h}{\beta_l} p_h(z) r_l(Q^*_l)$*, more productive banks have lower default probabilities even across markets, i.e.*

$$\hat{z}_i > \hat{z}_{i+1} \quad \forall\, i \in \{1, ..., N\} : \; q^*_i > 0 \; . \tag{32}$$

Proof. Proof is in the appendix. □

4.2 Basel III equilibrium

Among other measures aimed at capital adequacy, the Basel III accord introduced the leverage ratio. The motives of the regulator were driven by macro- as well as micro-prudential considerations. In order to comply, banks need to back up 3% of their total exposure with Tier 1 equity capital. Total exposure includes on-balance as well as off-balance sheet assets. The leverage ratio capital constraint of Basel III is given by β according to

$$e \geq \beta \left(q_{h,i} + q_{l,i}\right) \quad \textit{where} \quad 0 < \beta_l < \beta < \beta_h < 1 \, . \tag{33}$$

Adding the leverage ratio to the risk-weighted capital constraint in bank i's optimization problem gives

$$\begin{aligned} \underset{q_i}{\text{Max}} \quad & V(q_i, z) = E[\max\{\Pi_i(q_i, z), 0\}] \\ \text{s.t.} \quad & V(q_i, z) \geq r_e e \; , \quad 0 \leq q_i \leq W_i \; , \\ & e \geq \beta_h q_{h,i} + \beta_l q_{l,i} \; , \quad e \geq \beta \left(q_{h,i} + q_{l,i}\right) \; . \end{aligned} \tag{34}$$

The additional constraint reduces the set of feasible strategies. The shaded area including the bounding line segments in fig. 3 illustrates the set of feasible strategies of bank i. Banks could implement one of seven strategies ($s \in \{l, h, rw, lr, v, uc, 0\}$). Of these full specialization on high-risk loans (h), a mixed strategy constrained by the risk-weighted

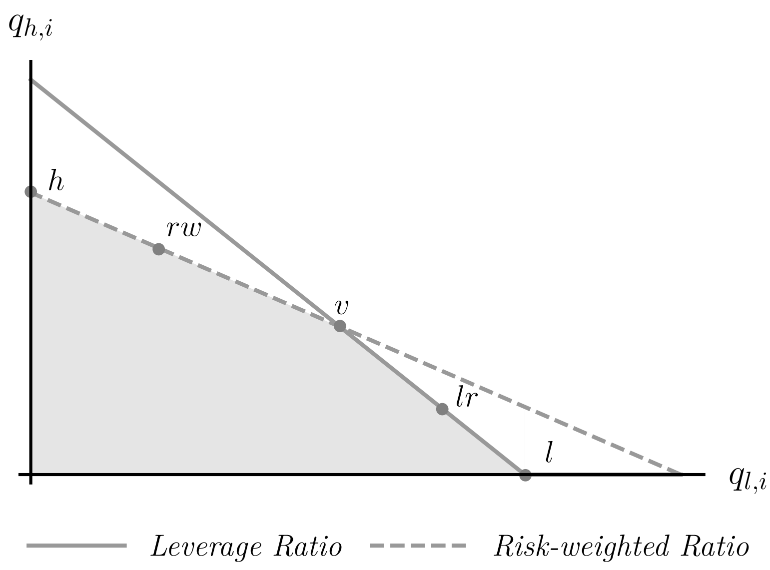

Figure 3: Feasible quantities under both capital requirements.

ratio (rw), an unconstrained strategy (uc), and the non-participation strategy (0) are known from the previous section. Specialization on low-risk loans as given in strategy l according to eq. (23) is not feasible under the leverage ratio. Banks that fully specialize in low-risk loans have to reduce the amount of loans offered because the relevant constraint will be given by the leverage ratio β and not the risk-weight β_l. Furthermore, banks could choose a mixed portfolio strategy where the leverage ratio is the binding constrained (lr). Lastly, as a special case of strategies lr and rw, banks could chose exactly the one mixed portfolio allocation which is constrained by the leverage ratio as well as the risk-weighted constraint. I call this strategy v as it appears on the vertex of the feasible set in fig. 3. These feasible strategies are denoted by

$$\begin{aligned} q_i^l &= \left(0, \frac{e}{\beta}\right) , \quad q_i^h = \left(\frac{e}{\beta_h}, 0\right) , \quad q_i^{rw} = \left(q_{h,i}^{rw}, q_{l,i}^{rw}\right) , q_i^{lr} = \left(q_{h,i}^{lr}, q_{l,i}^{lr}\right) , \\ q_i^v &= \left(q_{h,i}^v, q_{l,i}^v\right) , \quad q_i^{uc} = \left(q_{h,i}^{uc}, q_{l,i}^{uc}\right) , \quad q_i^0 = (0,0) . \end{aligned} \tag{35}$$

where $q_{\eta,i}^{uc}$ is defined in eq. (14), $q_{\eta,i}^{rw}$ is defined in eq. (23), $q_{\eta,i}^{lr}$ is defined in eq. (123) in the Appendix, and $q_{\eta,i}^v$ is

$$q_i^v = \left(\frac{(\beta - \beta_l)e}{\beta(\beta_h - \beta_l)}, \frac{(\beta_h - \beta)e}{\beta(\beta_h - \beta_l)}\right) . \tag{36}$$

Since the leverage ratio poses extra costs on banks specializing on low-risk loans, it sets incentives to shift the portfolio toward riskier assets. As mentioned, specialization on low-risk loans cannot be realized on the same scale as before. Therefore, entering into the high-risk market is now favorable for banks that previously specialized on low-risk loans. These banks change their strategy to strategy v where both constraints are binding (or the leverage ratio is not binding anymore). As we will see, for the remainder of banks it is still optimal to specialize in high-risk loans as long as it is feasible. As in Kiema and Jokivuolle (2014), in the Basel III equilibrium banks either choose this special mixed strategy or specialize. In the following I examine how this choice depends on banks' productivity and derive cutoff marginal costs for these choices and therewith for both loan markets.

Let $\widetilde{MC^l}$ denote the cutoff marginal costs between banks choosing strategy v and banks choosing strategy h. Since only banks that choose strategy v offer loans to low-risk entrepreneurs, $\widetilde{MC^l}$ defines the marginal costs of the bank with the lowest productivity that still participates in the low-risk market. Let $\widetilde{MC^l}$ be defined by

$$\begin{aligned} &V_i^v\left(q_i^v, z\right) \geq V_i^h\left(q^h, z\right) \qquad \forall\, z \;\wedge\; \forall\, i \in \{1, ..., N\} : MC_i \leq \widetilde{MC^l} \\ &\text{where} \quad \widetilde{MC^l} = \frac{\beta_h p_l(z) r_l(Q_l) - \beta_l p_h(z) r_h(Q_h)}{\beta_h - \beta_l}\,. \end{aligned} \tag{37}$$

Note that formally eq. (25) and eq. (37) are equal, hence the same equilibrium condition (eq. 26) applies. The value of the cutoff, however, changes since the Basel III equilibrium has different aggregate quantities and therefore different interest rates compared to the Basel II equilibrium.

Since this problem has two capital requirements, there are two shadow prices of being constrained. In addition to μ_i^s, let the shadow price of being constrained by the leverage ratio of bank i that chooses strategy s be denoted as λ_i^s. Strategy v is only feasible for banks with marginal costs below $\widetilde{MC^l}$ as long as shadow prices are non-negative, i.e. $\mu_i^v \geq 0$ and $\lambda_i^v \geq 0$. This is the case for all banks with marginal costs above $\widetilde{MC^l}$ if

$$\tfrac{\beta(\beta_h-\beta_l)}{\beta_h(\beta_h-\beta)}\left(\beta_l p_h(z) r_h(Q_h) - \beta_h p_l(z) r_l(Q_l)\right) + \tfrac{\beta_l(\beta-\beta_l)}{\beta_h(\beta_h-\beta)} p_h(z) r_h'(Q_h) e < p_l(z) r_l'(Q_l) e\,. \tag{38}$$

Strategies l and lr that both have a higher share of low-risk loans than strategy v are strictly dominated by strategy v irrespective of banks' marginal costs. Furthermore, $V_i(q^h,) \geq V_i(q^{rw}, z)$ for all banks with marginal costs higher than $\widetilde{MC^l}$. Therefore, these banks specialize in high-risk loans. Let the least productive bank that specializes on high-risk loans and is fully constrained by capital requirements define the cutoff $\widetilde{MC}^{\mu_h}$ as

$$\begin{aligned} &V_i\left(q^v, z\right) < V_i\left(q^h, z\right) \ \wedge \ \mu_i^h \geq 0 \qquad \forall z \ \wedge \ \forall\, i \in \{1, ..., N\} : \widetilde{MC}^l < MC_i \leq \widetilde{MC}^{\mu_h} \\ &\text{where} \quad \widetilde{MC}^{\mu_h} = p_h(z) r_h(Q_h) + \frac{e}{\beta_h} p_h(z) r_h'(Q_h) \ . \end{aligned} \tag{39}$$

As in the Basel II equilibrium, banks with marginal costs above $\widetilde{MC}^{\mu_h}$ can participate in the market as long as they meet their participation constraint. This constraint, as argued before, can be set arbitrarily including in such a way as to coincide with $\widetilde{MC}^{\mu_h}$. If this is not done, banks with marginal costs above $\widetilde{MC}^{\mu_h}$ that still participate could potentially choose among strategies lr, l, rw, or uc. Shadow prices of the strategies lr, l, rw would violate the non-negativity constraint if

$$-\beta_h \left(p_h(z) r_h(Q_h) - p_l(z) r_l(Q_l)\right) < p_h(z) r_h'(Q_h) e < -\beta \left(p_h(z) r_h(Q_h) - p_l(z) r_l(Q_l)\right) \tag{40}$$

so that these banks choose the unconstrained strategy. Eq. (40) implies that $\widetilde{MC}^{\mu_h} > p_l(z) r_l(Q_l)$. Therefore, banks with marginal costs higher than $\widetilde{MC}^{\mu_h}$ specialize in high-risk loans as well where $q_{h,i}^{uc}$ is defined in eq. (14) and $q_{l,i}^{uc} = 0$. Let the marginal costs of the least productive bank to do so denote the cutoff marginal costs in the high-risk market as $\widetilde{MC}^h$ which is defined by

$$V_i\left(q_i^{uc}, z\right) - r_e e \geq 0 \qquad \forall\, i \in \{1, ..., N\} : \widetilde{MC^{\mu_h}} < MC_i \leq \widetilde{MC}^h \tag{41}$$

Hence, banks with marginal costs below $\widetilde{MC}^l$ choose the exact same mixed strategy v where they offer low-risk as well as high-risk loans while all other banks that are able to meet their participation constraint specialize in providing high-risk loans. The Basel III equilibrium is illustrated in the lower half of Figure 4 and is summarized in the following proposition.

Proposition 3 (Basel III equilibrium)**.**
Consider the case with additive risk-weighted capital requirements and a leverage ratio. If eq. (26), (38), and (40) hold, then banks with marginal costs $MC_i \leq \widetilde{MC}^l$ hold a mixed portfolio while less productive banks with marginal costs $\widetilde{MC}^l < MC_i \leq \widetilde{MC}^h$ specialize in high-risk loans in equilibrium.

Proof. Proof is in the appendix. □

Note that the cutoffs defined above for the Basel III equilibrium are only formally the same as for the Basel II equilibrium in eq. (25), (28), and (30). Because the interest rates in both equilibria are not necessarily the same, the values of these cutoffs differ between the Basel II and Basel III equilibrium. In fact, the number of banks in the low-risk market can only increase and therefore the number of active banks in the high-risk market increases as well.

Corollary 1 (Change in market cutoff marginal costs)**.** Comparing the portfolio choices in the Basel II and Basel III equilibrium, the cutoffs for marginal costs increase, i.e.

$$\widetilde{MC^l}^{BaselII} < \widetilde{MC^l}^{BaselIII} \tag{42}$$

and

$$\widetilde{MC^h}^{BaselII} < \widetilde{MC^h}^{BaselIII} \tag{43}$$

Proof. Proof is in the appendix. □

Proposition 4 (Market share reallocation and average productivity)**.**
By tightening capital requirements through the introduction of a leverage ratio, market shares in the low-risk market are reallocated towards less productive banks while market shares in the high-risk market are reallocated towards more productive banks and less productive new entrants. Because of these entrants, the banking market has a lower average productivity.

Proof. Proof follows directly from Proposition 3 and Corollary 1. □

The results of Corollary 1 are illustrated in Figure 4. Taking the order of N banks according to their marginal costs, I distinguish six groups of banks according to whether they are affected or unaffected by the leverage ratio (i.e. whether they change their strategies between the Basel II and Basel III equilibrium) and whether they are constrained or unconstrained: (i - solid line segment) low-risk market incumbents, (ii - dashed) affected constrained high-risk market incumbents, (iii - solid) unaffected constrained high-risk market incumbents, (iv - dashdotted) affected unconstrained high-risk market incumbents, (v - solid) unaffected unconstrained high-risk market incumbents, and (vi - dotted) new entrants.

The most productive banks are the low-risk market incumbents (i). Their business model is affected directly by the leverage ratio. They react by shifting their portfolio and choosing the mixed strategy v. Thereby they reduce their supply of low-risk loans in order to compensate the additional cost of being constrained with higher loan rates which are available in the high-risk market. This in turn makes the low-risk market attractive for less productive banks that shift from a specialized high-risk into a mixed portfolio strategy (ii). The high-risk market gets more competitive as more productive banks enter it. In a Cournot-equilibrium with asymmetric costs, an increase in the number of banks in a market implies that supply is reduced and prices increase. This phenomenon is termed "anti-competitive" behavior by Amir and Lambson (2000).[18] Some specialized banks in the high-risk market are unaffected by the leverage ratio and do not change their strategy (iii), although they profit from the increase in the high-risk interest rate. Formerly unconstrained banks are able to increase their supply of loans so that some of them grow to point where they are constrained by the risk-weighted ratio (iv) and others grow as well but less (v). Finally, since expected revenue in the high-risk market is higher in the new equilibrium, new banks enter the high-risk market (vi). As a result, market shares are reallocated between heterogeneous banks. More productive banks lose market shares in the market for low-risk loans but gain shares in the other market. Less productive high-risk markets incumbents lose market shares.

The reallocation of market shares in the low-risk market implies that the average productivity of banks participating in that market decreases. On the other hand average productivity in the high-risk market might increase, i.e. if the number of new entrants is relatively small. In the unregulated equilibrium, the most productive banks dominate both markets. Hence, any capital requirement indirectly protects market shares of less productive banks in the affected market.

Another implication of the model is that the regulator faces a trade-off between ample credit supply and higher equity ratios. As mentioned above, in order to cope with the additional capital requirement banks reduce aggregate credit supply in both markets so that they are able to maintain profitability.

Lemma 4 (Effect on interest rates). *By tightening capital requirements through the in-*

[18] To rationalize this, consider that the competitive outcome is achievable in this model if the most productive bank 1 chooses to push every other bank out of the market by producing very high quantities at its marginal costs. Therefore, the more banks are active in equilibrium, the closer market outcomes are to monopoly outcomes. See sec. 5 for a discussion on how crucial the Cournot market is for the results.

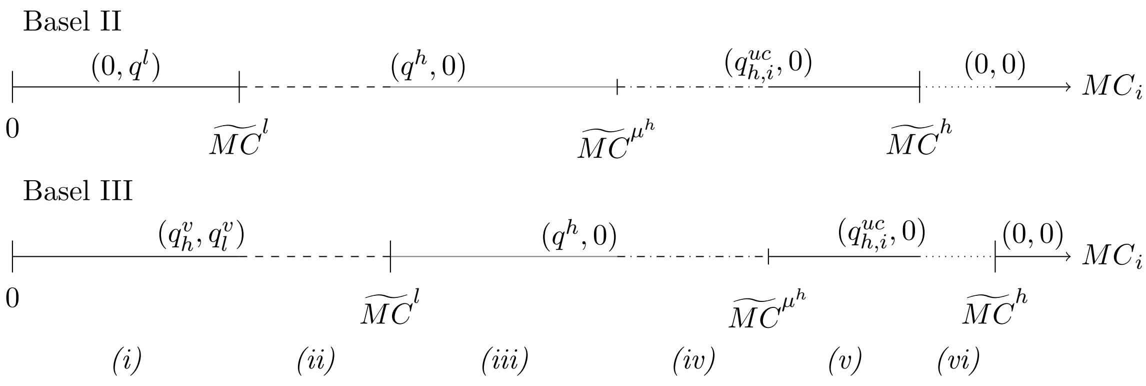

Figure 4: Optimal strategies and cutoff marginal costs in the Basel II equilibrium (upper line) and the Basel III equilibrium (lower line). Roman numbers on the bottom indicate groups of banks according to their change in strategy from the Basel II to Basel III equilibrium.

troduction of the leverage ratio, aggregate loan supply decreases and interest rates increase in both markets.

$$r_\eta(Q^{*BaselII}_\eta) < r_\eta(Q^{*BaselIII}_v) \quad \eta \in \{h, l\} \, . \tag{44}$$

Proof. Follows directly from Corollary 1. □

In terms of solvency, one might ask whether the risk-shifting of banks with high productivity increases their default probabilities as in Koehn and Santomero (1980) or if this effect is compensated by the increase of loss absorbing capital as in Acosta Smith et al. (2017). Besides the most productive banks that are directly affected by the leverage ratio and shift into the riskier loan class and reduce their leverage (group i), a subgroup of banks reacts in the opposite way (group ii). Because of the heterogeneity of banks, the effect of the leverage ratio differs between the six categories defined above. I focus on the first two groups (i) and (ii) because these change the portfolio composition of loans between the equilibria.

Lemma 5 (Risk shifting in the Basel III equilibrium)**.** *The risk-shifting channel introduced through the additional regulation by a risk insensitive capital requirement does not increase default probabilities as long as systematic risk realizes below a threshold $\tilde{z}$.*

Formally,

$$\exists\ \tilde{z}\ : 0 < \tilde{z} < \hat{z} \quad s.t. \quad \begin{cases} \dfrac{\partial \Pi_i}{\partial \gamma_i} \geq 0 \quad if \quad z \leq \tilde{z} \\ \dfrac{\partial \Pi_i}{\partial \gamma_i} < 0 \quad if \quad z > \tilde{z} \end{cases}$$
$$where \quad \hat{z} = \frac{-\zeta_h^2 + \zeta_l^2 + 2\ln\left(\frac{r_h(Q_h)}{r_l(Q_l)}\right)(1-\rho)}{2\sqrt{\rho}\,(\zeta_h - \zeta_l)}\ . \tag{45}$$

Proof. Formal proof is in the appendix. □

Additional to the risk-shifting channel, the total effect on default probabilities depends on an interest rate channel and a leverage channel. As interest rates increase, profits of all banks c.p. increase rendering them more resilient.[19] Since the most productive banks of group (i) increase the share of high-risk loans which offer higher yields, the interest rate channel has a positive effect in reducing their likeliness to default. According to the leverage channel, banks are c.p. less likely to default if they finance their assets with a higher equity share. Since equity is normalized among banks, banks of group (i) increase their individual leverage ratio by reducing debt which makes them more resilient to default. To sum up, as long as systematic risk realizes below the threshold $\tilde{z}$, default probabilities of the most productive banks decrease even though they shift their portfolio into the riskier loan class. The opposite holds for banks of group (ii) for the leverage as well as interest rate channel.[20]

4.3 Comparative Statics

Next I consider how the aforementioned effects change if a stricter leverage ratio is implemented. The level of the leverage ratio directly determines the optimal portfolio structure and quantities of the most productive banks, i.e. strategy v, and indirectly interest rates and cutoffs in equilibrium.

Proposition 5 (Comparative statics)**.**
A tightening of the leverage ratio increases the cutoff marginal costs of the low-risk market,

[19] Default probabilities of banks of group (iii) therefore decrease. These banks profit from increasing rates but do neither change their portfolio composition nor size nor leverage.

[20] Unconstrained banks of group (iv) and (v) encounter ambiguous effects: Their default probabilities are reduced by increasing interest rates but increased through higher leverage since these banks are able to expand.

and decreases the cutoff marginal costs of the high-risk market, i.e.

$$\frac{\partial \widetilde{MC}^{l}}{\partial \beta} > 0 \,, \quad \frac{\partial \widetilde{MC}^{\mu_h}}{\partial \beta} < 0 \,, \quad \frac{\partial \widetilde{MC}^{h}}{\partial \beta} < 0 \tag{46}$$

Therefore, average productivity in the low-risk market decreases while average productivity in the high-risk market as well as the overall banking market increases.

Proof. Proof is in the appendix. □

A stricter leverage ratio ensures that less new entrants with low productivity enter the high-risk market so that average productivity in the banking market decreases less compared to the introduction of a more lenient leverage ratio. The reallocation of market shares within each market gets stronger. Market shares on the high-risk market are reallocated more strongly toward banks with high productivity and market shares on the low-risk market are reallocated more strongly toward banks with low productivity, respectively.

Lemma 6 (Comparative effect on interest rates)**.** *A tightening of the leverage ratio increases supply of high-risk loans but decreases supply of low-risk loans, i.e.*

$$\frac{\partial r_h(Q_h)}{\partial \beta} < 0 \,, \quad \frac{\partial r_l(Q_l)}{\partial \beta} > 0 \,. \tag{47}$$

Proof. Proof is in the appendix. □

Since banks with high productivity have to reduce a higher amount of debt with stricter leverage ratio, the supply of low-risk loans decreases even more than compared with a more lenient ratio while the supply of high-risk loans decreases less. On the other hand, this implies a stricter leverage ratio checks the increase of the interest rate for high-risk loans. Regarding the effect on banks' solvency, the positive effect of the interest rate channel is weakened while clearly the positive effect of the leverage channel is strengthened. Consequently, a tightening of the leverage ratio does not necessarily decrease default probabilities.

5 Discussion

The model highlights how regulation naturally interferes with regular market forces and thus creates side effects on market structure with potential repercussions on financial stability. Productivity, irregardless of whether it stems from advantages in technology or information, influences banks' strategies and price setting. And ultimately, it influences market structure. As discussed in section 1, empirical findings advocate the underlying idea in my model that differences between banks matter for market outcomes.

A limitation to the model surely is the assumption that equity is fixed and the same amount for all banks. This serves to make banks comparable at some level and illustrates especially in the constrained equilibria the optimal portfolio choice for a given unit of equity. When in fact, productivity advantages and intangible charter value should be priced in equity markets in a way that more productive banks find it easier to refinance themselves. Instead of rebalancing their portfolio towards riskier assets, banks could as well increase equity in reaction to the leverage ratio. Indeed, banks raised equity ever since the ratio was announced and monitored (Basel Committee on Banking Supervision, 2016; Acosta Smith et al., 2017). Investors should have been aware that the capital was needed to comply to tightened regulatory guidelines. However, in this model the problem for more productive banks is moving from the product to the equity market. Loosening constraints by raising equity allows banks to move closer to an unregulated equilibrium where productivity sponsors market shares and size. Consequently, if a leverage ratio were to be binding for any bank at all, it still were binding for the more productive banks even if they do not change their portfolio composition as a response.

Another critical assumption is Cournot competition. It implies that lower concentration comes along with less competitive outcomes (Amir and Lambson, 2000). Therefore, the set-up of the model is related to Efficient Structure theories. As discussed in section 1, empirical support can be found for efficient structure as well as structure-conduct-performance hypothesis. For the sake of this model, the Efficient Structure like results are desirable since they imply the trade-off between productivity and regulation as the leverage ratio which can be seen as a tax on bank size. Furthermore, the study of Cournot competition allows me to derive the general equilibrium type effects of capital regulation, such as the rise of interest rates and the ensuing reallocation of market shares and decrease in average productivity, which constitute the main contribution of my model. Nevertheless, Cournot competition might misrepresent actual competition in loan markets where banks seem to quote prices rather than quantities. Hence Bertrand-type competition should yield a more realistic picture. However, Schliephake and Kirstein (2013) show that a two-stage game where banks first chose their capacity and then compete in Bertrand-style renders the same results as Cournot competition, especially if their capacity is capital constrained.

The focus of my work lies on the evaluation of capital requirements. Apart from the unintended side effects on the productivity distribution, the model also points out a positive effect on banks that are not directly affected by the introduction of a leverage ratio. These banks profit indirectly through the increasing interest rates which c.p. increases their distance to default. Nevertheless, this effect hinges on exactly this anti-competitive behavior brought about through the assumption of Cournot competition. In a competitive setting where banks cannot influence market loan rates, it is reasonable to conjecture that less productive banks would exit the market if new regulation causes additional costs. In fact, this is the case in the model when moving from the unregulated equilibrium to the Basel II equilibrium. But since banks are already constrained when the leverage ratio is introduced, they can circumvent incurring the costs of being regulated by adapting their business model and entering the high-risk market.

6 Conclusion

I study the optimal strategy choice under competing minimum capital requirements for heterogeneous banks. My model points to the fact that productivity influences banks' exposures to risk systematically so that regulation indirectly affects certain types of banks. As a result, capital requirements shape the market structure in banking.

The model shows that the introduction of the leverage ratio in combination with an existing risk-weighted ratio directly affects banks with high productivity. This is because their productivity advantage induces them to chose a less risky strategy under risk-weighted regulation which can be operated at a higher scale. They react to the leverage ratio by entering into the high-risk loan market. However, this higher share of high-risk loans in their portfolios does not increase their default probabilities, at least not as long as systematic risk is moderate. It induces a reallocation of market shares from more to less productive banks in the low-risk market. Average productivity in the low-risk market falls. These could be viewed as possible side effects of the current regulation. On the other hand, market shares in the high-risk market are distributed among a larger number of banks, including banks with high productivity. Compared to the Basel II equilibrium where high-risk loans are concentrated on low-productivity banks, this dispersion identified in my analysis highlights an unintended benefit of the new capital regulation regime if diversity is considered to enhance financial stability (Wagner, 2010).

Under the revision of the regulatory framework caused by the financial crisis numerous new instruments were implemented and discussed. It is important to consider the differential treatment caused by the interplay of different measures. As my model shows, the interaction of capital requirements can have important distributional effects. The results could apply to other measures. For example, capital requirements on operational risk charge banks based on their gross income. While gross income is used as a proxy of risk caused by complexity, it is reasonable to assume that gross income depends on productivity as well. Productivity is hard to measure. Yet the model illustrates that it can create a positive charter value in an imperfect competitive environment. Since it might be a difficult to impossible task to formulate any requirements contingent on productivity in order to regulate heterogeneous banks, capital regulation should at least contemplate possible channels between productivity and risk. One approach taken by policy makers is to make regulation proportional (Restoy, 2019). If risk measures are positively correlated to productivity measures, regulating these risks turns intangible charter value into observable capital. Generally, the banking market would be more transparent but banks might turn out not necessarily safer while market shares might be reshuffled. If on the other hand risk measures are negatively correlated to productivity, regulating these risks is more than called for. By using approaches with heterogeneous instead of representative banks, further theoretical work could systematically address the complex relationship between risk, capital, and productivity.

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Appendix

A.1 Proof of Lemma 1

Proof. This proof applies the results of Vives (2001b) and checks whether the conditions formulated therein are met in all games. According to Vives (2001b) Theorem 2.1, a Nash equilibrium for a game with strategy set Ω_i, payoffs V_i, and players $i \in \{1, \ldots, N\}$ exists, if

a) strategy sets Ω_i are non-empty, convex, and compact subsets of Euclidean space, and

b) payoff V_i is continuous in the actions of all firms and

c) quasi-concave in its own action.

a) The strategy set of bank i consists of all possible quantities of loans. The model facilitates the view of a bank to a simple loan generating and deposit taking intermediary and therefore abstracts from other financial products where negative positions would be attainable. A potential strategy is therefore non-negative and the strategy set focuses on the upper right quadrant of $\mathbb{R}^2$ which is a non-empty convex set and subset of Euclidean space. Since zero is included in the strategy set, it is closed. Given a capacity limit $0 \leq q_i \leq W_i$, the set is bounded. The Heine-Borel theorem states that any bounded and closed subset of Euclidean space is also compact. Consequently, the first condition is met by an unregulated market.

The capital requirements essentially lower the upper bound on the strategy set. Both constraints are linear and define a triangle in $\mathbb{R}^2$, which is convex. Figure 3 illustrates both constraints. In the case of joint regulation with both constraints, the strategy set is an intersection of the two strategy sets of the preceding games which are both convex. Hence, their intersection is convex as well. In all constrained cases, they include the upper bound and zero as the lower bound. Consequently, strategy sets of the constrained games are non-empty, convex, and compact subsets of Euclidean space. Let the strategy set Ω_i be defined as

$$
\begin{aligned}
&\textit{(without constraints)} && \Omega_i = \{q_i \mid 0 \leq q_i \leq W_i\} \\
&\textit{(risk-weighted)} && \Omega_i = \{q_i \mid 0 \leq \beta_h q_{h,i} + \beta_l q_{l,i} \leq e\} \\
&\textit{(both constraints)} && \Omega_i = \{q_i \mid 0 \leq \max\left[\beta_h q_{h,i} + \beta_l q_{l,i}, \beta(q_{h,i} + q_{l,i})\right] \leq e\}
\end{aligned} \tag{48}
$$

b) The payoff function of bank i is given as

$$
\begin{aligned}
V_i(q_i, z) &= E[max\{\Pi_i(q_i, z), 0\}] \\
&= \int_{-\infty}^{\hat{z}(q_i)} \left(p_h(z) r_h(Q_h) - MC_i\right) q_{h,i} + \left(p_l(z) r_l(Q_l) - MC_i\right) q_{l,i} \; d\Phi(z)
\end{aligned} \tag{49}
$$

where continuity follows from the continuity of its components and the existence of the integral

$$
\int_{-\infty}^{\hat{z}(q_i)} PD_\eta(z) \; d\Phi(z) = \int_{-\infty}^{\hat{z}(q_i)} \Phi\left(\frac{\zeta_\eta + \sqrt{\rho} z}{\sqrt{1-\rho}}\right) \phi(z) \; dz \; . \tag{50}
$$

The inverse demand functions $r(Q)$ are continuous by definition and q_i itself is continuous. Hence their product and difference is. Adding constraints was shown to alter the strategy space but not the payoff function. Therefore, the second condition for the existence of an equilibrium is fulfilled in all scenarios.

c) Profits are quasi-concave with respect to banks' own strategy choices, if all principal minors of the bordered Hessian matrix of $V_i(q_i, z)$ are of alternating signs. That is $V_i(q_i, z)$ is quasiconcave with respect to q_i if

$$
(-1)^r det H_r \geq 0 \quad \forall r = 1, 2 \tag{51}
$$

where H_r, the rth-order Bordered Hessians of $V(q_i, z)$ holding Q_{-i} constant, are

$$
H_1 = \begin{pmatrix} 0 & \frac{\partial V_i}{\partial q_{l,i}} \\ \frac{\partial V_i}{\partial q_{l,i}} & \frac{\partial^2 V_i}{\partial q_{l,i}^2} \end{pmatrix}, \; H_2 = \begin{pmatrix} 0 & \frac{\partial V_i}{\partial q_{l,i}} & \frac{\partial V_i}{\partial q_{h,i}} \\ \frac{\partial V_i}{\partial q_{l,i}} & \frac{\partial^2 V_i}{\partial q_{l,i}^2} & \frac{\partial^2 V_i}{\partial q_{l,i} \partial q_{h,i}} \\ \frac{\partial V_i}{\partial q_{h,i}} & \frac{\partial^2 V_i}{\partial q_{h,i} \partial q_{l,i}} & \frac{\partial^2 V_i}{\partial q_{h,i}^2} \end{pmatrix} . \tag{52}
$$

In the following I drop subscript i for brevity. The first principal minor is non-positive by construction and given as

$$detH_1 = -\left(\frac{\partial V}{\partial q_l}\right)^2 \leq 0 \; . \tag{53}$$

The second principal minor is

$$detH_2 = \frac{\partial V}{\partial q_l}\frac{\partial V}{\partial q_h}\frac{\partial^2 V}{\partial q_l \partial q_h} + \frac{\partial V}{\partial q_l}\frac{\partial V}{\partial q_h}\frac{\partial^2 V}{\partial q_h \partial q_l} - \frac{\partial^2 V}{\partial q_l^2}\left(\frac{\partial V}{\partial q_h}\right)^2 - \frac{\partial^2 V}{\partial q_h^2}\left(\frac{\partial V}{\partial q_l}\right)^2 \tag{54}$$

which can be rewritten as

$$\frac{1}{-\frac{\partial \Pi}{\partial z}}\left(\frac{\partial \Pi}{\partial q_h}\frac{\partial V}{\partial q_l} - \frac{\partial \Pi}{\partial q_l}\frac{\partial V}{\partial q_h}\right)^2 - \int_{-\infty}^{\hat{z}} \frac{\partial^2 \Pi}{\partial q_h^2} d\Phi(z)\left(\frac{\partial V}{\partial q_l}\right)^2 - \int_{-\infty}^{\hat{z}} \frac{\partial^2 \Pi}{\partial q_l^2} d\Phi(z)\left(\frac{\partial V}{\partial q_h}\right)^2 . \tag{55}$$

Given that

$$\frac{\partial^2 \Pi}{\partial q_\eta^2} = 2p_\eta \frac{\partial r_\eta}{\partial q_\eta} + p_\eta q_\eta \frac{\partial^2 r_\eta}{\partial q_\eta^2} \leq 0 \quad \forall q_\eta \tag{56}$$

due to the assumptions of concave inverse demand in eq. (2) so that the integral of $\Pi''(q_i)$ is non-positive, and

$$\frac{\partial \Pi}{\partial z} = -r_h(Q_h)q_h \frac{\partial PD_h}{\partial z} - r_l(Q_l)q_l \frac{\partial PD_l}{\partial z} \leq 0 \tag{57}$$

with

$$\frac{\partial PD_\eta}{\partial z} = \sqrt{\frac{\rho}{1-\rho}}\,\varphi\left(\frac{\zeta_\eta + \sqrt{\rho}z}{\sqrt{1-\rho}}\right) > 0 \; , \tag{58}$$

we see that $detH_2 \geq 0$. Hence, V_i is quasi-concave with respect to q_i. Constraints on the strategy set in form of capital requirements do not alter the objective function so that the third condition for existence is fulfilled in all scenarios. We conclude that at least one Nash-equilibrium must exist in each game. □

A.2 Proof of Lemma 2

Proof. I show that $\Pi_i(q_i, z)$ defined in eq. (6) has a unique root with respect to z which was defined as $\hat{z}_i$. This is because first $\Pi_i(q_i, z)$ decreases in z as shown in eq. (57) and second it is monotone decreasing due to the monotonicity of the CDF in $PD_\eta(z)$. Further, for all banks that actively produce in equilibrium, s.t. $q_{\eta,i} \geq 0$, we know

$$\lim_{z \to -\infty} \Pi_i(q_i, z) = 1 r_h(Q_h) q_{h,i} + 1 r_l(Q_l) q_{l,i} - MC_i(q_{h,i} + q_{l,i}) \geq 0 \tag{59}$$

and

$$\lim_{z \to \infty} \Pi_i(q_i, z) = -MC_i(q_{h,i} + q_{l,i}) \leq 0 \tag{60}$$

Hence, for any given strategy there is a unique unit root for $\Pi_i(q_i, z)$ which is illustrated in fig. 1.

To proof that $\hat{z}_i > \hat{z}_{i+1}$, consider the contradiction in assuming $\hat{z}_i < \hat{z}_{i+1}$. Then

$$G_\eta(\hat{z}_i) MC_i < G_\eta(\hat{z}_{i+1}) MC_{i+1} \quad \forall \eta \in \{h, l\} \tag{61}$$

and then according to eq. (14)

$$q_{\eta,i} > q_{\eta,i+1} \tag{62}$$

so that

$$\Pi_i(q_i, MC_i, z) > \Pi_{i+1}(q_{i+1}, MC_{i+1}, z) \; \forall z \in (\infty, -\infty) \tag{63}$$

and which implies $\hat{z}_i > \hat{z}_{i+1}$ due to the unique mapping between Π and z shown above contradicting what was assumed.

□

A.3 Proof of Proposition 1

Proof. The Karush-Kuhn-Tucker conditions to the optimization problem given in eq. (12) for bank i in each market η are

$$\frac{\partial V_i}{\partial q_{\eta,i}} \leq 0 \, , \tag{64}$$

$$q_{\eta,i} \frac{\partial V_i}{\partial q_{\eta,i}} = 0 \, , \tag{65}$$

$$V_i(q_i) - r_e e \geq 0 \tag{66}$$

$$W_i - q_{\eta,i} \geq 0 \tag{67}$$

$$q_{\eta,i} \geq 0 \tag{68}$$

where

$$\frac{\partial E[\Pi_i(q_i,z)]}{\partial q_{\eta,i}} = \underbrace{(E[\Pi_i(q_i,\hat{z})])}_{=0 \text{ by def. of } \hat{z}} \frac{\partial \hat{z}(q_i)}{\partial q_{\eta,i}} + \int_{-\infty}^{\hat{z}(q_i)} \frac{\partial \Pi_i(q_i,z)}{\partial q_{\eta,i}}\, d\Phi(z) = 0\ . \tag{69}$$

From eq. (65) and (68), we know that banks either produce nothing or, if they supply a positive amount of loans, marginal expected profits must be zero. Hence by solving eq. (64) for $q_{\eta,i}$ in equality, we get bank i's best reply function in market η. The integral to be solved is

$$\int_{-\infty}^{\hat{z}(q_i)} (1 - PD_\eta(z))\, r_\eta(Q_\eta) + (1 - PD_\eta(z)) \frac{\partial r_\eta}{\partial q_{\eta,i}} q_{\eta,i} - MC_i\, d\Phi(z)\ . \tag{70}$$

Equation 69 is equivalent to

$$\left(r_\eta(Q_\eta) + \frac{\partial r_\eta}{\partial q_{\eta,i}} q_{\eta,i} \right) - \frac{\int_{-\infty}^{\hat{z}(q_i)} MC_i d\Phi(z)}{\int_{-\infty}^{\hat{z}(q_i)} 1 - PD_\eta(z)\, d\Phi(z)} = 0\ . \tag{71}$$

Using function G defined in 15 as

$$G_\eta(\hat{z}) = \frac{\Phi(\hat{z}_i)}{\int_{-\infty}^{\hat{z}_i} p_\eta(z)\phi(z)\, dz} \tag{72}$$

eq. (71) becomes

$$\left(r_\eta(Q_\eta) + \frac{\partial r_\eta}{\partial q_{\eta,i}} q_{\eta,i} \right) - G_\eta(\hat{z}_i) MC_i = 0\ . \tag{73}$$

Solving for $q_{\eta,i}$ gives

$$q_{\eta,i} = \frac{r_\eta(Q_\eta) - G_{\eta,i}(\hat{z}_i) MC_i}{-r'_\eta}\ . \tag{74}$$

Summing eq. (73) over all $\nu_\eta \in \{1, \dots, N\}$ for which $q_{\eta,i} > 0$, and solving for $-r'_\eta$ gives

$$-r'_\eta = \frac{\nu_h r_\eta(Q_\eta) - \sum_{i=1}^{\nu_h} G_{\eta,i}(\hat{z}_i) MC_i}{Q_\eta} \tag{75}$$

Inserting the above into eq. (74) gives

$$q_{\eta,i} = \frac{r_\eta(Q_\eta) - G_{\eta,i}(\hat{z}_i) MC_i}{\nu_h r_\eta(Q_\eta) - \sum_{i=1}^{\nu_h} G_{\eta,i}(\hat{z}_i) MC_i} Q_\eta \tag{76}$$

Because $p_h(z)r_h(Q_h) > p_l(z)r_l(Q_l)$ by assumption, so that $p_h(\hat{z}_i)r_h(Q_h) > p_l(\hat{z}_i)r_l(Q_l)$. Hence if $p_l(\hat{z}_i)r_l(Q_l) - \Phi(\hat{z}_i)MC_i > 0$, then $p_h(\hat{z}_i)r_h(Q_h) - \Phi(\hat{z}_i)MC_i > 0$ as well so that if $q_{l,i} > 0$, then $q_{h,i} > 0$. On the other hand if $p_h(\hat{z}_i)r_h(Q_h) - \Phi(\hat{z}_i)MC_i > 0$ and therefore $q_{h,i} > 0$ it must not be that $q_{l,i} > 0$. There are some banks with $p_h(\hat{z}_i)r_h(Q_h) > \Phi(\hat{z}_i)MC_i > p_l(\hat{z}_i)r_l(Q_l)$.

Hence, any one bank has three strategies: First,if it is able to offer low-risk loans at a profit, it supplies high-risk loans as well. Second, if a bank is able to provide high-risk loans profitably, it could still incur costs that are too high to participate in the low-risk market. And third, a bank cannot participate in neither of the markets. I now consider the optimal strategy of bank i and bank $i+1$. Condition eq. (17) which states that

$$\frac{G_h(\hat{z}_i) - G_h(\hat{z}_{i+1})}{G_h(\hat{z}_{i+1})} < \frac{MC_{i+1} - MC_i}{MC_i} \tag{77}$$

implies that

$$\begin{aligned} G_h(\hat{z}_i)MC_i &< G_h(\hat{z}_{i+1})(MC_i + (MC_{i+1} - MC_{i+1})) \\ G_h(\hat{z}_i)MC_i &< G_h(\hat{z}_{i+1})MC_{i+1} \end{aligned} \tag{78}$$

and hence $q^*_{h,i} > q^*_{h,i+1}$. If this is true for an arbitrary bank i, it is true for bank 1. So bank 1 offers high-risk and low-risk loans and has the highest market shares in both markets. Then there is a bank ν_l with $G_l(\hat{z}_1)MC_1 < G_l(\hat{z}_{\nu_l})MC_{\nu_l} < r^*_l(Q^*_l)$ that offers both types of loans but is the bank with the highest marginal costs in the low-risk market such that there exists bank $\nu_l + 1$ with $G_l(\hat{z}_1)MC_1 < r^*_l(Q^*_l) < G_l(\hat{z}_{\nu_l+1})MC_{\nu_l+1} < r^*_h(Q^*_h)$ that cannot offer low-risk loans. Similarly, there exists bank $\nu_h + 1$ with $r^*_l(Q^*_l) < r^*_h(Q^*_h) < G_h(\hat{z}_{\nu_h+1})MC_{\nu_h+1}$.

From Lemma 1 we know an equilibrium must exist and is defined in 1. Hence, if there is an equilibrium and eq. (17) holds, optimal strategies of banks must be defined as

$$q^*_i(Q^*_{-i}) = \begin{cases} \left(\frac{r^*_h(Q^*_h) - G_h(\hat{z}_i)MC_i}{n_h r^*_h(Q^*_h) - \sum_{i=1}^{n_h} G_h(\hat{z}_i)MC_i} Q^*_h, \frac{r^*_l(Q^*_l) - G_l(\hat{z}_i)MC_i}{n_l r^*_l(Q^*_l) - \sum_{i=1}^{n_l} G_l(\hat{z}_i)MC_i} Q^*_l\right) & \forall i \leq \nu_l \\ \left(\frac{r^*_h(Q^*_h) - G_h(\hat{z}_i)MC_i}{n_h r^*_h(Q^*_h) - \sum_{i=1}^{n_h} G_h(\hat{z}_i)MC_i} Q^*_h, 0\right) & \forall i \textit{ with } \nu_l < i \leq \nu_h \\ (0,0) & \forall i \textit{ with } \nu_h < i. \end{cases} \tag{79}$$

□

A.4 Proof of Proposition 2

Proof. Let γ_i be defined as the share of high-risk loans to total loans in bank i's portfolio and let $q_i(\gamma_i)$ be defined as total loan supply of bank i, so that

$$\gamma_i = \frac{q_{h,i}}{q_{h,i} + q_{l,i}} = \frac{q_{h,i}}{q_i(\gamma_i)} \; . \tag{80}$$

Let $\varphi_{\eta,i}$ be defined as the marginal return of loan η for bank i as

$$\varphi_{\eta,i}(z) = (p_\eta(z) r_\eta(Q_\eta) - MC_i) \tag{81}$$

First, focusing only on strategies where the capital requirement is binding, i.e. $e = \beta_h q_{h,i} + \beta_l q_{l,i}$, then bank i's expected profit can be expressed in terms of the portfolio allocation using eq. (81) as

$$\begin{aligned} V(\gamma_i, z) &= E[\max\{\Pi_i(\gamma_i, z), 0\}] \\ V(\gamma_i, z) &= E[\max\{\gamma_i q_i(\gamma_i)\varphi_{h,i}(z) + (1-\gamma_i) q_i(\gamma_i)\varphi_{l,i}(z) + r_d e \;, 0 \;\}] \\ &= \int_{-\infty}^{\hat{z}_i} \gamma_i q_i(\gamma_i)(\varphi_{h,i} + r_d \beta_h) + (1-\gamma_i) q_i(\gamma_i)(\varphi_{l,i} + r_d \beta_l) \; d\Phi(z) \end{aligned} \tag{82}$$

In other words I want to proof that specialization is preferred over a mixed strategy which means

$$E[\max\{\Pi_i(1,z), \Pi_i(0,z), 0\}] \geq E[\max\{\Pi_i(\gamma_i, z), 0\}] \; . \tag{83}$$

Now, note that according to the definition of the low-risk market cutoff in eq. (25) $\max\{\Pi_i(1,z), \Pi_i(0,z)\} = \Pi_i(0,z)$ whenever $MC_i \leq \widetilde{MC}^l$, and $\max\{\Pi_i(1,z), \Pi_i(0,z)\} = \Pi_i(1,z)$ whenever $MC_i > \widetilde{MC}^l$, respectively. Let us therefore consider these two cases.

First, consider the case of banks that would rather specialize in low-risk than high-risk loans, i.e. $MC_i \leq \widetilde{MC}^l$. These banks prefer specialization over a mixed portfolio strategy if

$$E[\max\{\Pi_i(0,z), 0\}] \geq E[\max\{\Pi_i(\gamma_i, z), 0\}] \; . \tag{84}$$

If $\max\{\Pi_i(0,z), 0\} = 0$, then $\max\{\Pi_i(\gamma_i, z), 0\} = 0$, so that the above simplifies to

$E[0] \geq E[0]$ which is true. To see the first step, consider that $\max\{\Pi_i(0,z),0\} = 0$ implies that $\varphi_{l,i}(z) \leq 0$ and $MC_i \leq \widetilde{MC}^l$ implies $\beta_l\varphi_{h,i}(z) \leq \beta_h\varphi_{l,i}(z)$, so that $\varphi_{h,i}(z) \leq 0$ as well and hence $\Pi_i(\gamma_i,z) \leq 0$.

If $\max\{\Pi_i(0,z),0\} = \Pi_i(0,z)$, then $\max\{\Pi_i(\gamma_i,z),0\} = \Pi_i(\gamma_i,z)$, so that the above simplifies to $E[\Pi_i(0,z)] \geq E[\Pi_i(\gamma_i,z)]$ which is true because $\Pi_i(0,z) \geq \Pi_i(\gamma_i,z)$ for all z and the expectations operator is linear. To see the first step, consider that $\max\{\Pi_i(0,z),0\} = \Pi_i(0,z)$ implies that $\varphi_{l,i}(z) \geq 0$ and $p_l(z)r_l \leq p_h(z)r_h$ implies $\varphi_{h,i}(z) \geq 0$ as well and hence $\Pi_i(\gamma_i,z) \geq 0$. For the second step, consider

$$\begin{aligned} q_i(0)\varphi_{l,i}(z) &\geq q_i(\gamma_i)\left(\gamma_i\varphi_{h,i}(z) + (1-\gamma_i)\varphi_{l,i}(z)\right) \\ \frac{q_i(0)}{q_i(\gamma_i)} &\geq \frac{\gamma_i\varphi_{h,i}(z) + (1-\gamma_i)\varphi_{l,i}(z)}{\varphi_{l,i}(z)} \\ \frac{\beta_h}{\beta_l}\varphi_{l,i}(z) &\geq \varphi_{h,i}(z) \end{aligned} \tag{85}$$

which is equivalent to the case when $MC_i \leq \widetilde{MC}^l$ which was assumed.

Second, consider the case of banks that would rather specialize in high-risk than low-risk loans, i.e. $MC_i > \widetilde{MC}^l$. These banks prefer specialization over a mixed portfolio strategy if

$$E[\max\{\Pi_i(1,z),0\}] \geq E[\max\{\Pi_i(\gamma_i,z),0\}]\,. \tag{86}$$

Similarly to the argument above one can show that if $\max\{\Pi_i(1,z),0\} = 0$, then $\max\{\Pi_i(\gamma_i,z),0\} = 0$, so that the above simplifies to $E[0] \geq E[0]$ which is true. To see the first step, consider that $\max\{\Pi_i(1,z),0\} = 0$ implies that $\varphi_{h,i}(z) \leq 0$ and $p_l(z)r_l \leq p_h(z)r_h$ implies $\varphi_{l,i}(z) \leq 0$ as well and hence $\Pi_i(\gamma_i,z) \leq 0$.

If $\max\{\Pi_i(1,z),0\} = \Pi_i(1,z)$, then $\max\{\Pi_i(\gamma_i,z),0\} = 0$ or $\max\{\Pi_i(\gamma_i,z),0\} = \Pi_i(\gamma_i,z)$. The first could happens if $\varphi_{l,i}(z) < 0$. Then eq. (86) simplifies to $E[\Pi_i(1,z)] \geq E[0]$ which is true since $E[\Pi_i(1,z)] \geq 0$ otherwise bank i would not offer loans at all. In the second case eq. (86) simplifies to $E[\Pi_i(1,z)] \geq E[\Pi_i(\gamma_i,z)]$ which is true because $\Pi_i(1,z) \geq \Pi_i(\gamma_i,z)$ for all z and the linearity of the expectations operator. To see that

$\Pi_i(1,z) \geq \Pi_i(\gamma_i,z)$, consider

$$\begin{aligned} q_i(1)\varphi_{h,i}(z) &\geq q_i(\gamma_i)\left(\gamma_i\varphi_{h,i}(z) + (1-\gamma_i)\varphi_{l,i}(z)\right) \\ \frac{q_i(1)}{q_i(\gamma_i)} &\geq \frac{\gamma_i\varphi_{h,i}(z) + (1-\gamma_i)\varphi_{l,i}(z)}{\varphi_{h,i}(z)} \\ \varphi_{h,i}(z) &\geq \frac{\beta_h}{\beta_l}\varphi_{l,i}(z) \end{aligned} \tag{87}$$

which is equivalent to the case when $MC_i > \widetilde{MC}^l$ which was assumed.

Until here, the analysis is an extension of Lemma 1 in Repullo and Suarez (2004). Building on their proof, one could have stated that $V(\gamma_i, z)$ is convex for $MC_i \leq \widetilde{MC}^l$, and again that $V(1-\gamma_i, z)$ is convex for $MC_i > \widetilde{MC}^l$ which implies that specialization dominates mixed strategies. The problem at hand is however slightly more complex. The differences in marginal costs result in differences in optimality and feasibility of strategies. Let us therefore first consider under which conditions the above derived optimal specialization strategies are feasible to which banks.

Let me introduce the Lagrange function of the problem in eq. (22) using μ_i as the Lagrange parameter for the capital constraint and ν_i as the Lagrange parameter for the participation constraint as

$$\mathcal{L}(q_i) = V(q_i,z) - \mu_i\left(\beta_h q_{h,i} + \beta_l q_{l,i} - e\right) - \nu_i\left(V(q_i,z) - r_e e\right) \tag{88}$$

where $q_i \geq 0$, $\mu_i \geq 0$, and $\nu_i \geq 0$. First, I focus only on strategies and banks where the participation constraint is met, i.e. $V(q_i^s, z) > r_e e$ and $\nu_i = 0$. Strategies h and l where the capital constraint is binding, i.e. $\beta_h q_{h,i} + \beta_l q_{l,i} = e$, are feasible as long as $\mu_i^\eta \geq 0$ where

$$\mu_i^\eta = \frac{1}{\beta_\eta}\frac{\partial V(q_i,z)}{\partial q_{\eta,i}} = \frac{1}{\beta_\eta}\, E\left[\max\left\{\frac{\partial \Pi(q_i,z)}{\partial q_{\eta,i}}, 0\right\}\right] \tag{89}$$

Now since q_i^h and q_i^l are both independent of MC_i, and $\frac{\partial \mu_i^s}{\partial MC_i} < 0$, there is a unique cutoff $\widetilde{MC}^{\mu_s}$ which defines the least productive bank for which strategy s is feasible and which can be found by solving

$$\int_{-\infty}^{\hat{z}_i} \frac{\partial \Pi(q_i,z)}{\partial q_{\eta,i}} d\Phi(z) = \int_{-\infty}^{\hat{z}_i} p_\eta(z)\left(r_\eta(Q_\eta) + r'_\eta(Q_\eta)q^\eta\right) - MC_i d\Phi(z) = 0\ . \tag{90}$$

So that specialization strategies are feasible, we need three conditions. First, $\widetilde{MC}^l$ must be positive. This is the case if eq. (26) holds. Second, $\widetilde{MC}^{\mu_l} > \widetilde{MC}^{l}$, which is true if eq. (27) holds. Third, $\widetilde{MC}^{\mu_h} > \widetilde{MC}^{\mu_{rw}}$ which holds under eq. (29).

Eq. (29) usefully implies that

$$\begin{aligned} p_l r_l(Q_l^*) &< p_h r_h(Q_h^*) + \frac{e}{\beta_h} p_h r_h'(Q_h^*) \\ p_l r_l(Q_l^*) &< \widetilde{MC}^{\mu^h} \end{aligned} \tag{91}$$

so that if banks choose the unconstrained strategy, they specialize in high-risk loans and are not able to supply low-risk loans profitably. Hence, given eq. (26),(27), and (29) optimal strategies in equilibrium are

$$(q_{h,i}^*, q_{l,i}^*) = \begin{cases} (0, q_l^l) & \text{if} \quad MC_i \leq \widetilde{MC}^{l} \\ (q_h^h, 0) & \text{if} \quad \widetilde{MC}^{l} < MC_i \leq \widetilde{MC}^{\mu^h} \\ (q_{h,i}^{uc}(MC_i), 0) & \text{if} \quad \widetilde{MC}^{\mu^h} < MC_i \leq \widetilde{MC}^{h} . \end{cases} \tag{92}$$

For completeness, the optimal mixed strategy constrained by the risk-weighted capital requirement is given by

$$\begin{aligned} q_{h,i}^{rw} &= \frac{(p_l r_l(Q_l) - MC_i) - \frac{\beta_l}{\beta_h}(p_h r_h(Q_h) - MC_i) + p_l r_l'(Q_l)\frac{e}{\beta_l}}{\frac{\beta_h}{\beta_l} p_l r_l'(Q_l) + \frac{\beta_l}{\beta_h} p_h r_h'(Q_h)} \\ q_{l,i}^{rw} &= \frac{(p_h r_h(Q_h) - MC_i) - \frac{\beta_h}{\beta_l}(p_l r_l(Q_l) - MC_i) + p_h r_h'(Q_h)\frac{e}{\beta_h}}{\frac{\beta_h}{\beta_l} p_l r_l'(Q_l) + \frac{\beta_l}{\beta_h} p_h r_h'(Q_h)} . \end{aligned} \tag{93}$$

□

A.5 Proof of Lemma 3

Proof. First, I show that within each strategy, banks with lower marginal costs have higher critical values and therefore lower default probabilities. This follows from the characteristics of the profit function $\Pi_i(q_i, z)$ derived in the proof of Lemma 2, i.e. profits are monotonic decreasing in z and have a unique unit root $\hat{z}$. Further, since $\Pi_i(q_i^\eta, z) > \Pi_{i+1}(q_{i+1}^\eta, z)$ for all z because $q_i^\eta = q_{i+1}^\eta$ in equilibrium, $\Pi_i(q_i^\eta, \hat{z}_{i+1}) > \Pi_{i+1}(q_{i+1}^\eta, \hat{z}_{i+1}) = 0$ and therefore $\hat{z}_i > \hat{z}_{i+1}$.

For the specialized strategies, we can solve $\Pi_i(q_i, z) = 0$ for $\hat{z}_i^\eta$ which is the critical value of bank i if it specializes on strategy η. Given equilibrium strategies and outcomes we get

$$\begin{aligned}(1 - PD_\eta(\hat{z}_i^\eta))\; r_\eta(Q_\eta^*) q_\eta^{\eta *} &- MC_i q_\eta^{\eta *} + r_d e = 0 \\ \left(1 - \Phi\left(\frac{\zeta_\eta + \sqrt{\rho}\; \hat{z}_i^\eta}{\sqrt{1-\rho}}\right)\right) r_\eta(Q_\eta^*) \frac{e}{\beta_\eta} &- MC_i \frac{e}{\beta_\eta} + r_d e = 0\,.\end{aligned} \tag{94}$$

Rearranging gives

$$\hat{z}_i^\eta = \frac{\sqrt{1-\rho}}{\sqrt{\rho}}\; \Phi^{-1}\left(1 - \frac{MC_i - r_d \frac{e}{q_\eta^{\eta *}}}{r_\eta(Q_\eta^*)}\right) - \frac{\zeta_\eta}{\sqrt{\rho}}\,. \tag{95}$$

Except MC_i, all parameters in eq. (95) are equal for banks with the same constrained equilibrium strategy. Taking the derivative with respect to MC_i gives

$$\frac{\partial \hat{z}_i^\eta}{\partial MC_i} = (-1) \frac{\sqrt{1-\rho}}{\sqrt{\rho}\; \phi\left(\Phi^{-1}\left(1 - \frac{MC_i - r_d \frac{e}{q_\eta^{\eta *}}}{r_\eta(Q_\eta^*)}\right)\right)} < 0 \tag{96}$$

where $\phi(x)$ is the PDF of the standard normal distribution.

For high-risk specialists that are not constrained (strategy uc), the parameters MC_i and q_h^{uc*} change in eq. (95). Simplifying $\hat{z}_i^{uc} > \hat{z}_{i+1}^{uc}$ yields

$$(MC_{i+1} - MC_i) q_{h,i+1}^{uc*} q_{h,i}^{uc*} > r_d e (q_{h,i+1}^{uc*} - q_{h,i}^{uc*}) \tag{97}$$

which is always true since $q_{h,i+1}^{uc*} - q_{h,i}^{uc*} < 0$. Hence, when comparing different banks with the same strategy, we find that within each market banks with lower marginal costs have higher critical values and therefore lower default probabilities.

Next, I show that within the same bank and given $p_h(z) r_h(Q_h^*) < \frac{\beta_h}{\beta_l} p_h(z) r_l(Q_l^*)$, strategies with a higher share of high-risk loans have a higher default probability. Let us now compare default probabilities of different strategies for one bank i. If $p_h(z) r_h(Q_h^*) < \frac{\beta_h}{\beta_l} p_h(z) r_l(Q_l^*)$, then

$$1 - \frac{MC_i - r_d \beta_l}{r_l(Q_l^*)} > 1 - \frac{MC_i - r_d \beta_h}{r_h(Q_h^*)} \tag{98}$$

and hence

$$\Phi^{-1}\left(1-\frac{MC_i-r_d\beta_l}{r_l(Q_l^*)}\right)>\Phi^{-1}\left(1-\frac{MC_i-r_d\beta_h}{r_h(Q_h^*)}\right) \tag{99}$$

so that the right hand side in the following is negative which ensures that it is true that

$$\frac{\zeta_h-\zeta_l}{\sqrt{1-\rho}}>\Phi^{-1}\left(1-\frac{MC_i-r_d\beta_h}{r_h(Q_h^*)}\right)-\Phi^{-1}\left(1-\frac{MC_i-r_d\beta_l}{r_l(Q_l^*)}\right) \tag{100}$$

and thus

$$\hat{z}_i^l>\hat{z}_i^h\ . \tag{101}$$

Since we know that $\hat{z}_i^h>\hat{z}_{i+1}^h$, we can compare the default probabilities of the least productive bank in the low-risk market n_l (which has marginal cost just below or at the cutoff: $MC_{n_l}\leq\widetilde{MC}^l$) with the next bank n_{l+1} that is the most productive bank in the high-risk market with $MC_{n_l+1}>\widetilde{MC}^l$, and state that

$$\hat{z}_1^l>\cdots>\hat{z}_{n_l}^l>\hat{z}_{n_l}^h>\hat{z}_{n_l+1}^h>\cdots>\hat{z}_{n_h}^{uc}\ . \tag{102}$$

□

A.6 Proof of Proposition 3

Proof. The proof is structured similarly as the proof of Proposition 2. First, I focus on strategies where banks are constrained by the leverage ratio. These are banks with $MC_i<\widetilde{MC}^l$. For them I show that strategy v (which is constrained by the leverage ratio and the risk-weighted ratio) is optimal because

$$E[\max\{\Pi_i(\gamma_i^v,z),0\}]\geq E[\max\{\Pi_i(1,z),\Pi_i(\gamma_i^{rw},z),\Pi_i(\gamma_i^{lr},z)\Pi_i(0,z),0\}]\ . \tag{103}$$

First, whenever $E[\max\{\Pi_i(\gamma_i^v,z),0\}]=0$, any expected profit is 0. Hence, it suffices to show

$$E[\Pi_i(\gamma_i^v,z)]\geq E[\max\{\Pi_i(1,z),\Pi_i(\gamma_i^{rw},z),\Pi_i(\gamma_i^{lr},z)\Pi_i(0,z)\}]\ . \tag{104}$$

Comparing the payoff of strategies v and l gives

$$\begin{aligned} \Pi_i^v(q^v, z) &> \Pi_i^l(q^l, z) \\ p_h(z)r_h q_h^v + p_l(z)r_l q_l^v - MC_i(q_h^v + q_l^v) &> p_l(z)r_l q^l - MC_i q^l \\ p_h(z)r_h q_h^v - p_l(z)r_l(q_i^l - q_l^v) &> 0 \\ (p_h(z)r_h - p_l(z)r_l)q_h^v &> 0\ . \end{aligned} \tag{105}$$

Note that for all strategies constrained by the leverage ratio eq. (33) holds with equality so that bank i's costs are equal for strategies l,lr, and v. Furthermore, since $q_l^l = \frac{e}{\beta}$, from eq. (33) follows that $q_l^l - q_l^v = q_h^v$. Comparing the payoff of strategies v and lr gives

$$\begin{aligned} \Pi_i^v(q^v, z) &> \Pi_i^{lr}(q^{lr}, z) \\ p_h(z)r_h q_h^v + p_l(z)r_l q_l^v - MC_i(q_h^v + q_l^v) &> p_h(z)r_h q_h^{lr} + p_l(z)r_l q_l^{lr} - MC_i(q_h^{lr} + q_l^{lr}) \\ p_h(z)r_h(q_h^v - q_h^{lr}) - p_l(z)r_l(q_l^{lr} - q_l^v) &> 0 \\ (p_h(z)r_h - p_l(z)r_l)(q_h^v - q_h^{lr}) &> 0 \end{aligned} \tag{106}$$

For the last step, reckon that the leverage ratio constraint in eq. (33) holds with equality for strategies v and lr. Equation (106) and eq. (105) are true for all banks irregardless of MC_i. Hence, strategy v dominates strategies l and lr for any given z.

$$\begin{aligned} E[\Pi_i^v(q^v, z)] &> E[\Pi_i^l(q^l, z)] \qquad \forall MC_i \\ E[\Pi_i^v(q^v, z)] &> E[\Pi_i^{lr}(q^{lr}, z)] \qquad \forall MC_i \end{aligned} \tag{107}$$

Comparing strategy v to h gives the cutoff defined in eq. (37), and comparing it to strategy rw gives

$$\begin{aligned} \Pi_i^{rw}(q_i^{rw}, z) &< \Pi_i^v(q^v, z) \\ \varphi_{h,i}q_{h,i}^{rw} + \varphi_{l,i}q_{l,i}^{rw} &< \varphi_{h,i}q_h^v + \varphi_{l,i}q_l^v \\ \frac{\varphi_{h,i}}{\varphi_{l,i}} &< \frac{q_l^v - q_{l,i}^{rw}}{q_{h,i}^{rw} - q_h^v} \\ \frac{\varphi_{h,i}}{\varphi_{l,i}} &< \frac{\beta_h}{\beta_l} \end{aligned} \tag{108}$$

which gives the same cutoff as in eq. (37). For the last step, note that eq. (21) holds with equality for both strategies. Hence, strategy v only dominates strategies h and rw

if marginal costs are below the cutoff, i.e.

$$\begin{aligned} E[\Pi_i^v(q^v, z)] &> E[\Pi_i^h(q^h, z)] \qquad \forall MC_i : MC_i \leq \widetilde{MC}^l \\ E[\Pi_i^v(q^v, z)] &> E[\Pi_i^{rw}(q^{rw}, z)] \qquad \forall MC_i : MC_i \leq \widetilde{MC}^l \ . \end{aligned} \tag{109}$$

So to summarize, for banks with $MC_i \leq \widetilde{MC}^l$ the above shows that $E[\Pi_i(\gamma_i^v, z)] \geq E[\max\{\Pi_i(q^h, z), \Pi_i(q_i^{rw}, z), \Pi_i(q_i^{lr}, z)\Pi_i(q^l, z)\}]$. Further, I consider banks with $MC_i > \widetilde{MC}^l$. Comparing strategies h and rw gives

$$\begin{aligned} \Pi_i^{rw}(q_i^{rw}, z) &< \Pi_i^h(q^h, z) \\ \varphi_{h,i} q_{h,i}^{rw} + \varphi_{l,i} q_{l,i}^{rw} &< \varphi_{h,i} q_h^h \\ \frac{\varphi_{h,i}}{\varphi_{l,i}} &> \frac{q_{l,i}^{rw}}{q_{h,i}^{rw} - q_h^h} \\ \frac{\varphi_{h,i}}{\varphi_{l,i}} &> \frac{\beta_h}{\beta_l} \end{aligned} \tag{110}$$

which again gives the same cutoff as in eq. (37). Hence,

$$\begin{aligned} E[\Pi_i^h(q^h, z)] &> E[\Pi_i^{rw}(q_i^{rw}, z)] \qquad \forall i \in \{1, ..., N\} : MC_i > \widetilde{MC}^l \\ E[\Pi_i^h(q^h, z)] &> E[\Pi_i^v(q_i^v, z)] \qquad \forall i \in \{1, ..., N\} : MC_i > \widetilde{MC}^l \ . \end{aligned} \tag{111}$$

To summarize, for banks with $MC_i \leq \widetilde{MC}^l$, the above shows that $E[\Pi_i(q^h, z)] \geq E[\max\{\Pi_i(q_i^{rw}, z), \Pi_i(q_i^v, z)\Pi_i(q^l, z)\}]$ and from before we know $E[\Pi_i(q_i^v, z)]$ is greater than $E[max\{\Pi_i(q_i^{lr}, z)\Pi_i(q^l, z)\}]$ for any bank.

Let me introduce the Lagrange function of the problem in eq. (34) using μ_i as the Lagrange parameter for the risk-weighted capital constraint, λ_i as the Lagrange parameter for the leverage ratio capital constraint, and ν_i as the Lagrange parameter for the participation constraint as

$$\mathcal{L}(q_i) = V(q_i, z) - \mu_i \left(\beta_h q_{h,i} + \beta_l q_{l,i} - e\right) - \lambda_i \left(\beta(q_{h,i} + q_{l,i}) - e\right) - \nu_i \left(V(q_i, z) - r_e e\right) \tag{112}$$

where $q_i \geq 0$, $\mu_i \geq 0$, $\lambda_i \geq 0$, and $\nu_i \geq 0$. First, I focus only on strategies and banks where the participation constraint is met, i.e. $V(q_i^s, z) > r_e e$ and $\nu_i = 0$. A strategy s is only feasible if $\mu_i^s \geq 0$ and $\lambda_i^s \geq 0$.

For the relevant strategies v and h this means

$$\mu_i^h \geq 0 \; \forall i \in \{1, ..., N\} : MC_i \leq \widetilde{MC}^{\mu^h} \text{ where } \widetilde{MC}^{\mu^h} > 0 \tag{113}$$

$$\lambda_i^v \geq 0 \; \forall i \in \{1, ..., N\} : MC_i \leq \widetilde{MC}^{\lambda^v} \text{ where } \widetilde{MC}^{\lambda^v} > 0 \tag{114}$$

$$\mu^v \geq 0 \; \forall i \in \{1, ..., N\} \tag{115}$$

where

$$\widetilde{MC}^{\mu^h} = p_h(z) r_h + \frac{e}{\beta_h} p_h(z) r_h' \tag{116}$$

$$\widetilde{MC}^{\lambda^v} = \frac{\beta_h p_l(z) r_l - \beta_l p_h(z) r_h}{(\beta_h - \beta_l)} + \frac{\beta_h(\beta_h - \beta)}{\beta(\beta_h - \beta_l)^2} p_l(z) r_l' e - \frac{\beta_l(\beta - \beta_l)}{\beta(\beta_h - \beta_l)^2} p_h(z) r_h' e \tag{117}$$

and $\mu^v = 0$ for all banks with $MC_i \leq \widetilde{MC}^l$ that chose strategy v because of the strategic complementarity condition of eq. (112) with respect to μ.

Thirdly, strategies v and h should be viable for all banks for whom these strategies are profit maximizing. That is the case if

$$\widetilde{MC}^l < \widetilde{MC}^{\lambda^v} < \widetilde{MC}^{\mu^h} \tag{118}$$

$$\widetilde{MC}^{\mu^h} > \max\left[\widetilde{MC}^{\mu^{rw}}, \widetilde{MC}^{\lambda^l}, \widetilde{MC}^{\lambda^{lr}}, p_l(z) r_l\right] . \tag{119}$$

The conditions given in eq. (113), (114), (115), (118), and (119) simplify to eq. (38) and (40) in the following way: Given (113) and (114), $\widetilde{MC}^l < \widetilde{MC}^{\lambda^v}$ in (118) is true. Given $\widetilde{MC}^{\lambda^v} < \widetilde{MC}^{\mu^h}$ in (118), (113) is true. If (114) and

$$-\beta_h(p_h(z) r_h - p_l(z) r_l) < p_h(z) r_h' e \, , \tag{120}$$

then $\widetilde{MC}^{\mu^h} > p_l(z) r_l$ in (119) which itself implies $\widetilde{MC}^{\mu^h} > \widetilde{MC}^{\lambda^l}$, and $\widetilde{MC}^{\mu^h} > \widetilde{MC}^{\mu^{rw}}$ in (119). If (120) and

$$p_h(z) r_h' e < -\beta(p_h(z) r_h - p_l(z) r_l) \, , \tag{121}$$

then $\widetilde{MC}^{\mu^h} > \widetilde{MC}^{\lambda^{lr}}$ in (119). To sum up, condition (38) is equal to eq. (114), and eq. (120) and (121) combine to condition (40) which is stricter than (115) and $\widetilde{MC}^{\lambda^v} < \widetilde{MC}^{\mu^h}$ in (118).

Hence, given eq. (114), (120), and (121) optimal strategies in equilibrium are

$$(q_{h,i}^*, q_{l,i}^*, 0, \lambda_i^*) = \begin{cases} (q_h^v, q_l^v, \mu_i^v, \lambda_i^v(MC_i)) & \text{if} \quad MC_i \leq \widetilde{MC}^l \\ \left(q^h, 0, \mu_i^h(MC_i), 0\right) & \text{if} \quad \widetilde{MC}^l < MC_i \leq \widetilde{MC}^{\mu^h} \\ \left(q_{h,i}^{uc}(MC_i), 0, 0, 0\right) & \text{if} \quad \widetilde{MC}^{\mu^h} < MC_i \leq \widetilde{MC}^h \\ (0,0,0,0) & \text{if} \quad \widetilde{MC}^h < MC_i \end{cases} \tag{122}$$

For completeness, the optimal mixed strategy constrained by the leverage ratio requirement is given by

$$\begin{aligned} q_{h,i}^{lr} &= \frac{-\beta \left(p_h(z) r_h(Q_h) - p_l(z) r_l(Q_l)\right) + p_l(z) r_l'(Q_l) e}{\beta \left(p_l(z) r_l'(Q_l) + p_h(z) r_h'(Q_h)\right)} \\ q_{l,i}^{lr} &= \frac{\beta \left(p_h(z) r_h(Q_h) - p_l(z) r_l(Q_l)\right) + p_h(z) r_h'(Q_h) e}{\beta \left(p_l(z) r_l'(Q_l) + p_h(z) r_h'(Q_h)\right)} \end{aligned} \tag{123}$$

□

A.7 Proof of Corollary 1

Proof. I proof Corollary 1 by contradiction. Assume the cutoff $\widetilde{MC}^l$ decreases. It implies that the number of banks participating in low-risk market decreases. Then fewer banks produce a smaller quantity each so that the total supply of low-risk loans decreases. Note that these banks previously produced $q_l^l = \frac{e}{\beta_l}$ and now produce $q_l^v = \frac{(\beta_h - \beta)e}{\beta(\beta_h - \beta_l)} < q_l^l$. Hence, the interest rate on low-risk loans increases. From eq. (37) follows that the interest rate on high-risk loans must increase as well (and even more) otherwise the cutoff would not decrease as was assumed.

Due to eq. (2) the interest rate on high-risk loans only increases if total supply decreases. On the other hand an increase of r_h implies that the cutoffs $\widetilde{MC}^h$ and $\widetilde{MC}^{\mu_h}$ both increase while $\widetilde{MC}^l$ decreases. Thus, the number of specialized banks in the high-risk market increases and more productive banks with strategy v enter the high-risk market. All in all, this implies that the aggregate supply of high-risk loans must increase which contradicts the necessary decrease of aggregate supply such that the interest rate could rise. Hence, the cutoff $\widetilde{MC}^l$ cannot decrease but has to increase.

Assume further the cutoff $\widetilde{MC}^h$ decreases. Then the interest rate on high-risk loans necessarily decreases and aggregate supply increases. That is

$$\begin{aligned} Q_h^{*B2} &< Q_h^{*B3} \\ n_l^{B3}(1+\frac{q_h^v}{q_h^h}) - n_l^{B2} &< (n_h^{B3} - n_h^{B2}) \end{aligned} \tag{124}$$

which cannot be true since the right hand side is negative if the cutoff decreases, as was assumed, while the left hand side is positive because the cutoff in the low-risk market increase as was shown earlier. Hence, the cutoff in the high-risk market must increase as well.

□

A.8 Proof of Lemma 5

Proof. We rewrite eq. (6) by defining the share of high-risk loans in bank i's portfolio as $\gamma_i = \frac{q_{h,i}}{d_i+e}$ as

$$\Pi_i(\gamma_i, q_i, z) = (p_h r_h(Q_h)\gamma_i + p_l(z) r_l(Q_l)(1-\gamma_i) - MC_i)\,(q_{h,i} + q_{h,i}) + r_d e\ . \tag{125}$$

The effect of a higher share of high-risk loans on default probabilities is implicitly defined by

$$\frac{\partial \Pi_i}{\partial \gamma_i} = (q_h^v + q_l^v)\,(r_h(Q_h)(1 - PD_h(z)) - r_l(Q_l)(1 - PD_l(z))) \tag{126}$$

which could be either negative or positive depending on z in the following way:

$$\begin{aligned} &lim_{z\to-\infty} \quad r_h(Q_h)(1-PD_h(z)) - r_l(Q_l)(1-PD_l(z)) = r_h(Q_h) - r_l(Q_l) \\ &lim_{z\to\infty} \quad r_h(Q_h)(1-PD_h(z)) - r_l(Q_l)(1-PD_l(z)) = 0 \\ &\qquad\qquad\quad r_h(Q_h)(1-PD_h(0)) - r_l(Q_l)(1-PD_l(0)) = p_h r_h(Q_h) - p_l r_l(Q_l) \end{aligned} \tag{127}$$

This means that the effect is positive for non-positive z and vanishes for very high z. But the effect can be negative, because $\frac{\partial \Pi_i}{\partial \gamma_i}$ has a local minimum given at $\hat{z}$ defined by

$$\frac{\partial^2 \Pi_i}{\partial \gamma_i \partial z} = 0 \quad \Leftrightarrow \quad \hat{z} = \frac{-\zeta_h^2 + \zeta_l^2 + 2\ln(\frac{r_h}{r_l})(1-\rho)}{2\sqrt{\rho}(\zeta_h - \zeta_l)}\ . \tag{128}$$

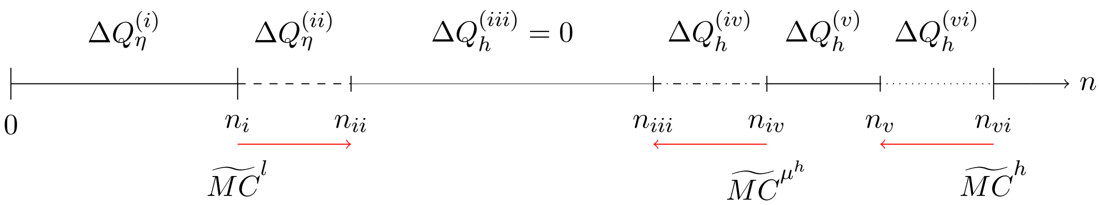

Figure 5: Illustration of notation for proof of Proposition 5. $\Delta Q_\eta^{(g)}$ is the change in aggregate supply of loans of type η for banks of group $g \in \{i, ii, iii, iv, v, vi\}$ induced by an increase of β. n_g indicates the index of the bank with the highest marginal costs within group g. For example, $\Delta Q_h^{(ii)} = (n_{ii} - n_i)(q_h^v - q_h^h)$, i.e. when the leverage ratio is tightened, a number of $(n_{ii} - n_i)$ banks reduce their supply by $(q_h^v - q_h^h) < 0$ because they choose the vertex strategy instead of the specialization on loans of type h.

Therefore, as $z \to \infty$, $\frac{\partial \Pi_i}{\partial \gamma_i}$ must approach the limit 0 from below implying

$$\exists\ \tilde{z}\ : 0 < \tilde{z} < \hat{z} \quad s.t. \quad \begin{cases} \frac{\partial \Pi_i}{\partial \gamma_i} \geq 0 & \text{if} \quad z \leq \tilde{z} \\ \frac{\partial \Pi_i}{\partial \gamma_i} < 0 & \text{if} \quad z > \tilde{z} \end{cases} \tag{129}$$

□

A.9 Proof of Proposition 5

Proof. First I show that the cutoff in the low-risk market $\widetilde{MC}^l$ increases by contradiction. Imagine the cutoff decreases. Because $\frac{\partial q_l^v}{\partial \beta} < 0$, each bank in the low-risk market provides smaller quantities and there are less banks active. Then clearly aggregate supply of low-risk loans (Q_l) decreases. This implies that the interest rate for low-risk loans increases. Consequently, the interest rate for high-risk loans must increase as well, otherwise the cutoff cannot decrease. But the interest rate for high-risk loans cannot increase because all these changes taken together imply that the aggregate supply of high-risk loans (Q_h) must increase and hence the interest rate cannot increase: Banks with the vertex strategy produce a higher amount ($\frac{\partial q_h^v}{\partial \beta} > 0$), there are more banks that choose to specialize in high-risk loans and additional banks entering with small quantities in the high-risk market. Hence, $\widetilde{MC}^l$ cannot decrease.

Next I show that the cutoffs in the high-risk market $\widetilde{MC}^h$ and $\widetilde{MC}^{\mu_h}$ decrease by contradiction using the fact that $\widetilde{MC}^l$ increases. To ease notation, let $\Delta Q_\eta^{(g)}$ be defined as the change in aggregate supply of loans of type η for banks of group $g \in \{i, ii, iii, iv, v, vi\}$

induced by an increase of β. n_g indicates the index of the bank with the highest marginal costs within group g. These groups are illustrated in Fig. 5. Imagine the cutoffs would increase. This necessitates that the interest rate for high-risk loans increases and aggregate supply decreases. Given the movement in the cutoffs we get the following condition for this scenario.

$$\begin{aligned} -\Delta Q_h^{(ii)} &> \Delta Q_h^{(i)} + \Delta Q_h^{(iv)} + \Delta Q_h^{(v)} + \Delta Q_h^{(vi)} \\ n_{ii} &> n_i \left(1 + \frac{\beta_h}{\beta(\beta_h - \beta)}\right) + \frac{\beta\beta_h(\beta_h - \beta_l)}{\beta_l(\beta_h - \beta)e} \left(\Delta Q_h^{(iv)} + \Delta Q_h^{(v)} + \Delta Q_h^{(vi)}\right) \end{aligned} \tag{130}$$

We get a second condition ensuring that the interest rate for low-risk loans increases as well otherwise $\widetilde{MC}^l$ could never increase.

$$\begin{aligned} -\Delta Q_l^{(i)} &> \Delta Q_l^{(ii)} \\ n_{ii} &< n_i \left(1 + \frac{\beta_h}{\beta(\beta_h - \beta)}\right) \end{aligned} \tag{131}$$

We see that both conditions can never be true at the same time because $\Delta Q_h^{(iv)} + \Delta Q_h^{(v)} + \Delta Q_h^{(vi)} > 0$. Hence cutoffs in the high-risk market $\widetilde{MC}^h$ and $\widetilde{MC}^{\mu_h}$ cannot increase. □

A.10 Proof of Lemma 6

Proof. Given Proposition 5 we know that cutoffs in the high-risk market must decrease which implies that the high-risk loan rate decreases. We can rewrite Equation (130) for the opposite case of decreasing interest rate and increasing aggregate demand as

$$\begin{aligned} \Delta Q_h^{(i)} &< -\Delta Q_h^{(ii)} - \Delta Q_h^{(iv)} - \Delta Q_h^{(v)} - \Delta Q_h^{(vi)} \\ n_i \left(1 + \frac{\beta_h}{\beta(\beta_h - \beta)}\right) &> n_{ii} - \frac{\beta\beta_h(\beta_h - \beta_l)}{\beta_l(\beta_h - \beta)e} \left(\Delta Q_h^{(iv)} + \Delta Q_h^{(v)} + \Delta Q_h^{(vi)}\right) . \end{aligned} \tag{132}$$

We know that Equation (132) must be true, so Equation (131) is true as well, because $-(\Delta Q_h^{(iv)} + \Delta Q_h^{(v)} + \Delta Q_h^{(vi)}) > 0$. Therefore, interest rates in the low-risk market increase and aggregate loan supply in the low-risk market decreases when the leverage ratio is tightened. □

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